Signals and Systems Review

Resource: MIT OCW 6.003

Dennis Freeman. *6.003 Signals and Systems.* Fall 2011. Massachusetts Institute of Technology: MIT OpenCourseWare, https://ocw.mit.edu. License: Creative Commons BY-NC-SA.

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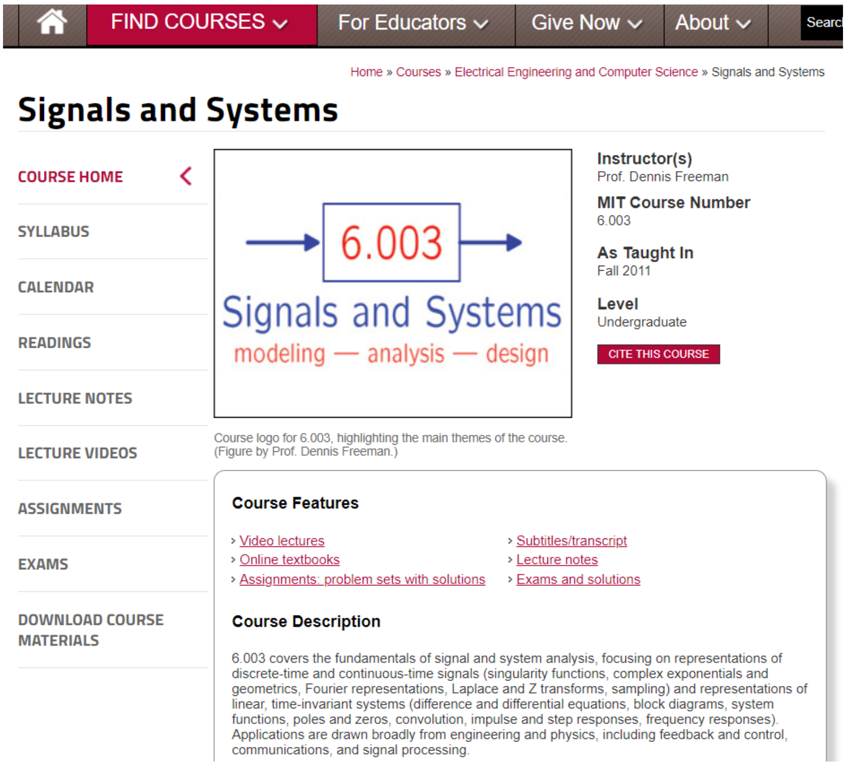
Webpage for MIT OCW 6.003

https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-003-signals-and-systems-fall-2011/

if you cannot find the contents you are looking for from the list below, you can always go to this webpage and find more! Note that there are videos and homework with solutions in addition to the lecture and notes. There are some matlab codes after each homework for advanced study.

Overview of the webpage:



The corresponding readings in the S&S book and relevant topics. The detailed contents are in the other file.

TOPICS	SECTIONS
Discrete-time (DT) systems	Chapters 1–4 of supplementary notes
Continuous-time (CT) systems	1.0–1.5, 2.4–2.5
Laplace transforms	9.1–9.10
Z transforms	10.1–10.10
Convolution	2.0–2.6
Frequency response	3.10, 6.0–6.2.3, 6.5–6.5.3
Feedback	11.0–11.2.3
CT Fourier series	3.0–3.5.9
CT Fourier transform	4.0–4.8
DT Fourier series	3.6–3.12
DT Fourier transform	5.0–5.9
Sampling	7.0–7.6
Modulation	8.0–8.9

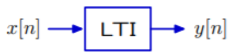
1. Convolution

location: lecture 8

The lecture uses an example and helps illustrate the definition with step-by-step analysis. In summary, the definition is:

Convolution

Response of an LTI system to an arbitrary input.



$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] \equiv (x*h)[n]$$

This operation is called **convolution**.

Notation

Convolution is represented with an asterisk.

$$\sum_{k=-\infty}^{\infty} x[k]h[n-k] \equiv (x*h)[n]$$

It is customary (but confusing) to abbreviate this notation:

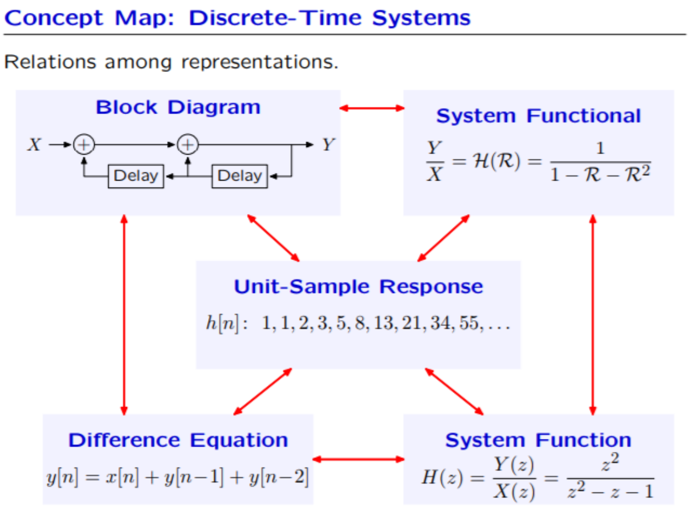
$(x*h)[n] = x[n]*h[n]$

Suggested HW problems: HW5 P1, P2, P3.

