ESE 531 Recitation 1

Recall a LTI system S:

$$y[n] = S\{x[n]\}$$

L: Linear

TI: time-invariant

$$\left\{\begin{array}{l}\text{scaling}\\ \text{addition}\\ \text{shift in time}\end{array}\right. \Rightarrow \left\{\begin{array}{l} S\{a\,x[n]\} = a\,y[n] \\ S\{x_1[n]+x_2[n]\} = y_1[n]+y_2[n] \\ S\{x[n-p]\} = y[n-p]\end{array}\right.$$

2.28

(a). $x_4[n] = \delta[n]$

If S is time-invariant: $S\{x_4[n-p]\} = y_4[n-p]$

$x_3[n] = \delta[n-1] + \cdots$

If S is TI: then $x_3[n]$ should produce:
$\delta[n-1] + \cdots$

But $y_3[1] = 0 \neq \delta[n-1] = 1 \Rightarrow$ not TI

(b). Construct a sequence from $x_1[n], x_2[n], x_3[n], x_4[n]$

$$x''[n] = x_1[n] + x_2[n] = 2x_3[n] + 2x_4[n]$$

$x'[n]$:

$$S\{x'[n]\} = S\{x_1[n]+x_2[n]\}$$

$$= S\{2x_3[n]+2x_4[n]\}$$

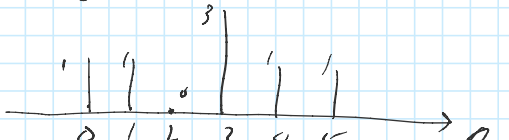
$$= S\{2x_3[n] + 2x_4[n]\}$$

$$S\{x_1[n] + x_2[n]\} = y_1[n] + y_2[n]$$

$\Updownarrow$ $\quad$ $=$ if S is LTI

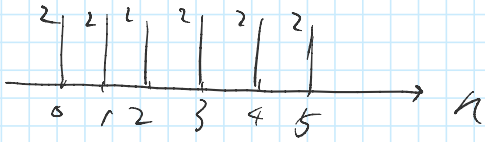
$$S\{2x_3[n] + 2x_4[n]\} = 2y_3[n] + 2y_4[n]$$

Check: $y_1[n] + y_2[n]$:



$\neq$

$2y_3[n] + 2y_4[n]$:



$\Rightarrow$ S is **not** LTI

(c).

$$\begin{cases} y_2[n] = S_2\{x_2[n]\} \\ y_3[n] = S_2\{x_3[n]\} \end{cases}, \quad S_2 \text{ is LTI}$$

Recall $h[n] = S_2\{\delta[n]\}$

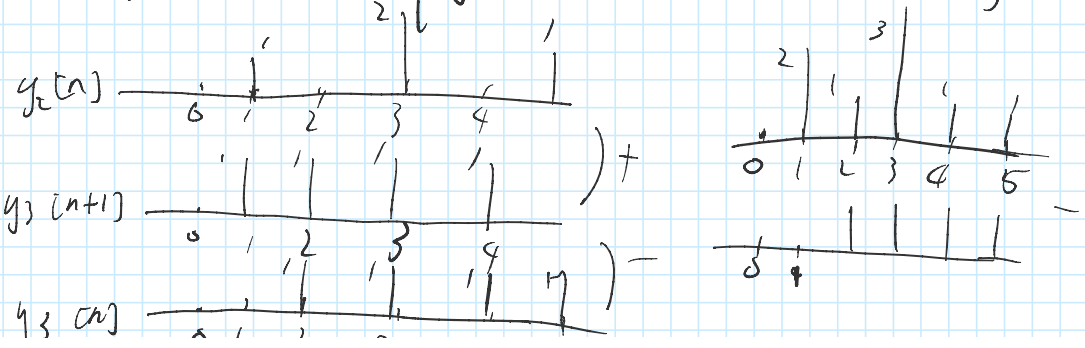
construct $\delta[n]$ from $x_2[n]$ & $x_3[n]$

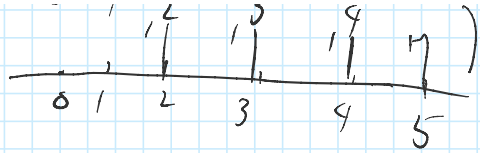
$$\begin{cases} x_2[n] = \delta[n] + \delta[n-2] & \text{---- ①} \\ x_3[n] = \delta[n-1] + \delta[n-2] & \text{---- ②} \end{cases}$$

$\Rightarrow$ $x_3[n+1] = \delta[n] + \delta[n-1]$ ---- ③

$$\delta[n] = \frac{1}{2}\{x_2[n] + x_3[n+1] - x_3[n]\}$$

$\Rightarrow$ $$h[n] = \frac{1}{2}\{y_2[n] + y_3[n+1] - y_3[n]\}$$



$y_3[n]$ 

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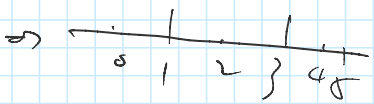
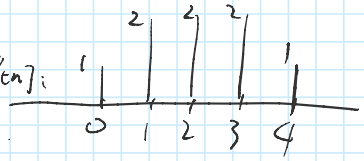
$\Rightarrow$ 

$\Rightarrow h[n] = \delta[n-1] + \delta[n-3]$

(d)

$$y_1[n] = S_3\{x_1[n]\}$$

Need to construct $x''[n]$: 

from $x_1[n]$ 

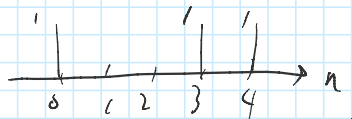
Notice $x''[n] = x_1[n] + x_1[n-2]$

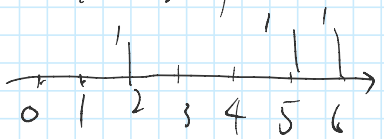
$$\Rightarrow y''[n] = S_3\{x''[n]\}$$

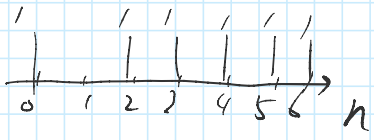
$$= S_3\{x_1[n] + x_1[n-2]\}$$

LTI $\Big\}$

$$= y_1[n] + y_1[n-2]$$

$y_1[n]$: 

$y_1[n-2]$: 

$+ \Rightarrow$ 

$$y''[n] = \delta[n] + \sum_{p=2}^{6} \delta[n-p]$$