ESE 531: Digital Signal Processing

Week 5

Lecture 9: February 14th, 2021

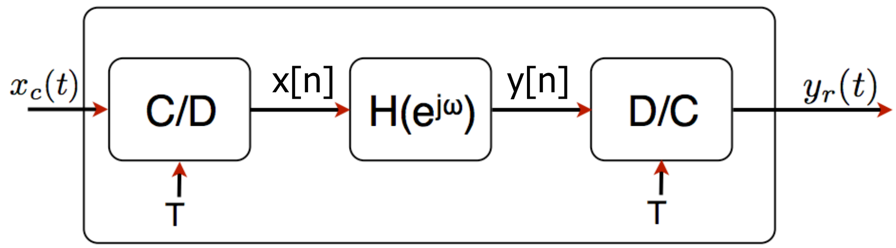
DT/CT Processing of CT/DT Signals

Lecture Outline

- DT processing of CT signals
  - Impulse Invariance
- CT processing of DT signals (why??)

Signal Processing

- Use theory of sampling (C/D) and reconstruction (D/C) to implement signal processing systems
- Two cases:
  - Discrete-time processing of continuous-time signals



Signal Processing

- Use theory of sampling (C/D) and reconstruction (D/C) to implement signal processing
- Two cases:
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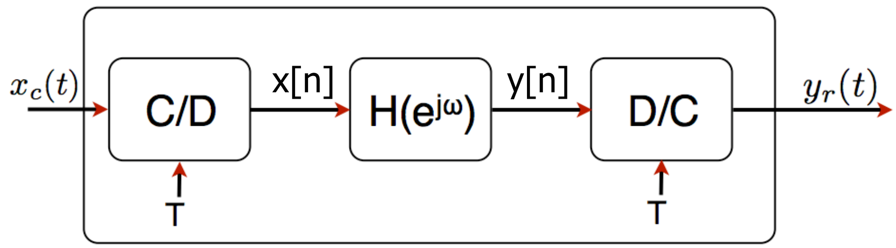
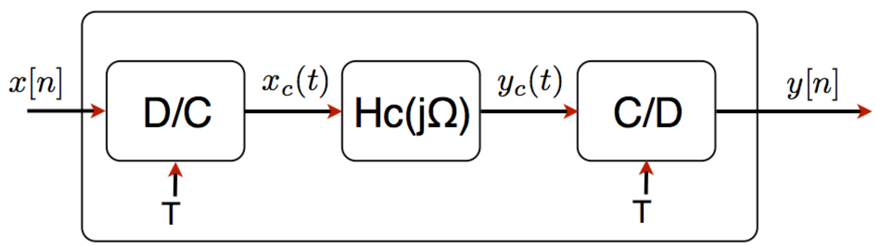
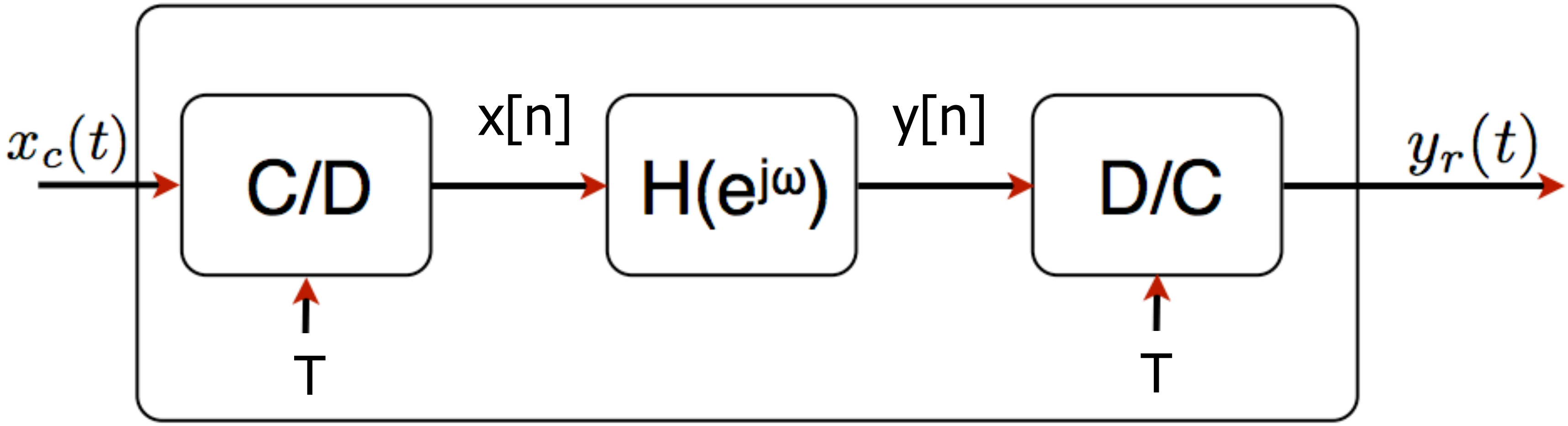


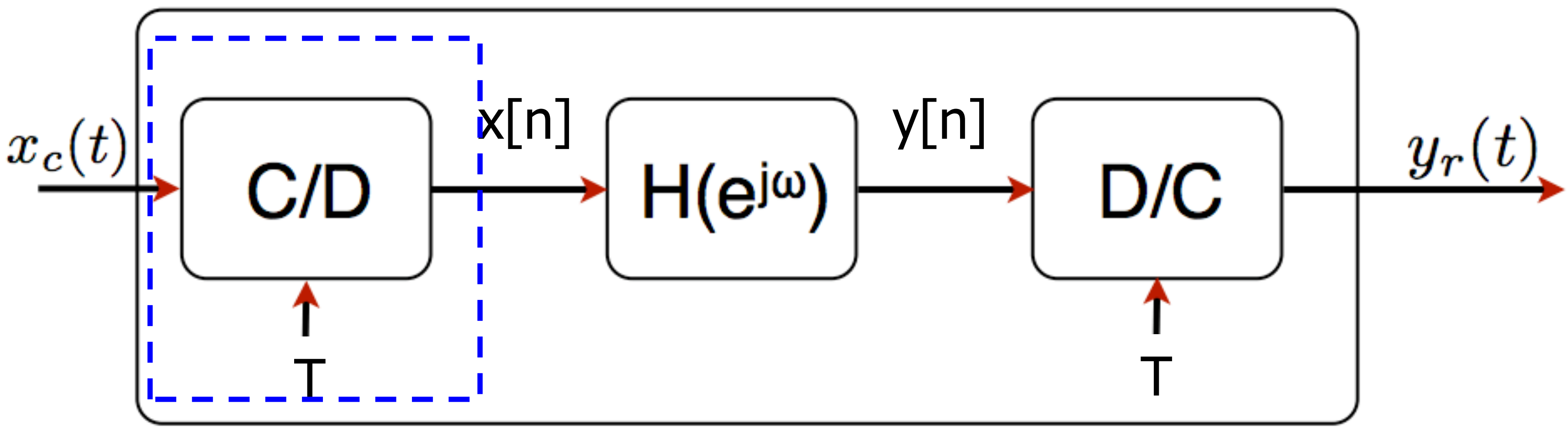
  - Continuous-time processing of discrete-time signals



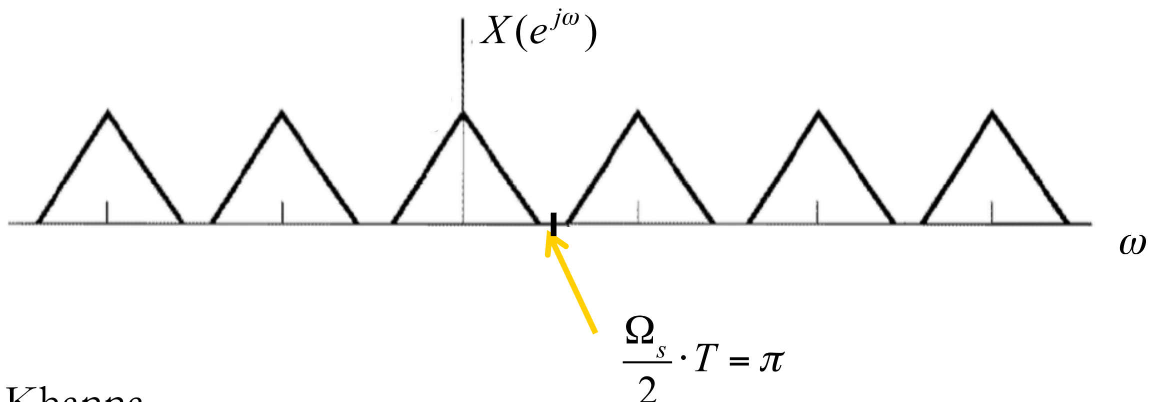
Discrete-Time Processing of Continuous Time



