ⓘ Students have either already taken or started taking this quiz, so be careful about editing it. If you change any quiz questions in a significant way, you may want to consider regrading students who took the old version of the quiz.

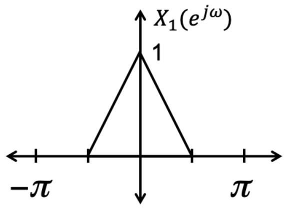
Points 100 Published

Details | Questions

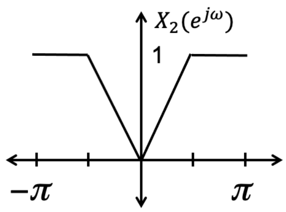
Q1 Pick 1 questions, 15 pts per question

Q1

Suppose that you know $x_1[n]$, the sequence whose transform is given below as $X_1\left(e^{j\omega}\right)$.



Given the DTFT below, $X_2\left(e^{j\omega}\right)$, find $x_2[n]$ in terms of $x_1[n]$. You should not have to compute explicitly any transforms or inverse transforms.

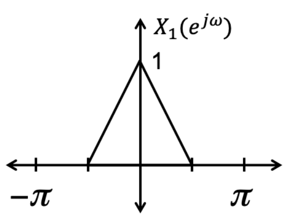


SOLUTION:

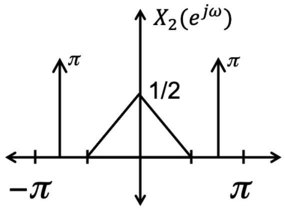
$$X_2(e^{j\omega}) = 1 - X_1(e^{j\omega})$$
$$x_2[n] = \delta[n] - x[n]$$

Q1

Suppose that you know $x_1[n]$, the sequence whose transform is given below as $X_1\left(e^{j\omega}\right)$.



Given the DTFT below, $X_2\left(e^{j\omega}\right)$, find $x_2[n]$ in terms of $x_1[n]$. You should not have to compute explicitly any transforms or inverse transforms.

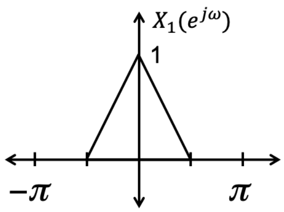


SOLUTION:

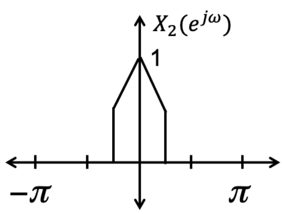
$$X_2(e^{j\omega}) = \frac{1}{2}X_1(e^{j\omega}) + \pi\delta(\omega - \frac{3\pi}{4}) + \pi\delta(\omega + \frac{3\pi}{4})$$
$$x_2[n] = \frac{1}{2}x_1[n] + cos(\frac{3\pi}{4}n)$$

Q1

Suppose that you know $x_1[n]$, the sequence whose transform is given below as $X_1\left(e^{j\omega}\right)$.



Given the DTFT below, $X_2\left(e^{j\omega}\right)$, find $x_2[n]$ in terms of $x_1[n]$. You should not have to compute explicitly any transforms or inverse transforms.



SOLUTION:

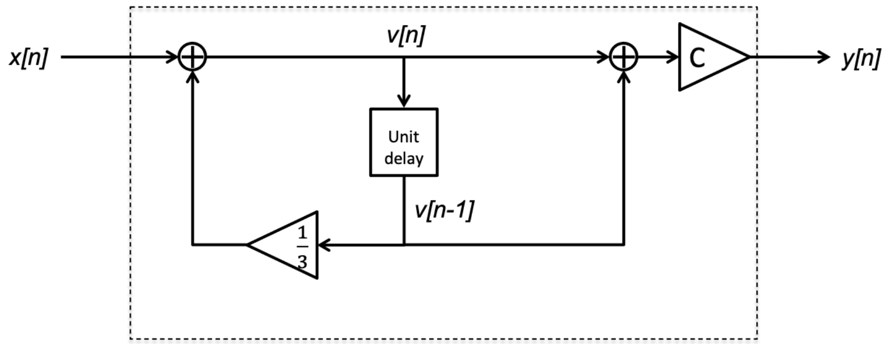
$X_2(e^{j\omega}) = X_1(e^{j\omega})\cdot$ ideal LPF with $\omega_c = \pi/4$

$$x_2[n] = x_1[n] * \frac{sin(\frac{\pi}{4}n)}{\pi n}$$

Q2 Pick 1 questions, 35 pts per question

Q2

A causal stable discrete-time linear time-invariant filter with input $x[n]$ and output $y[n]$ is governed by the following block diagram. The triangle block is an amplifier and just scales the input by the scale factor given in the triangle. The unit delay block delays the input by one sample. The summer output signal is the sum of all signal inputs where arrows indicate what are inputs and what are outputs.



