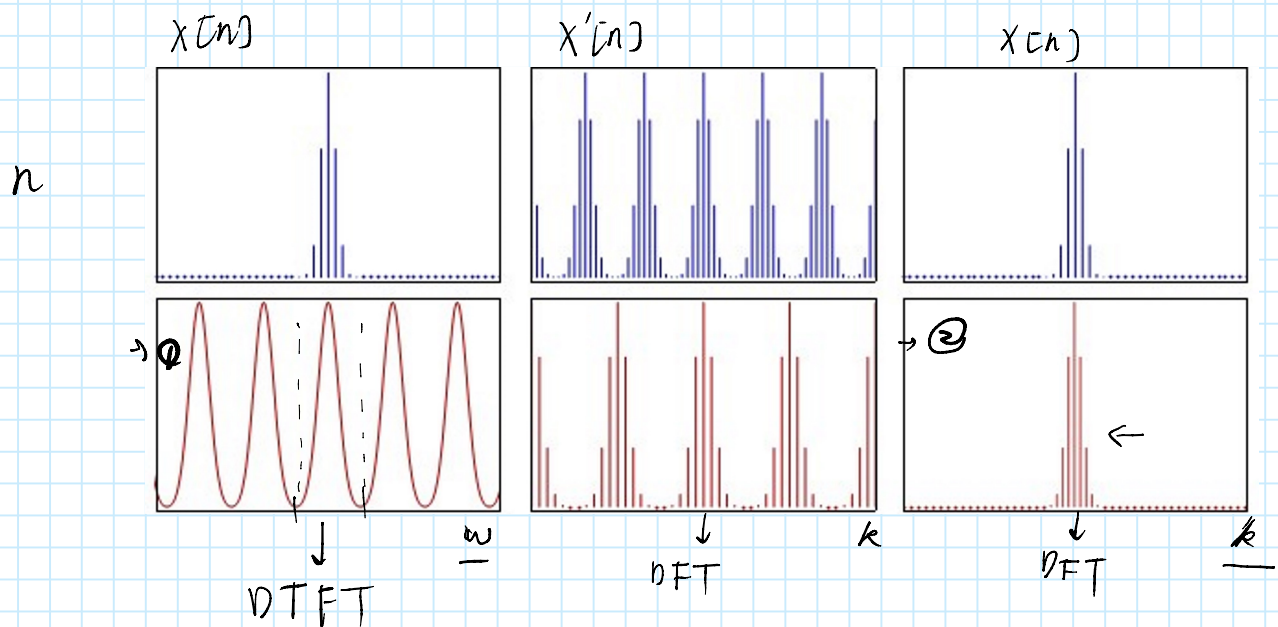


ESE 531 Recitation 11

Review on DTFT & DFT



$$\text{DTFT: } X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \quad \dots (I)$$

$$\text{DFT: } X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn}, \quad 0 \leq k \leq N-1 \quad \dots (II)$$

Compare Figure ① & ②, $\text{DFT}(I) \& (II)$.

→ DFT of a finite signal is the sampled version of DTFT within $[0, 2\pi)$

8.28

$$(a). \quad X[k] = \sum_{n=0}^5 x[n] W_6^{kn}, \quad 0 \leq k \leq 5$$

$$(b). \quad w[n] \xrightarrow{\text{IDFT}} W[k]$$

(b).

$$W[n] \stackrel{\text{IDFT}}{\leftarrow} W[k]$$

$$W[k] = \underbrace{X[k]} \times \underbrace{W_6^{-2k}}$$

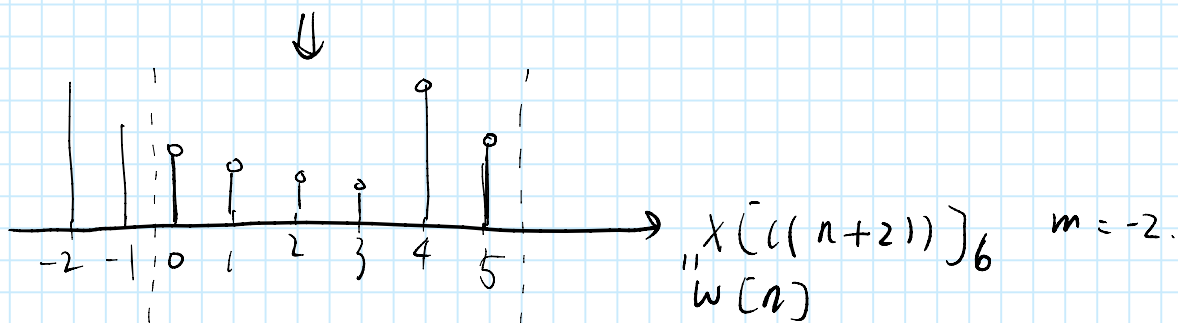
$$\underbrace{e^{-j\omega n_0} \times X(e^{j\omega})}_{\text{shift in time}}$$

$\Rightarrow W[n]$ will be shifted in time.