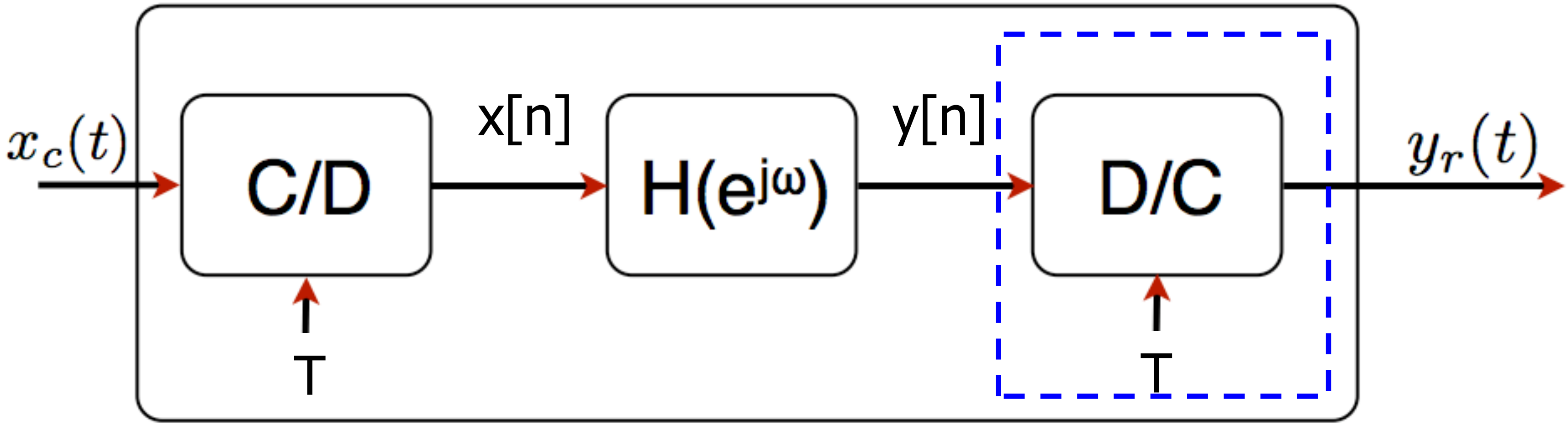
Discrete-Time Processing of Continuous Time

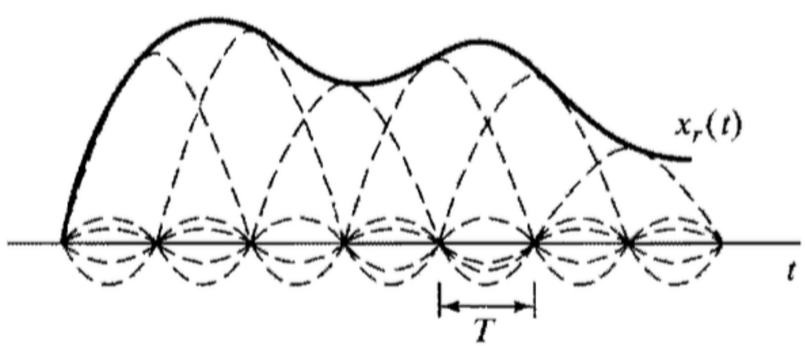


$$X(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left[j\left(\frac{\omega}{T} - \frac{2\pi k}{T}\right)\right]$$



Discrete-Time Processing of Continuous Time

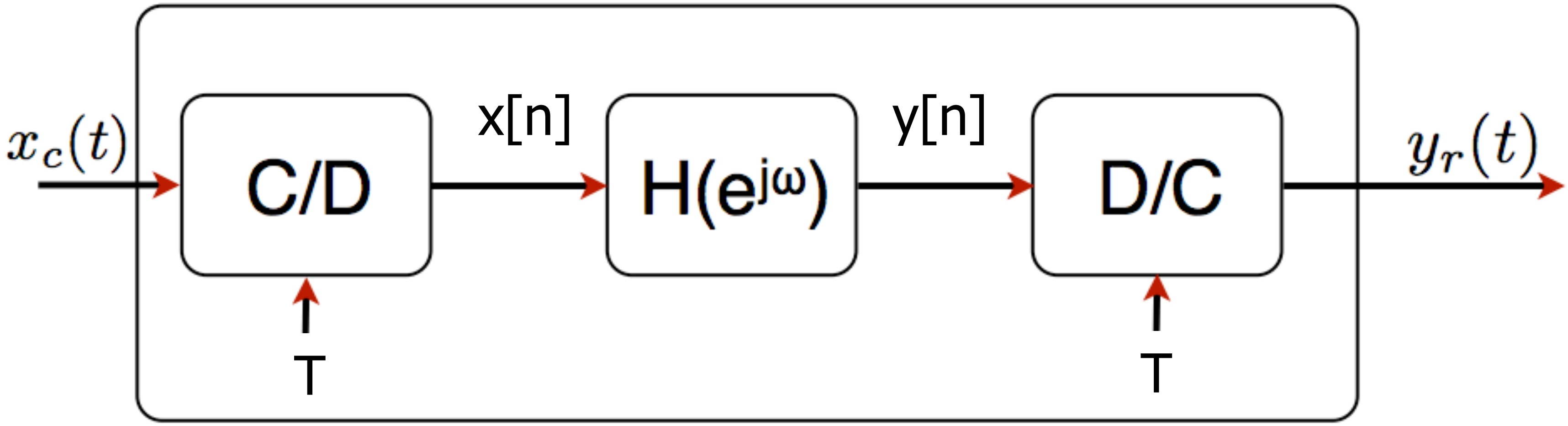




Sum of scaled shifted sincs

$$y_r(t) = \sum_{n=-\infty}^{\infty} y[n] \frac{\sin[\pi(t-nT)/T]}{\pi(t-nT)/T}$$

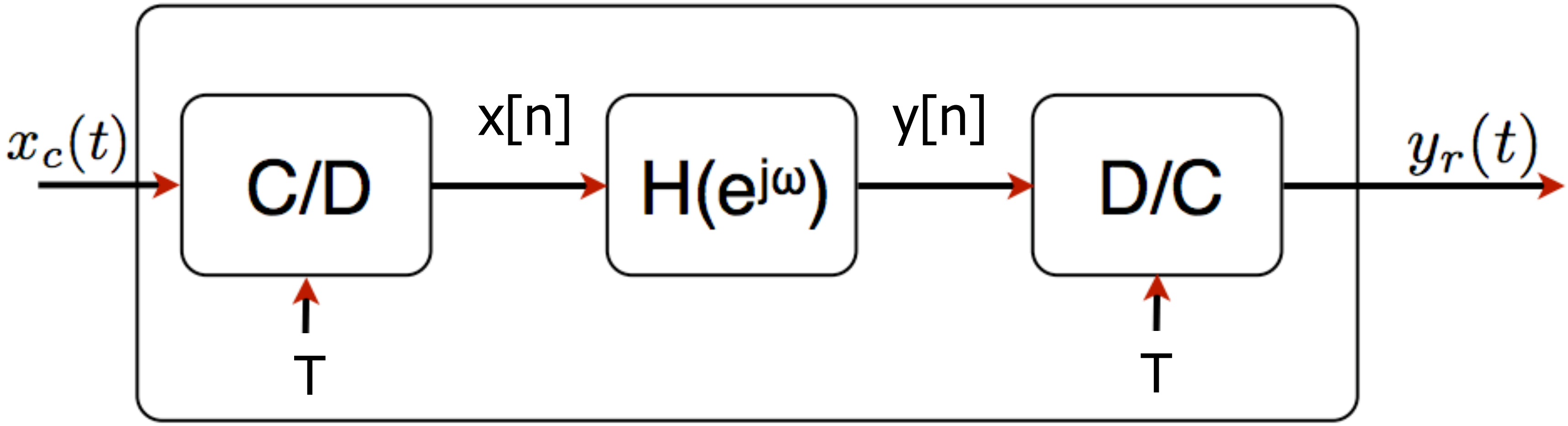
Discrete-Time Processing of Continuous Time



$$X(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left[j\left(\frac{\omega}{T} - \frac{2\pi k}{T}\right)\right] \qquad y_r(t) = \sum_{n=-\infty}^{\infty} y[n]\frac{\sin[\pi(t-nT)/T]}{\pi(t-nT)/T}$$

- If h[n]/H($e^{j\omega}$) is LTI
  - Is the whole system from $x_c(t) \rightarrow y_r(t)$ LTI?