From the diagram answer the following questions. Note that it might be easier to do these in a different order than listed below. You can do them in any order you decide, but make sure to label your answers on your work with the relevant letter in the list below.

a) Find the difference equation relating the input $x[n]$ and output $y[n]$. Your answer should not include $v[n]$.

b) Find the z-transform $H(z)$ describing the filter including the region of convergence

c) Draw the corresponding pole-zero diagram for $H(z)$.

d) Does the frequency response of the filter, $H\left(e^{j\omega}\right)$ exist? If so, what is it?

e) What is the best description of the frequency selectivity of this filter: lowpass, highpass, bandstop or bandpass? Explain your reasoning.

f) Find a numeric value of **C** that normalizes the magnitude response. (I.e. ensures that the maximum magnitude for all frequencies is unity.)

SOLTUION:

a)

Working backwards from part b:

$$y[n] - \frac{1}{3}y[n-1] = C(x[n] + x[n-1])$$

b)

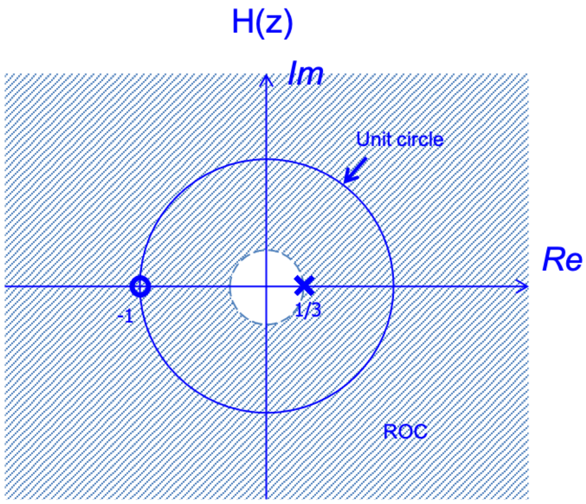
$$v[n] = x[n] + \frac{1}{3}v[n-1]$$
$$\frac{V(z)}{X(z)} = \frac{1}{1-\frac{1}{3}z^{-1}}$$
$$y[n] = C(v[n] + v[n-1])$$
$$\frac{Y(z)}{V(z)} = C(1+z^{-1})$$

Multiplying the two equations above:

$$H(z) = \frac{Y(z)}{X(z)} = C\frac{1+z^{-1}}{1-\frac{1}{3}z^{-1}}$$

We have a pole at $z = \frac{1}{3}$ and a zero at $z = -1$. Since this is causal, $ROC: |z| > \frac{1}{3}$.

c)



d)

Yes, the ROC includes the unit circle.

$$H(e^{j\omega}) = H(z)|_{z=e^{j\omega}} = C\frac{1+e^{-j\omega}}{1-\frac{1}{3}e^{-j\omega}}$$

e)

Lowpass, there is a zero at $z = -1$ which corresponds to $\omega = \pi$, meaning high frequencies are zeroed out.

f)

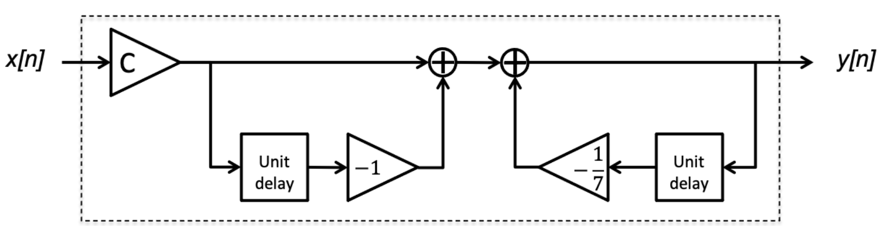
Max magnitude happens at $\omega = 0 \rightarrow z = 1$:

$$H(1) = C\frac{1+1^{-1}}{1-\frac{1}{3}1^{-1}} = 3C$$

Setting this magnitude equal to 1, gives C=1/3.

Q2

A causal stable discrete-time linear time-invariant filter with input $x[n]$ and output $y[n]$ is governed by the following block diagram. The triangle block is an amplifier and just scales the input by the scale factor given in the triangle. The unit delay block delays the input by one sample. The summer output signal is the sum of all signal inputs where arrows indicate what are inputs and what are outputs.



From the diagram answer the following questions. Note that it might be easier to do these in a different order than listed below. You can do them in any order you decide, but make sure to label your answers on your work with the relevant letter in the list below.

a) Find the difference equation relating the input $x[n]$ and output $y[n]$

b) Find the z-transform $H(z)$ describing the filter including the region of convergence

c) Draw the corresponding pole-zero diagram for $H(z)$.

d) Does the frequency response of the filter, $H\left(e^{j\omega}\right)$ exist? If so, what is it?

e) What is the best description of the frequency selectivity of this filter: lowpass, highpass, bandstop or bandpass? Explain your reasoning.

f) Find a numeric value of C that normalizes the magnitude response. (I.e. ensures that the maximum magnitude for all frequencies is unity.)