2. System functions/Impulse response

Location: lecture 5.

The following map illustrates the relationship between different representations clearly.



Following the definitions for Z transform, there are many examples with solutions to help better understand the system function.

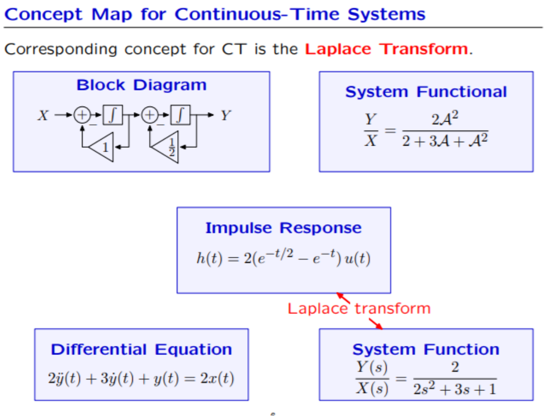
Location: lecture 4.

Steps leading to impulse response is in 'Elementary Building-Block Signals' from lecture 4 starting page 14. The unit-impulse signal acts as a pulse with unit area but zero width.

Suggested HW problems: HW4 P5; HW2 P5; Fall 2011 Quiz #1 P2; Quiz #1 (Spring 2010) P2

3. Laplace transform and properties

Location: lecture 6



Topics covered: Definition; Regions of Convergence; Time-Domain Interpretation of ROC; Properties of Laplace Transforms.

Properties of Laplace Transforms

Usefulness of Laplace transforms derives from its many properties.

Property	$x(t)$	$X(s)$	ROC
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	$\supset (R_1 \cap R_2)$
Delay by T	$x(t-T)$	$X(s)e^{-sT}$	R
Multiply by t	$tx(t)$	$-\frac{dX(s)}{ds}$	R
Multiply by $e^{-\alpha t}$	$x(t)e^{-\alpha t}$	$X(s+\alpha)$	shift R by $-\alpha$
Differentiate in t	$\frac{dx(t)}{dt}$	$sX(s)$	$\supset R$
Integrate in t	$\int_{-\infty}^{t} x(\tau)\,d\tau$	$\frac{X(s)}{s}$	$\supset \big(R \cap (\mathrm{Re}(s) > 0)\big)$
Convolve in t	$\int_{-\infty}^{\infty} x_1(\tau)x_2(t-\tau)\,d\tau$	$X_1(s)X_2(s)$	$\supset (R_1 \cap R_2)$

and many others!

Suggested HW problems: HW4 P1; Fall 2011 Quiz #1 P3; Quiz #1 (Spring 2010) P3

4. Frequency response/Bode plots

Location: lecture 9

Summary: frequency response is a different way to characterize a system.

Topics covered: Eigenfunctions; Complex Exponentials; Rational System Functions; Vector Diagrams; Frequency Response; Conjugate Symmetry.

Location: lecture 11

For detailed explanation and examples, please refer to the original lecture slides.

Bode Plot

The log of the magnitude is a sum of logs.

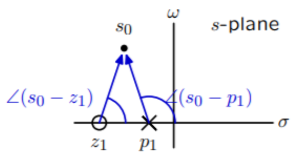
$$|H(s_0)| = \left| K \frac{\prod_{q=1}^{Q}(s_0 - z_q)}{\prod_{p=1}^{P}(s_0 - p_p)} \right| = |K| \frac{\prod_{q=1}^{Q}|s_0 - z_q|}{\prod_{p=1}^{P}|s_0 - p_p|}$$

$$\log|H(j\omega)| = \log|K| + \sum_{q=1}^{Q}\log|j\omega - z_q| - \sum_{p=1}^{P}\log|j\omega - p_p|$$

Bode Plot

The angle of a product is the sum of the angles.

$$\angle H(s_0) = \angle\left(K \frac{\prod_{q=1}^{Q}(s_0 - z_q)}{\prod_{p=1}^{P}(s_0 - p_p)} \right) = \angle K + \sum_{q=1}^{Q}\angle(s_0 - z_q) - \sum_{p=1}^{P}\angle(s_0 - p_p)$$



The angle of K can be 0 or π for systems described by linear differential equations with constant, real-valued coefficients.

$$|H(j\omega)|\text{[dB]} = 20\log_{10}|H(j\omega)|$$

Suggested HW problems: HW7 P2

5. Pole-zero plots

Location: lecture 3, 5.

Definition for poles are in lecture 3, (starting from page 23), followed by several example.

Also, on lecture 5 page 24-25, there are definitions abut poles and zeros, followed by examples for pole-zero plots.

Rational Polynomials

Applying the fundamental theorem of algebra and the factor theorem, we can express the polynomials as a product of factors.

$$H(z) = \frac{a_0 z^k + a_1 z^{k-1} + a_2 z^{k-2} + \cdots}{b_0 z^k + b_1 z^{k-1} + b_2 z^{k-2} + \cdots}$$

$$= \frac{(z - z_0)\ (z - z_1)\ \cdots\ (z - z_k)}{(z - p_0)\ (z - p_1)\ \cdots\ (z - p_k)}$$

where the roots are called **poles** and **zeros**.

Suggested HW problems: HW3 P5; HW4 P3