Discrete-Time Processing of Continuous Time



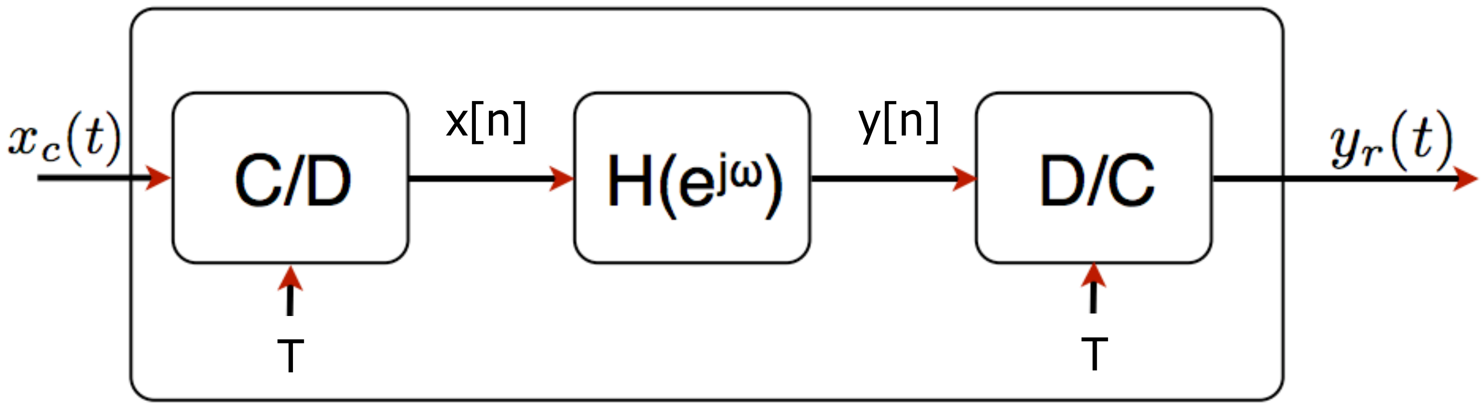
$$X(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left[j\left(\frac{\omega}{T} - \frac{2\pi k}{T}\right)\right] \qquad y_r(t) = \sum_{n=-\infty}^{\infty} y[n]\frac{\sin[\pi(t-nT)/T]}{\pi(t-nT)/T}$$

- If $x_c(t)$ is bandlimited by $\Omega_s/T=\pi/T$, then,

$$\frac{Y_r(j\Omega)}{X_c(j\Omega)} = H_{eff}(j\Omega) = \begin{cases} H(e^{j\omega})\Big|_{\omega=\Omega T} & |\Omega| < \Omega_s / T \\ 0 & else \end{cases}$$

Example 1

- Consider the following system



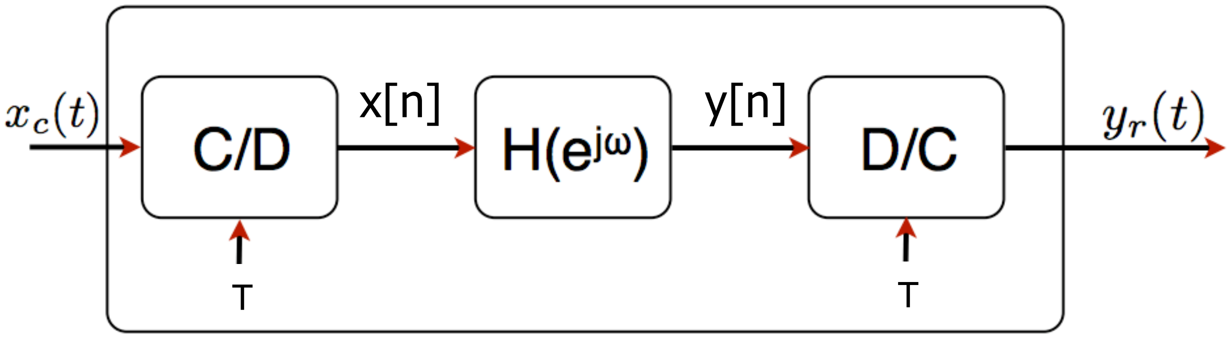
- Where

$$H(e^{j\omega}) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & \omega_c < |\omega| \le \pi \end{cases}$$

- What is the effective frequency response of the system? What happens to a signal bandlimited by Ω_N?

Example 2

- DT implementation of an ideal CT bandlimited differentiator



- The ideal CT differentiator is defined by

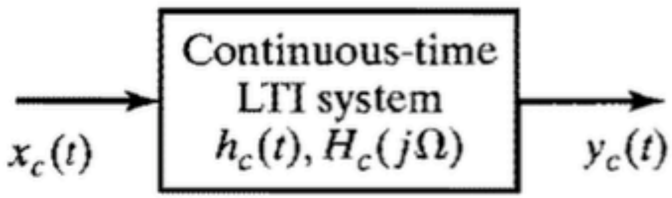
$$y_C(t) = \frac{d}{dt}[x_C(t)]$$

- With corresponding

$$H_C(j\Omega) = j\Omega$$

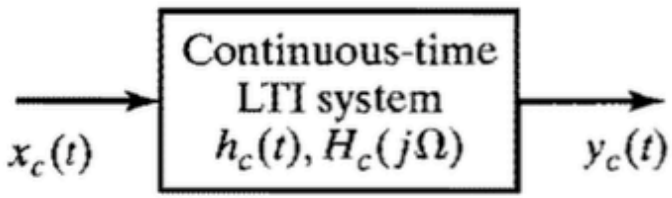
Impulse Invariance

- Want to implement continuous-time system…

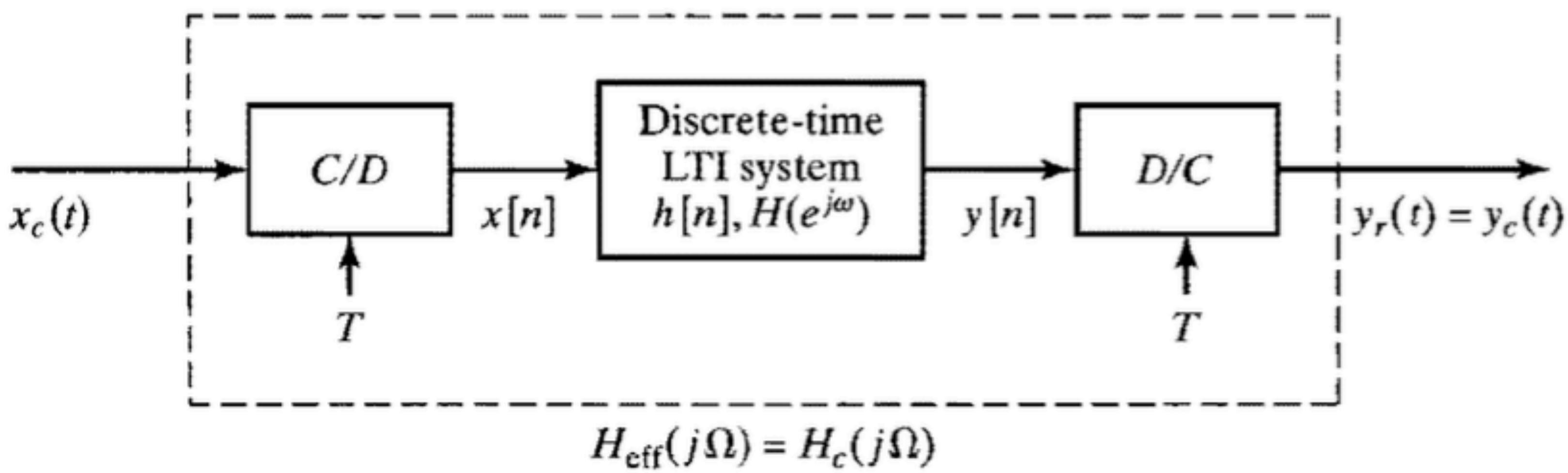


Impulse Invariance

- Want to implement continuous-time system…



- …in discrete-time



Impulse Invariance

- With $H_c(j\Omega)$ bandlimited, choose

$$H(e^{j\omega}) = H_c(j\Omega)\Big|_{\Omega=\frac{\omega}{T}}, \qquad |\omega| < \pi$$

- With the further requirement that T be chosen such that

$$H_c(j\Omega) = 0, \qquad |\Omega| \geq \pi / T$$

Impulse Invariance

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- With the further requirement that T be chosen such that

$$H_c(j\Omega) = 0, \qquad |\Omega| \geq \pi / T$$

$$\boxed{h[n] = T h_c(nT)}$$

Impulse Invariance

- Let,

$$h[n] = h_c(nT)$$

Impulse Invariance

- Let,

$$h[n] = h_c(nT)$$

- If sampling at Nyquist Rate then

$$H(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} H_c\left[j\left(\frac{\omega}{T} - \frac{2\pi k}{T}\right)\right]$$

Impulse Invariance

- Let,

$$H_c(j\Omega) = 0, \quad |\Omega| \geq \pi / T$$

$$h[n] = h_c(nT)$$

- If sampling at Nyquist Rate then

$$H(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} H_c\left[j\left(\frac{\omega}{T} - \frac{2\pi k}{T}\right)\right]$$

$$H(e^{j\omega}) = \frac{1}{T} H_c\left(j\frac{\omega}{T}\right), \qquad |\omega| < \pi$$