SOLUTION:

a)

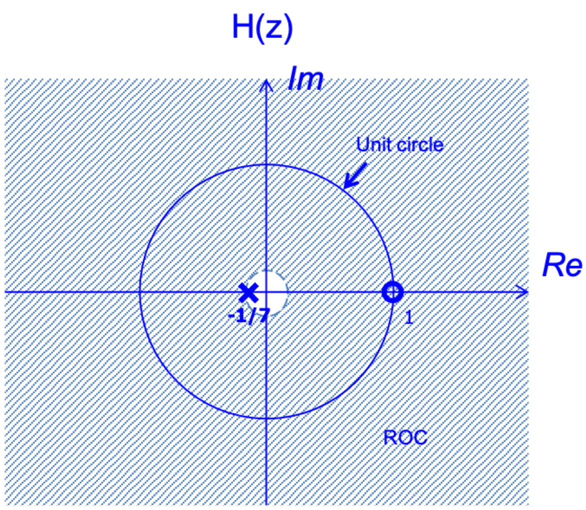
$$y[n] = -\frac{1}{7}y[n-1] + C(x[n] - x[n-1])$$

b)

$$H(z) = \frac{Y(z)}{X(z)} = C\frac{1-z^{-1}}{1+\frac{1}{7}z^{-1}}$$

We have a pole at $z = -\frac{1}{7}$ and a zero at $z = 1$. Since this is causal, $ROC : |z| > \frac{1}{7}$.

c)



d)

Yes, the ROC includes the unit circle.

$$H(e^{j\omega}) = H(z)|_{z=e^{j\omega}} = C\frac{1-e^{-j\omega}}{1+\frac{1}{7}e^{-j\omega}}$$

e)

Highpass, there is a zero at $z = 1$ which corresponds to $\omega = 0$, meaning low frequencies are zeroed out.

f)

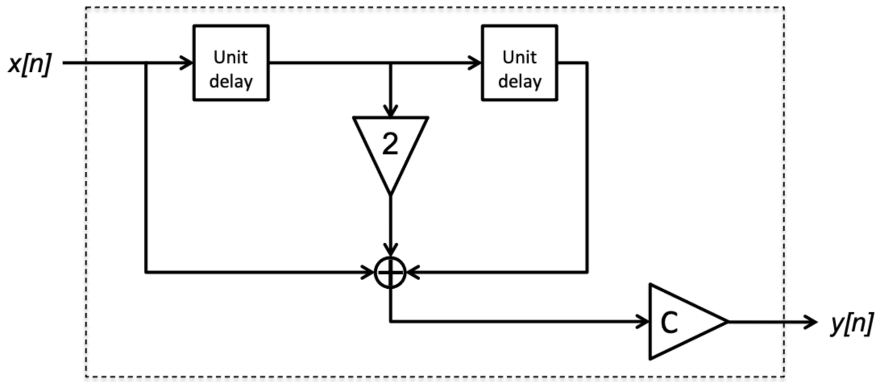
Max magnitude happens at $\omega = \pi \rightarrow z = -1$:

$$H(1) = C\frac{1+(-1)^{-1}}{1-\frac{1}{3}(-1)^{-1}} = \frac{7}{3}C$$

Setting this magnitude equal to 1, gives C=3/7.

Q2

A causal stable discrete-time linear time-invariant filter with input $x[n]$ and output $y[n]$ is governed by the following block diagram. The triangle block is an amplifier and just scales the input by the scale factor given in the triangle. The unit delay block delays the input by one sample. The summer output signal is the sum of all signal inputs where arrows indicate what are inputs and what are outputs.



From the diagram answer the following questions. Note that it might be easier to do these in a different order than listed below. You can do them in any order you decide, but make sure to label your answers on your work with the relevant letter in the list below.

a) Find the difference equation relating the input $x[n]$ and output $y[n]$

b) Find the z-transform $H(z)$ describing the filter including the region of convergence

c) Draw the corresponding pole-zero diagram for $H(z)$.

d) Does the frequency response of the filter, $H\left(e^{j\omega}\right)$ exist? If so, what is it?

e) What is the best description of the frequency selectivity of this filter: lowpass, highpass, bandstop or bandpass? Explain your reasoning.

f) Find a numeric value of $\mathbf{C}$ that normalizes the magnitude response. (I.e. ensures that the maximum magnitude for all frequencies is unity.)