Impulse Invariance

- Let,

$$H_c(j\Omega)=0, \qquad |\Omega| \geq \pi/T$$

$$h[n]=T\,h_c(nT)$$

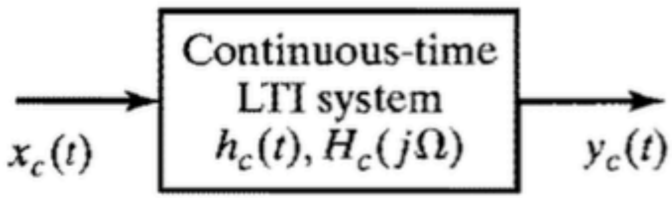
- If sampling at Nyquist Rate then

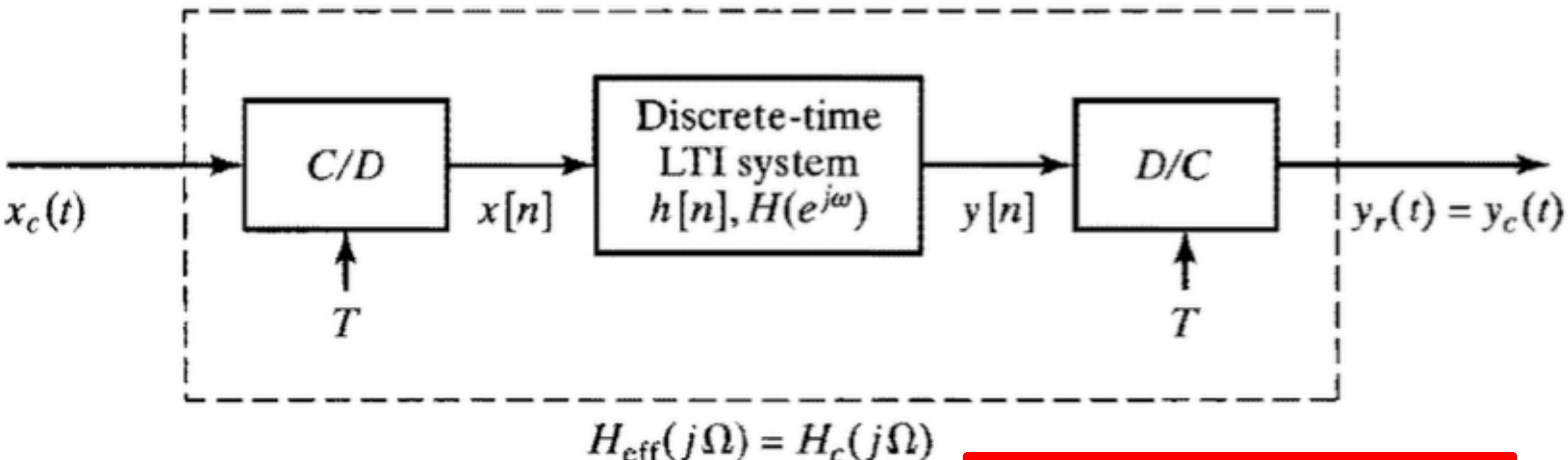
$$H(e^{j\omega})=\frac{T}{T}\sum_{k=-\infty}^{\infty}H_c\left[j\left(\frac{\omega}{T}-\frac{2\pi k}{T}\right)\right]$$

$$H(e^{j\omega})=\frac{T}{T}H_c\left(j\frac{\omega}{T}\right), \qquad |\omega|<\pi$$

Impulse Invariance

- Want to implement continuous-time system in discrete-time

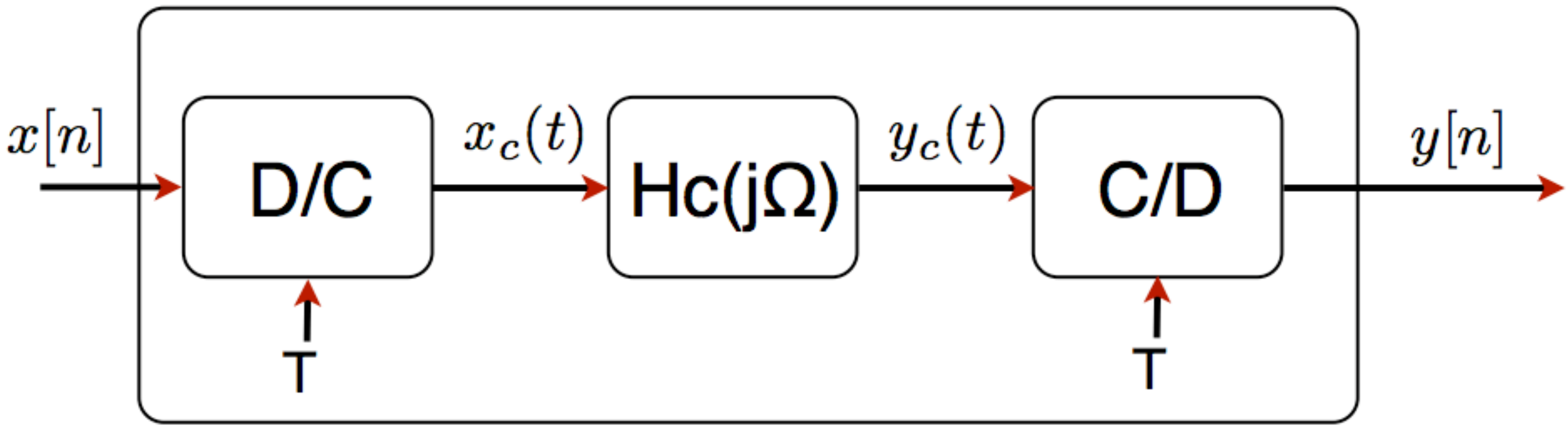




$$h[n] = Th_c(nT)$$

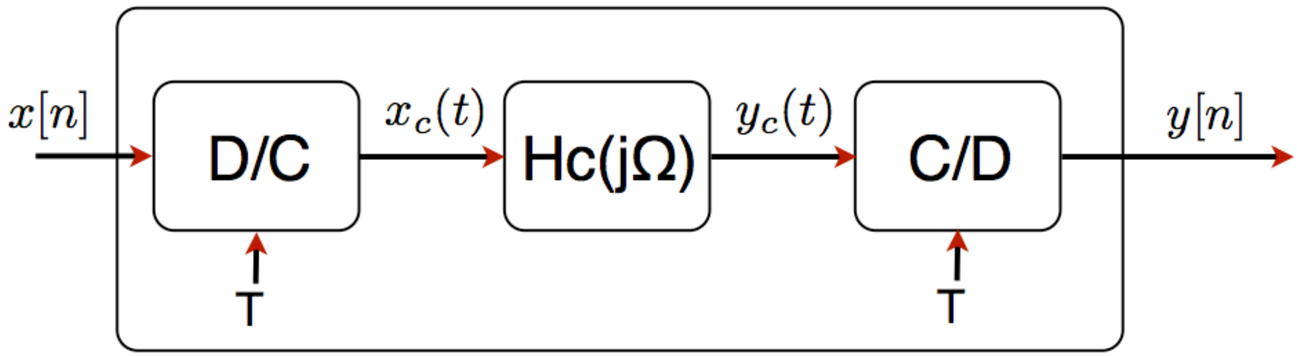
Continuous-Time Processing of Discrete-Time

- Useful to interpret DT systems with no simple interpretation in discrete time



Is the effective $H(e^{j\omega})$ LTI?

Continuous-Time Processing of Discrete-Time

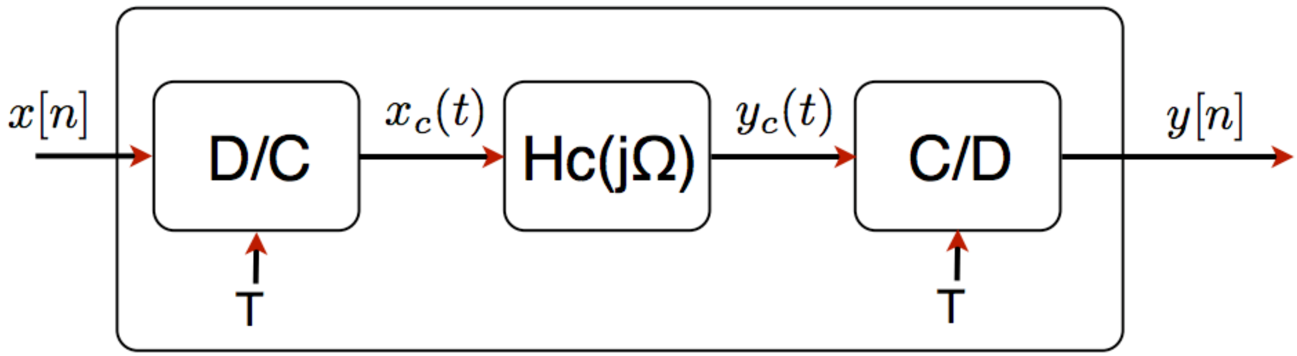


$$X_c(j\Omega) = \begin{cases} TX(e^{j\Omega T}) & |\Omega| < \pi / T \\ 0 & else \end{cases}$$

Also bandlimited

$$Y_c(j\Omega) = H_c(j\Omega) X_c(j\Omega)$$

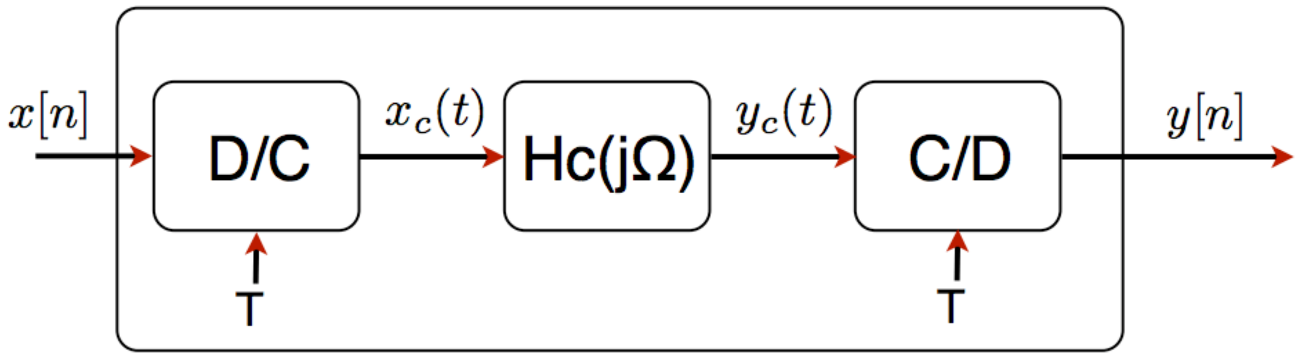
Continuous-Time Processing of Discrete-Time



$$Y_c(j\Omega) = H_c(j\Omega)X_c(j\Omega)$$

$$Y(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} Y_c\left[j\left(\Omega - k\Omega_s\right)\right]\Big|_{\Omega=\omega/T} = \frac{1}{T}Y_c(j\Omega)\Big|_{\Omega=\omega/T}$$

Continuous-Time Processing of Discrete-Time



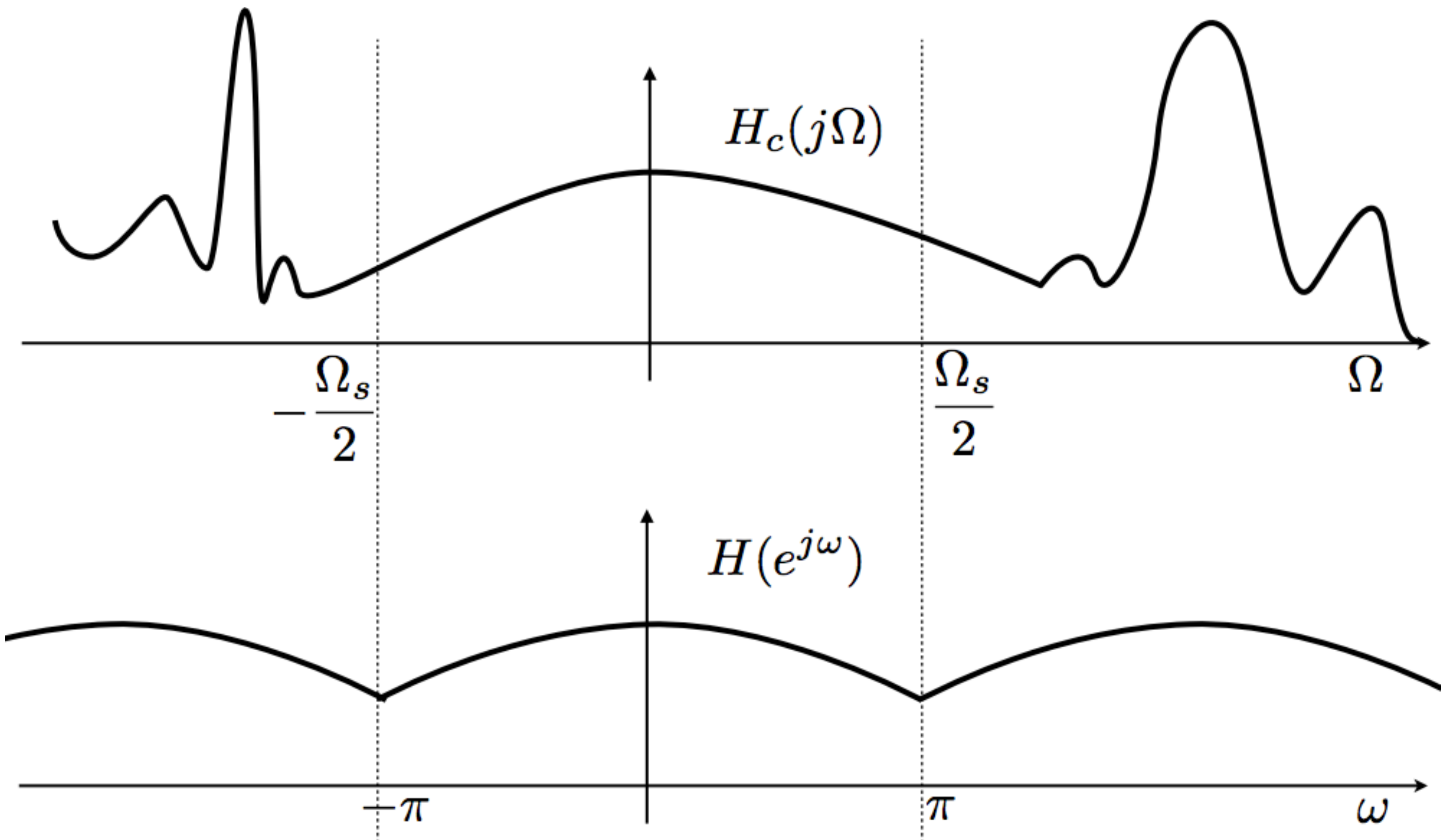
$$Y_c(j\Omega) = H_c(j\Omega)X_c(j\Omega) \qquad Y(e^{j\omega}) = \frac{1}{T}Y_c(j\Omega)\Big|_{\Omega=\omega/T}$$

$$Y(e^{j\omega}) = \frac{1}{T}H_c(j\Omega)\Big|_{\Omega=\omega/T} X_c(j\Omega)\Big|_{\Omega=\omega/T}$$

$$= \frac{1}{T}H_c(j\Omega)\Big|_{\Omega=\omega/T}(TX(e^{j\omega})) \qquad |\omega| < \pi$$

$$H(e^{j\omega})$$

Example



$H_c(j\Omega)$
$-\frac{\Omega_s}{2}$
$\frac{\Omega_s}{2}$
Ω
$H(e^{j\omega})$
$-\pi$
π
ω