SOLUTION:

a)

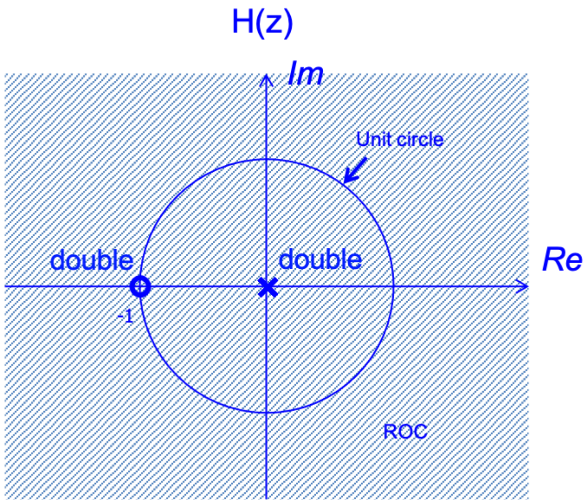
$$y[n] = C(x[n] + 2x[n-1] + x[n-2])$$

b)

$$H(z) = \frac{Y(z)}{X(z)} = C(1 + 2z^{-1} + z^{-2})$$

We have a double pole at $z = 0$ and a double zero at $z = -1$. This means the entire z-plane is the ROC and $z \neq 0$.

c)



d)

Yes, the ROC includes the unit circle.

$$H(e^{j\omega}) = H(z)|_{z=e^{j\omega}} = C(1 + 2e^{-j\omega} + e^{-j2\omega})$$

e)

Lowpass, there is a zero at $z = -1$ which corresponds to $\omega = \pi$, meaning high frequencies are zeroed out.

f)

Max magnitude happens at $\omega = 0 \rightarrow z = 1$:

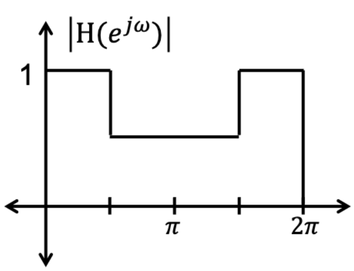
$$H(1) = C(1 + 2(1)^{-1} + (1)^{-2}) = 4C$$

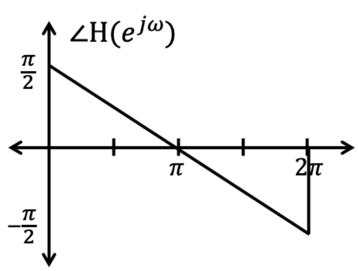
Setting this magnitude equal to 1, gives C=1/4.

Q3 Pick 1 questions, 15 pts per question

Q3

The signal $x[n] = \cos\left(\frac{\pi}{5}n\right)$ is input to a digital filter with frequency response magnitude and phase as shown:





find the output $y[n]$ of this system.

SOLUTION:

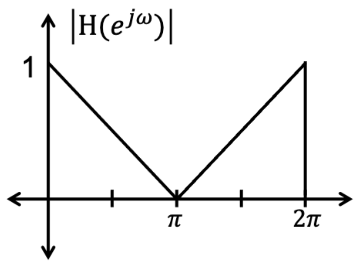
$|H(e^{j\frac{\pi}{5}})| = 1$

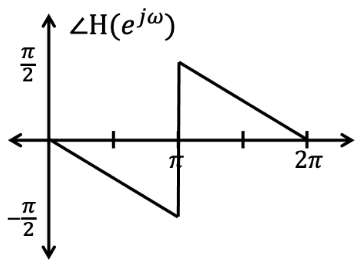
$\angle H(e^{j\frac{\pi}{5}}) = \frac{2\pi}{5}$

$y[n] = cos\left(\frac{\pi}{5}n + \frac{2\pi}{5}\right)$

Q3

The signal $x[n] = \cos\left(\frac{2\pi}{3}n\right)$ is input to a digital filter with frequency response magnitude and phase as shown:





find the output $y[n]$ of this system.

SOLUTION:

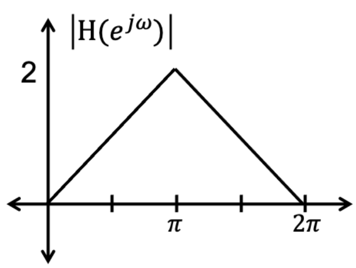
$|H(e^{j\frac{2\pi}{3}})| = \frac{1}{3}$

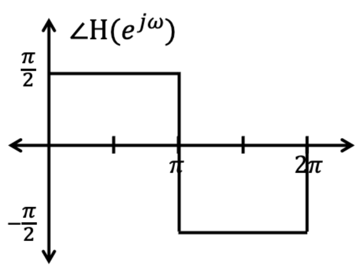
$\angle H(e^{j\frac{2\pi}{3}}) = -\frac{\pi}{3}$

$y[n] = \frac{1}{3}cos\left(\frac{2\pi}{3}n - \frac{\pi}{3}\right)$

Q3

The signal $x[n] = \cos\left(\frac{\pi}{3}n\right)$ is input to a digital filter with frequency response magnitude and phase as shown:





find the output $y[n]$ of this system.