Example: Non-integer Delay

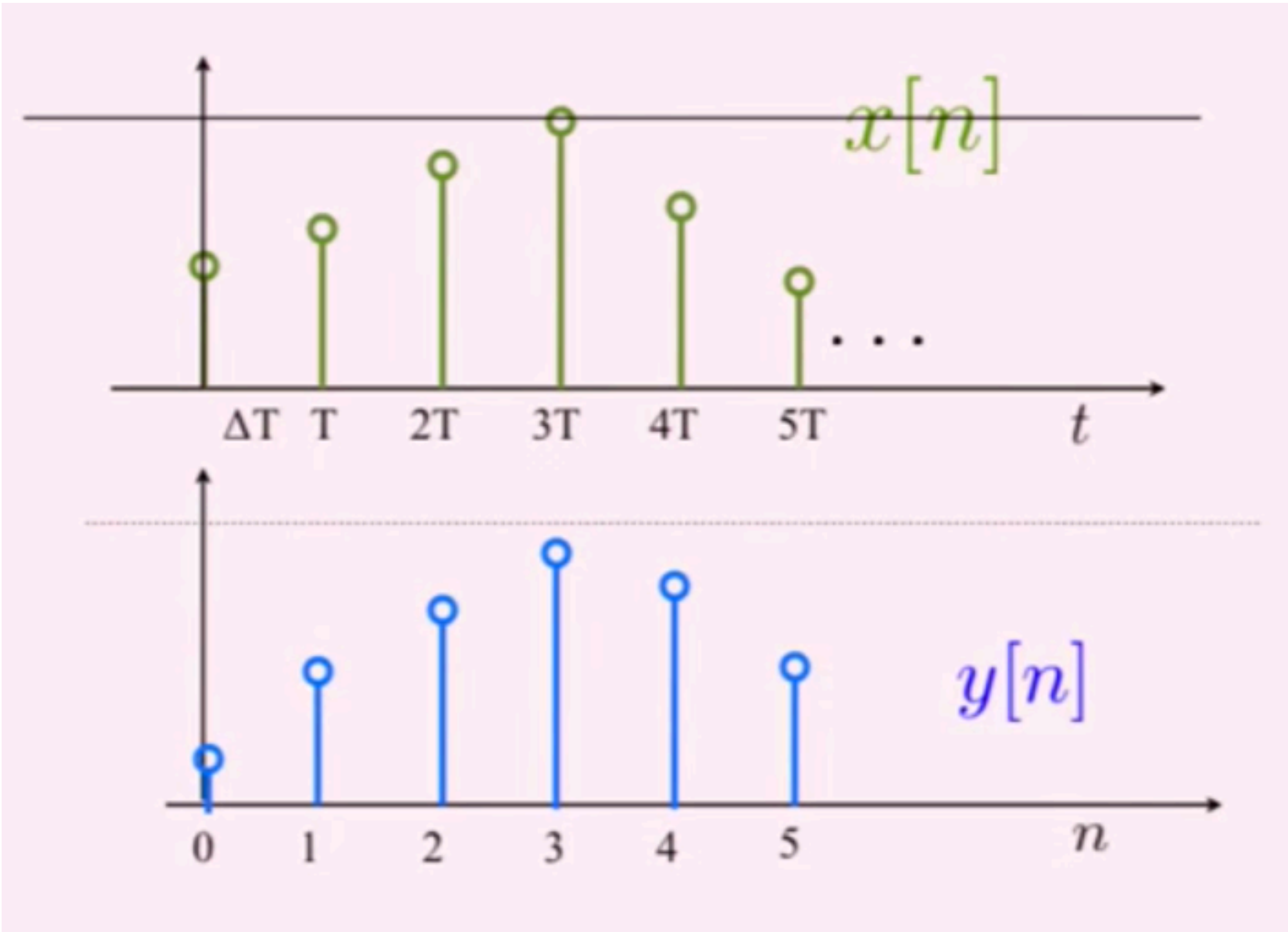
- What is the time domain operation when Δ is non-integer? I.e Δ=1/2

$$H(e^{j\omega}) = e^{-j\omega\Delta}$$

$$\delta[n] \leftrightarrow 1$$

$$\delta[n-n_d] \leftrightarrow e^{-j\omega n_d}$$

Example: Non-integer Delay



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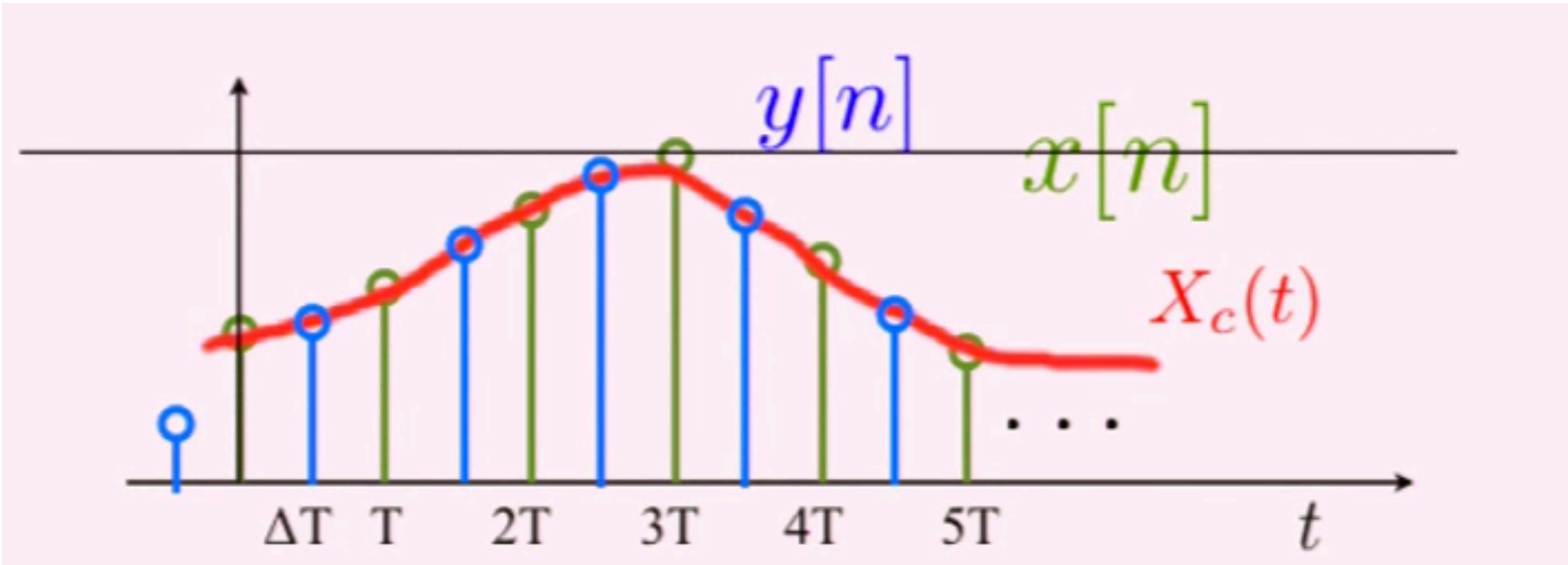

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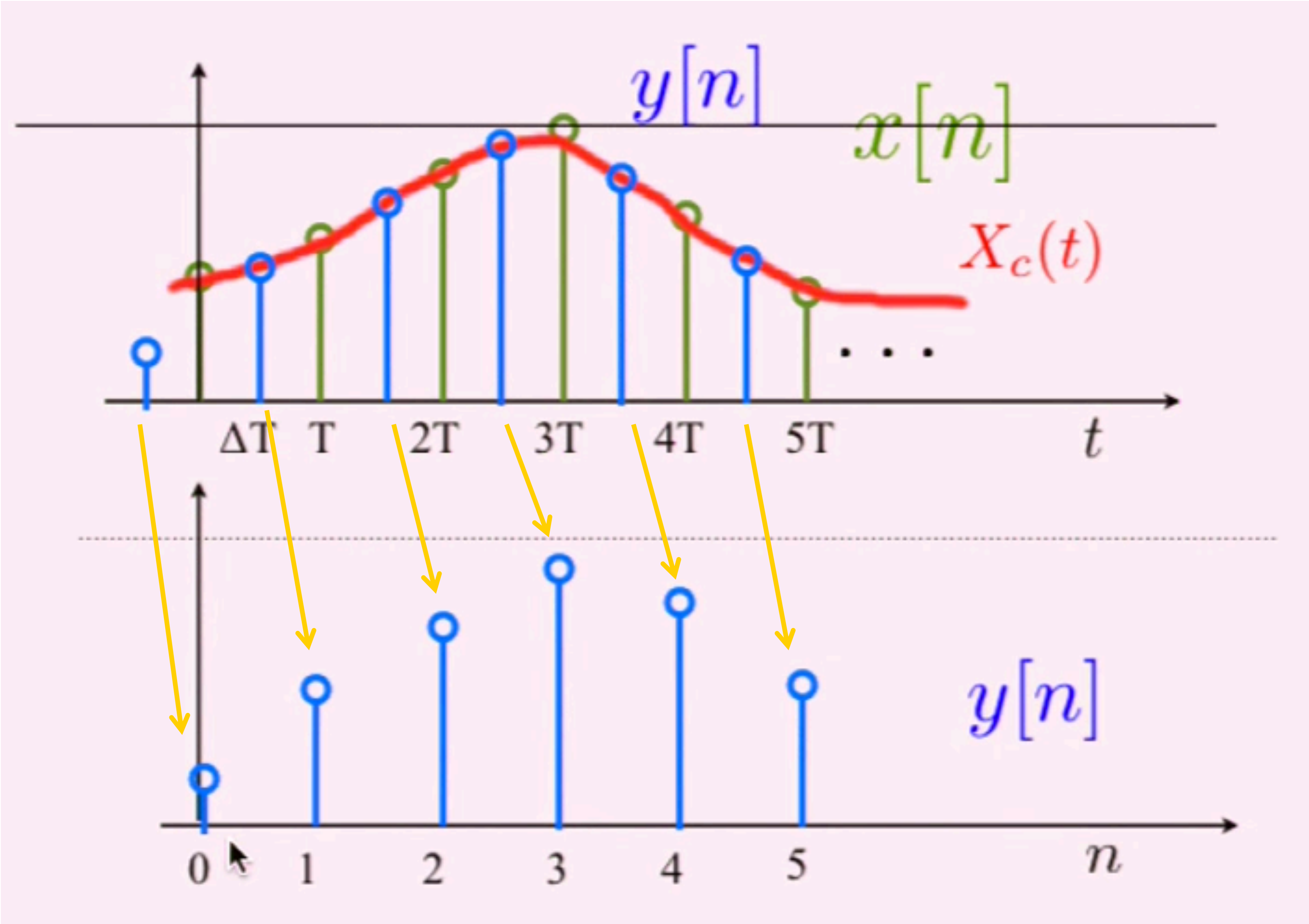

Example: Non-integer Delay

- What is the time domain operation when Δ is non-integer? I.e Δ=1/2

$$H(e^{j\omega}) = e^{-j\omega\Delta}$$

Let: $H_c(j\Omega) = e^{-j\Omega\Delta T}$ <u>delay of ΔT in continuous time</u>

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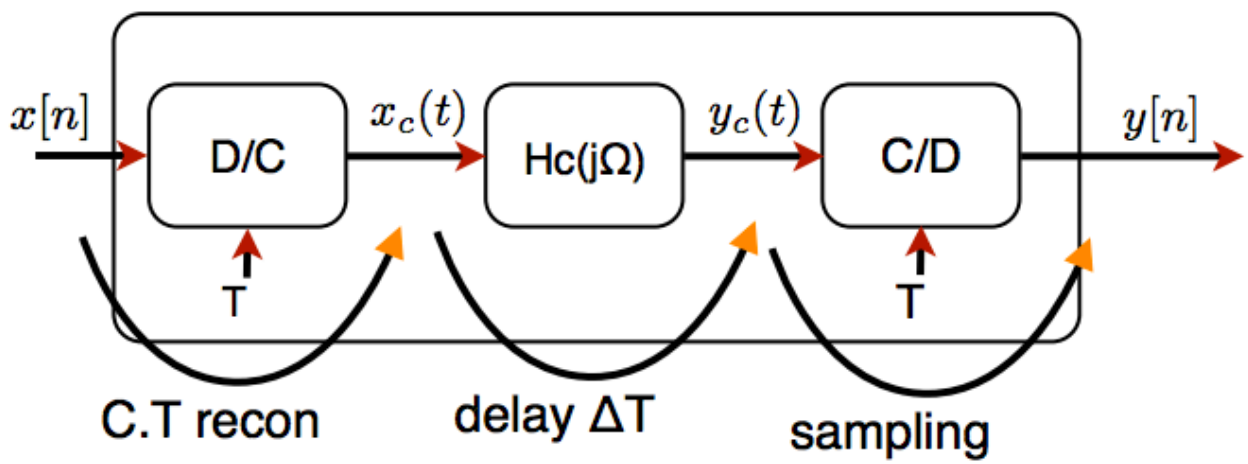
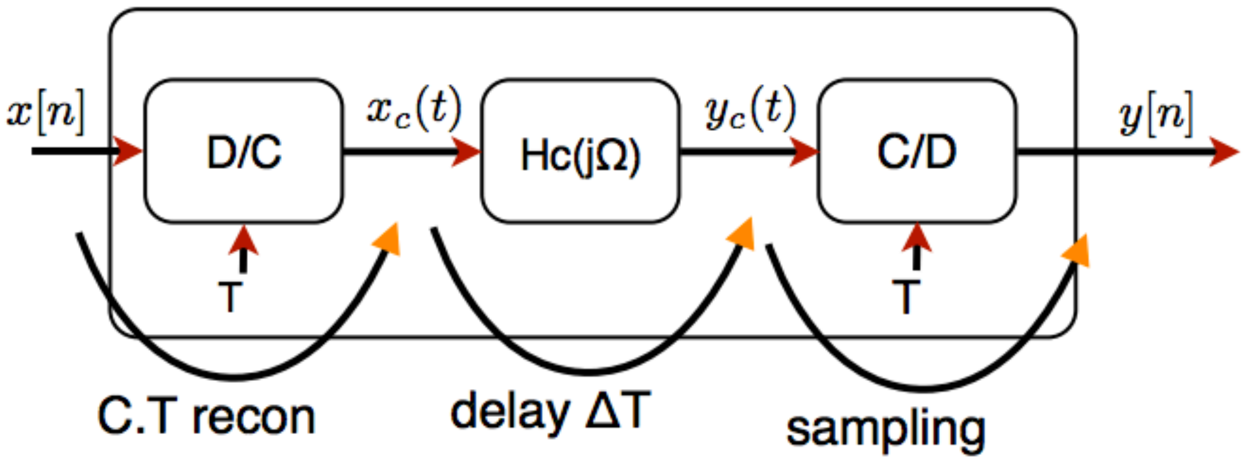


Example: Non-integer Delay

- The block diagram is for interpretation/analysis only



$$x_c(t) = \sum_k x[k]\operatorname{sinc}\left(\frac{t-kT}{T}\right)$$

Example: Non-integer Delay

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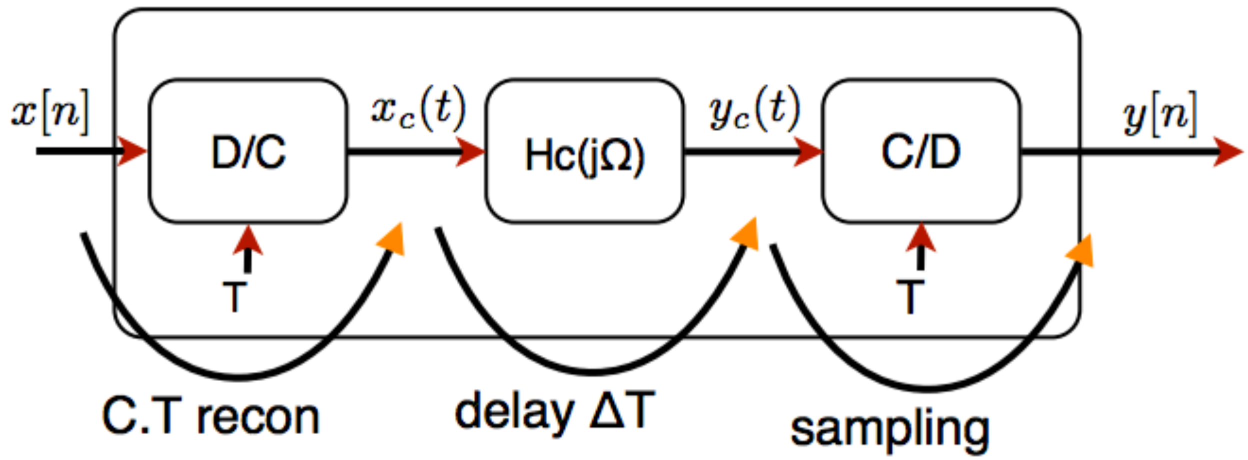

$$x_c(t) = \sum_k x[k]\,\text{sinc}\left(\frac{t - kT}{T}\right)$$

$$y_c(t) = x_c(t - T\Delta)$$

Example: Non-integer Delay

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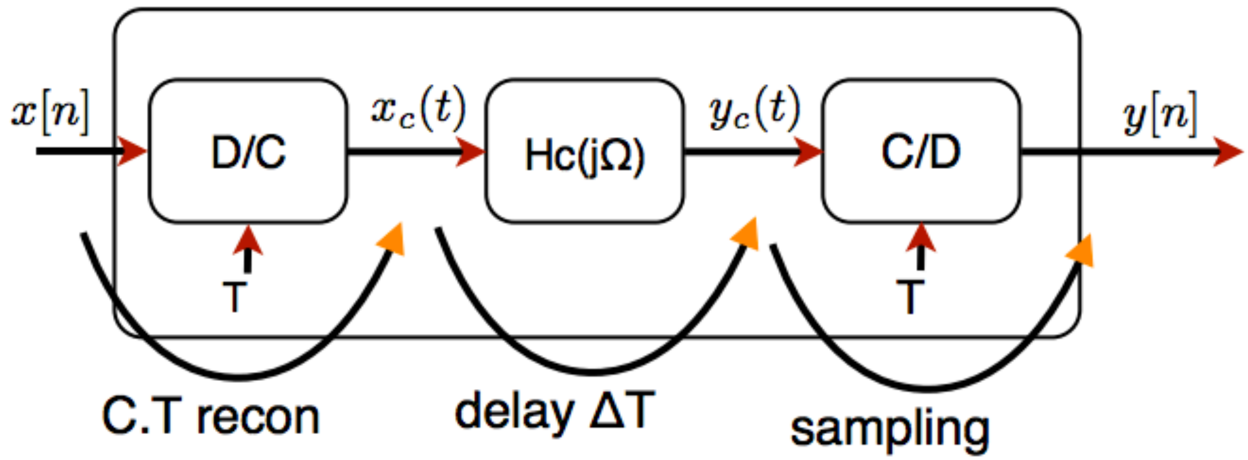


$$x_c(t) = \sum_k x[k] \operatorname{sinc}\left(\frac{t - kT}{T}\right) \qquad y[n] = y_c(nT)$$