SOLUTION:

$|H(e^{j\frac{\pi}{3}})| = \frac{2}{3}$

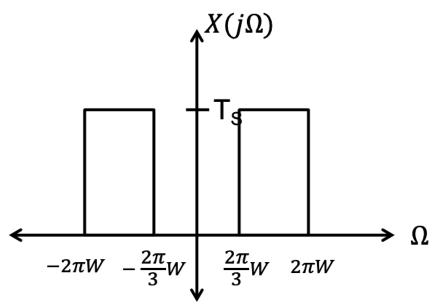
$\angle H(e^{j\frac{\pi}{3}}) = \frac{\pi}{2}$

$y[n] = \frac{2}{3}cos\left(\frac{\pi}{3}n + \frac{\pi}{2}\right)$

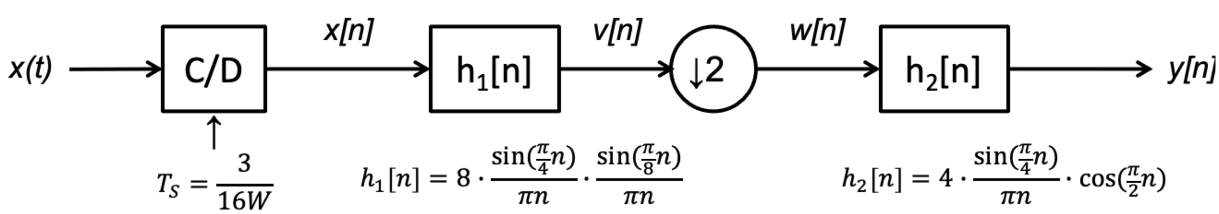
Q4 Pick 1 questions, 35 pts per question

Q4

Consider the analog input signal $x(t)$ whose Continuous Time Fourier Transform (CTFT), $X(j\Omega)$, is plotted below.



The input $x(t)$, which is both real-valued and even-symmetric, is sampled with a sampling period T_S to create the discrete-time signal $x[n] = x(nT_S)$ which is then input into a cascaded system of filters and re-sampling blocks shown below.

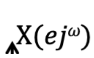


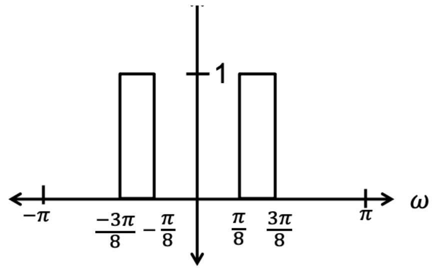
Sketch the frequency responses of all discrete time signals and filters:
$X(e^{j\omega}), H_1(e^{j\omega}), V(e^{j\omega}), W(e^{j\omega}), H_2(e^{j\omega}), Y(e^{j\omega})$

Make sure to label all axes and indicate heights and relevant frequencies. Be sure to specify a full 2π period of each frequency response.

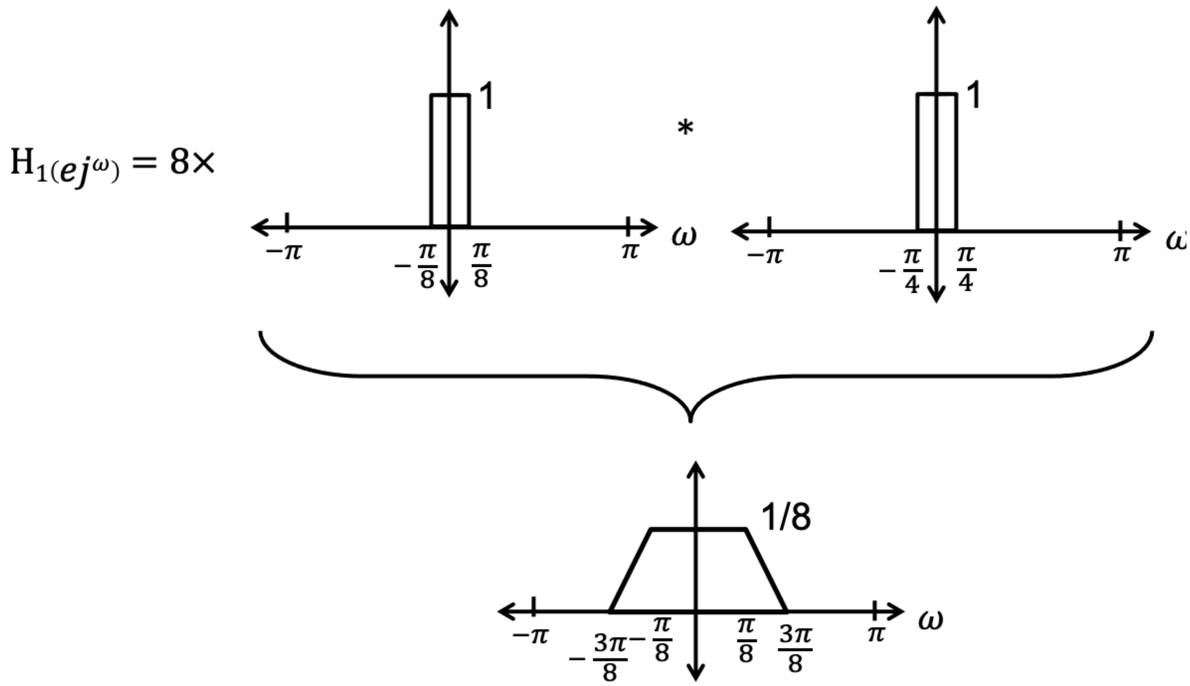
SOLUTION:

After sampling:

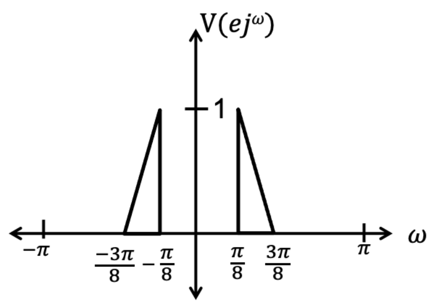


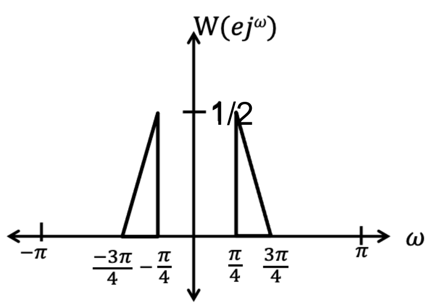


H1 is two LPFs convolved with each other:

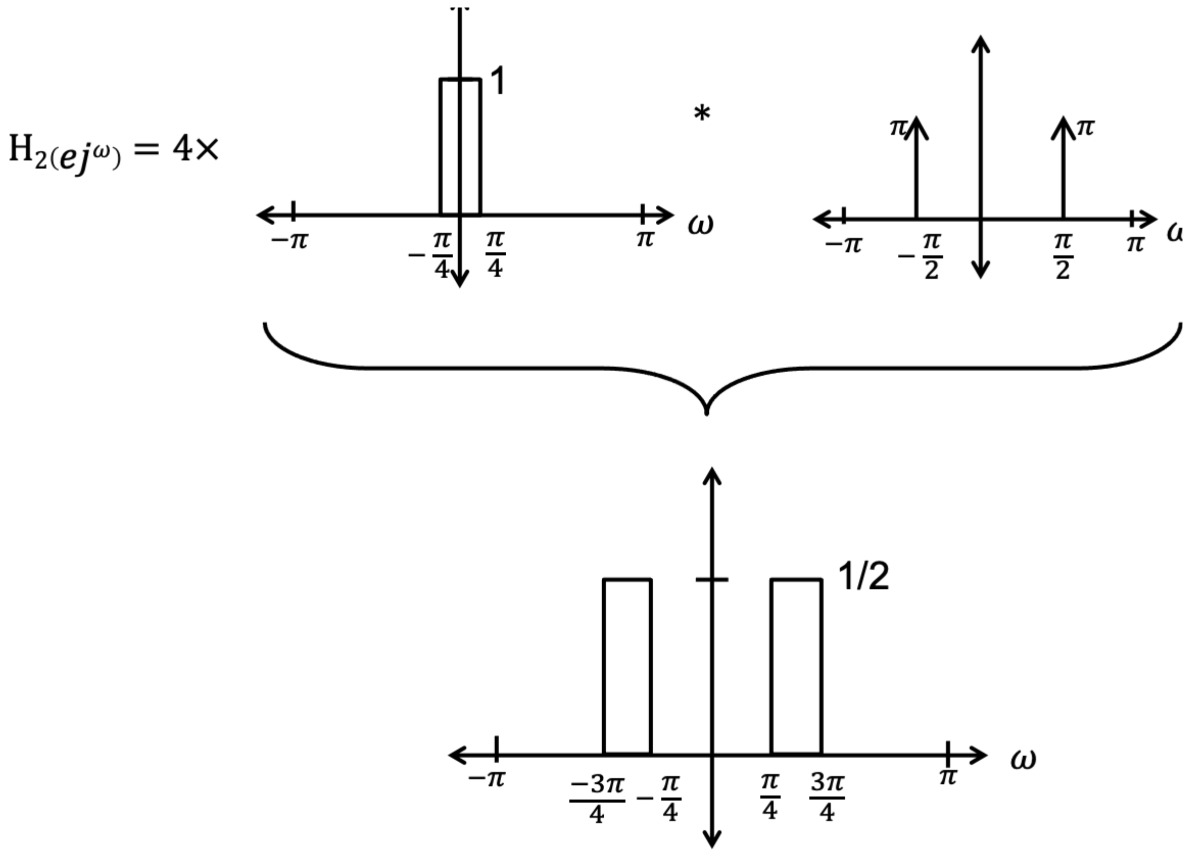


After filtering and upsampling:

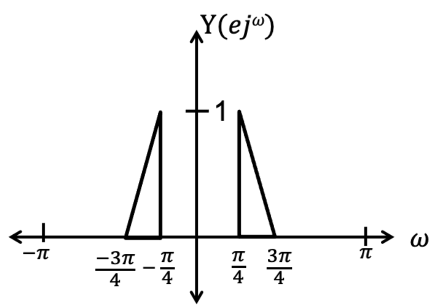




H2 is an LPF convolved with a cosine resulting in a BPF:

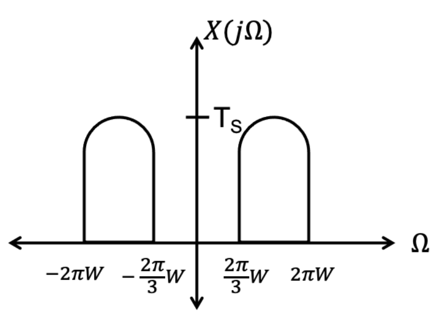


Finally the output:



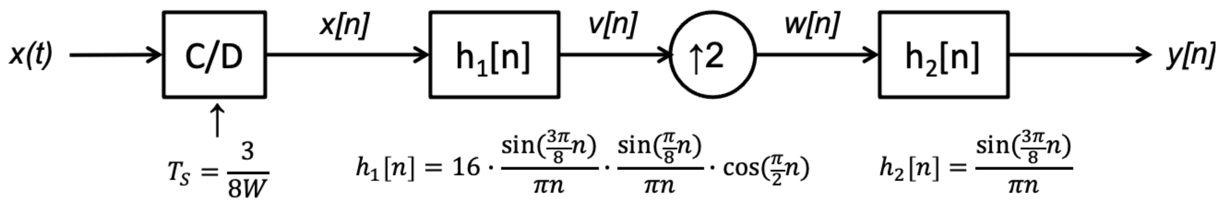
Q4

Consider the analog input signal $x(t)$ whose Continuous Time Fourier Transform (CTFT), $X(j\Omega)$, is plotted below.



The input $x(t)$, which is both real-valued and even-symmetric, is sampled with a sampling period T_S to create the discrete-time signal

$x[n] = x(nT_S)$ which is then input into a cascaded system of filters and re-sampling blocks shown below.

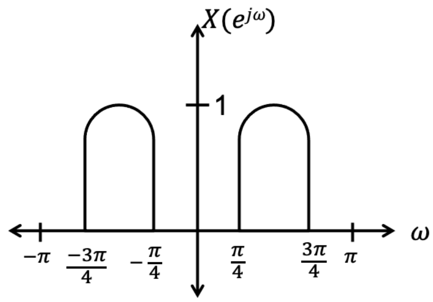


Sketch the frequency responses of all discrete time signals and filters:
$X(e^{j\omega}),\ H_1(e^{j\omega}),\ V(e^{j\omega}),\ W(e^{j\omega}),\ H_2(e^{j\omega}),\ Y(e^{j\omega})$

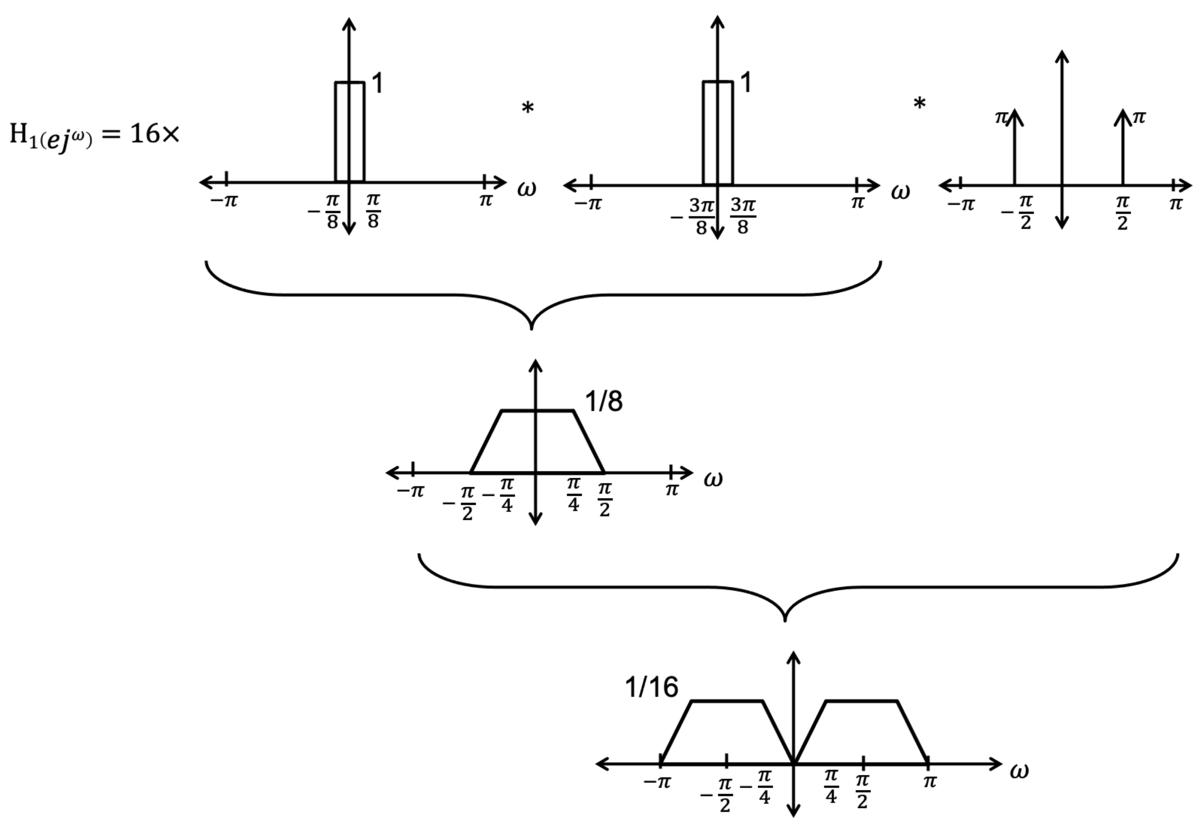
Make sure to label all axes and indicate heights and relevant frequencies. Be sure to specify a full 2π period of each frequency response.