$$y_c(t) = x_c(t - T\Delta)$$

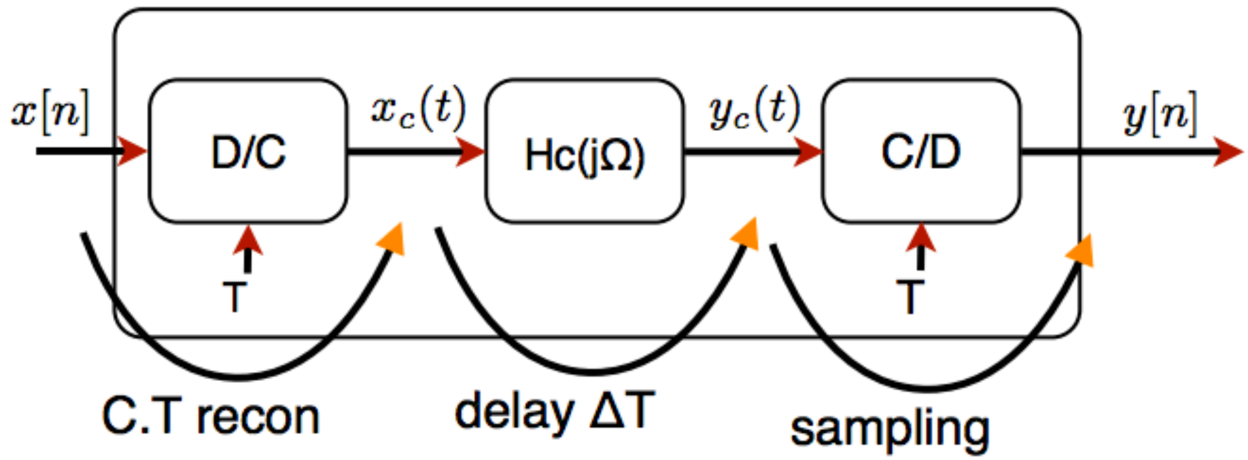
Example: Non-integer Delay

- The block diagram is for interpretation/analysis only

$$x_c(t) = \sum_k x[k] \operatorname{sinc}\left(\frac{t-kT}{T}\right) \Rightarrow y_c(t) = x_c(t - T\Delta) \Rightarrow y[n] = y_c(nT)$$

Example: Non-integer Delay

- The block diagram is for interpretation/analysis only



$$y[n] = y_c(nT) = x_c(nT - T\cdot\Delta) \qquad x_c(t) = \sum_k x[k]\operatorname{sinc}\left(\frac{t-kT}{T}\right)$$

$$x_c(nT - T\cdot\Delta) = \sum_k x[k]\operatorname{sinc}\left(\frac{nT - T\cdot\Delta - kT}{T}\right) = \sum_k x[k]\operatorname{sinc}(n - \Delta - k)$$

Example: Non-integer Delay

- Delay system has an impulse response of a sinc with a continuous time delay

$$y[n] = \sum_k x[k]\operatorname{sinc}\left(n - \Delta - k\right)$$

$$= x[n] * \operatorname{sinc}\left(n - \Delta\right)$$

$$\Rightarrow h[n] = \operatorname{sinc}(n - \Delta)$$

Example: Non-integer Delay

- What is the time domain operation when Δ is non-integer? I.e Δ=1/2

$$H(e^{j\omega}) = e^{-j\omega\Delta}$$

$$\delta[n] \leftrightarrow 1$$

$$\delta[n-n_d] \leftrightarrow e^{-j\omega n_d}$$

$$\Rightarrow h[n] = \text{sinc}(n-\Delta)$$

Example: Non-integer Delay

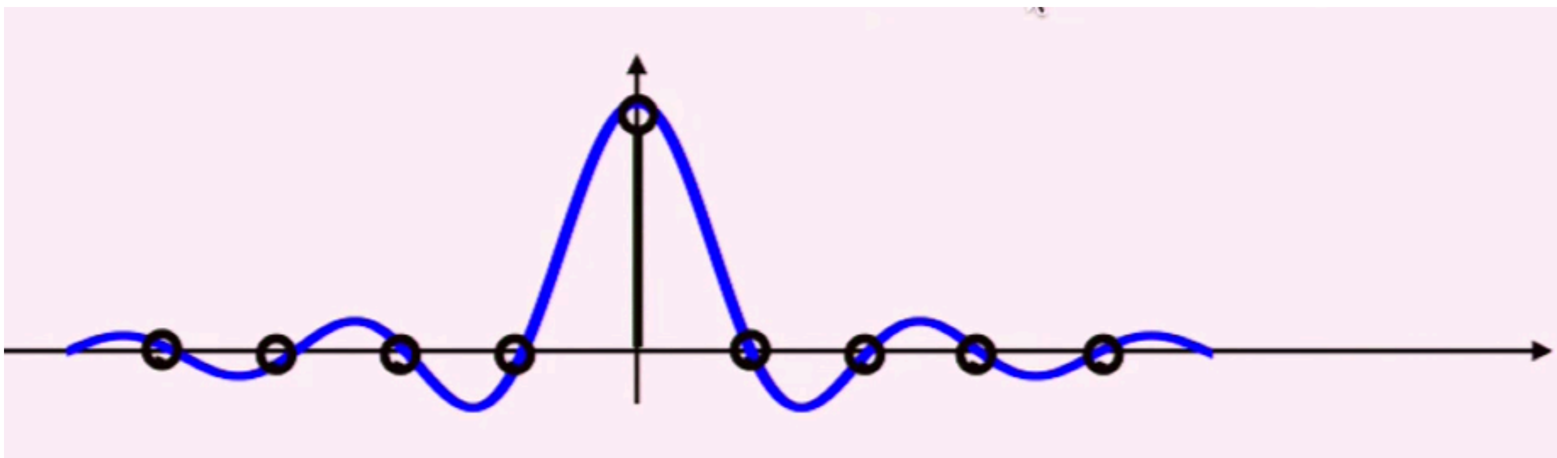
- My non-integer delay system has an impulse response of a sinc with a continuous time delay

$$H(e^{j\omega}) = e^{-j\omega\Delta} \longleftrightarrow h[n] = \text{sinc}(n - \Delta)$$

Example: Non-integer Delay

- My non-integer delay system has an impulse response of a sinc with a continuous time delay

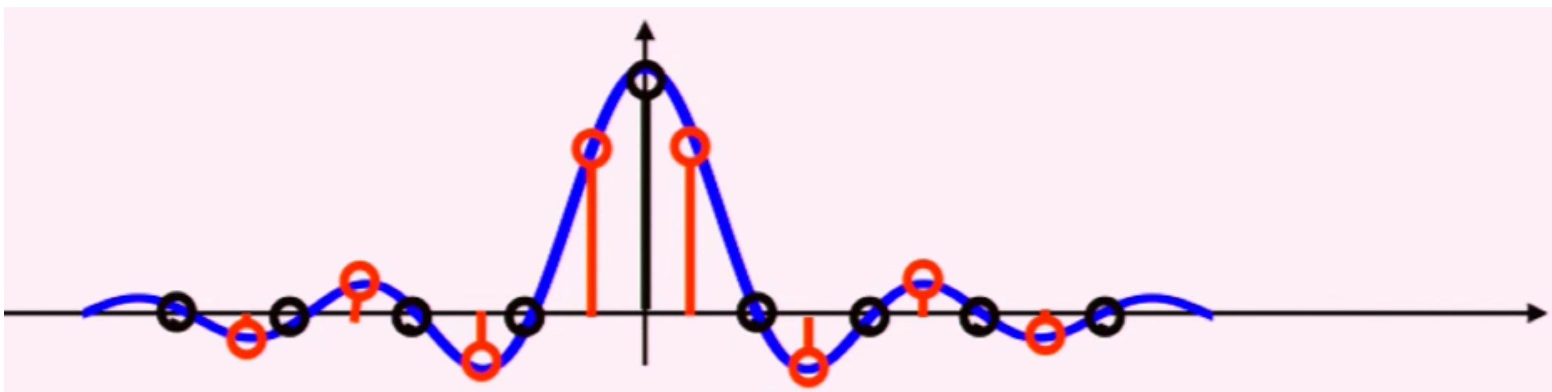
$$H(e^{j\omega}) = e^{-j\omega\Delta} \longleftrightarrow h[n] = \mathrm{sinc}(n - \Delta)$$



Example: Non-integer Delay

- My non-integer delay system has an impulse response of a sinc with a continuous time delay

$$H(e^{j\omega}) = e^{-j\omega\Delta} \longleftrightarrow h[n] = \mathrm{sinc}(n - \Delta)$$



Big Ideas

- DT processing of CT
  - Effectively LTI if no aliasing
- CT processing of DT
  - Always LTI
  - Useful for interpretation

Admin

- HW 3 due Monday