SOLUTION:

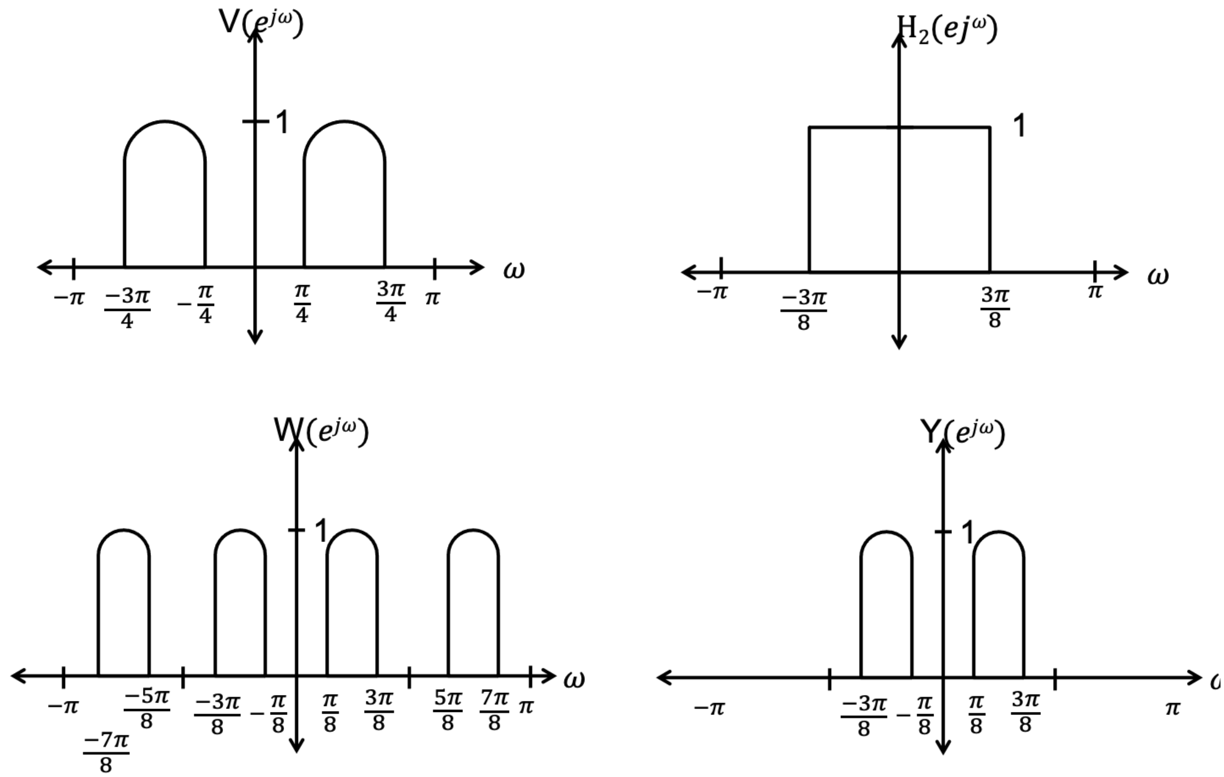
After sampling:



H1 is two LPFs convolved with each other that are then convolved with a cosine:



After filtering, upsampling. H2 is just an LPF resulting in the final output.



⣿ **Q5** 0 pts

Upload a single file (,pdf preferred) of your answers with work here for grading and partial credit.

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