But note that this shift is circular.

$$\Rightarrow \underline{X[(N-m)]_N}$$

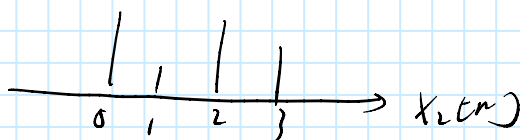
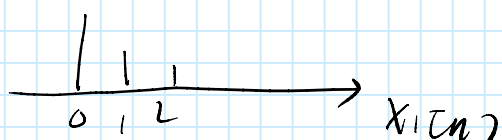
Shift term: $W_6^{-2k} \Rightarrow$ circular shift to the left of $X[n]$ by 2.

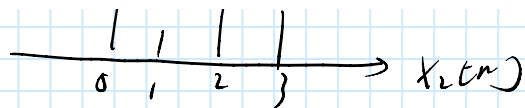
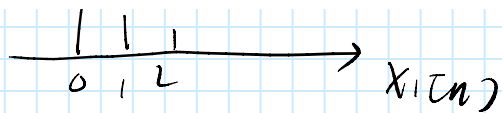


(c).

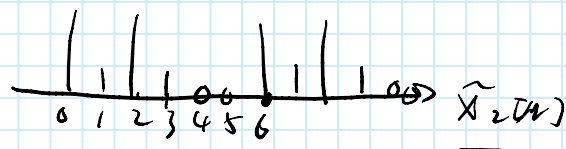
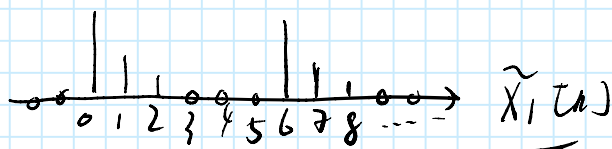
Review: Circular convolution

$$X_3[n] = \sum_{m=0}^{N-1} \underbrace{X_1[m]} \underbrace{\tilde{X}_2[n-m]} \quad 0 \leq n \leq N-1$$





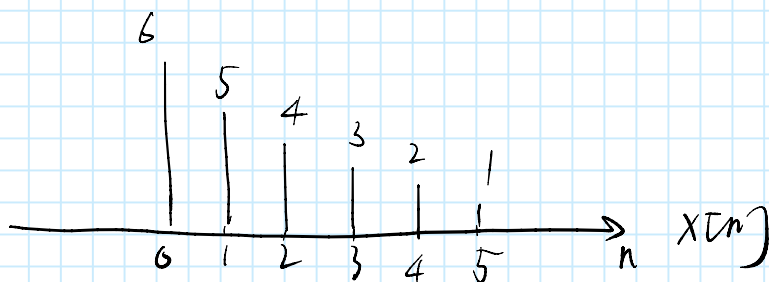
Step 1. Make $x_1[n]$ & $x_2[n]$ periodic, with period N .



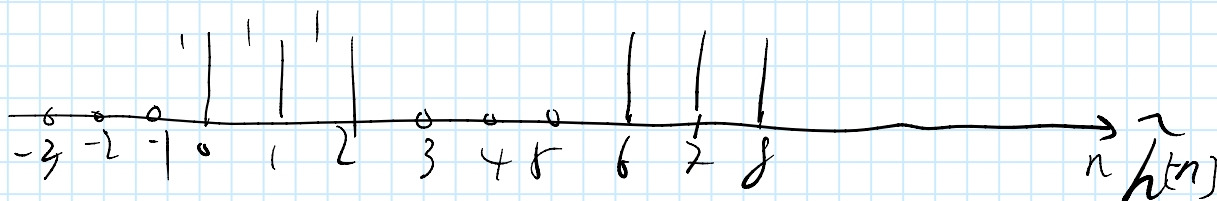
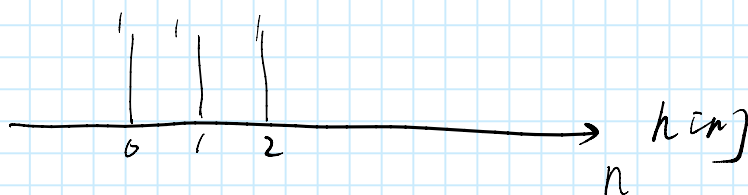
Step 2:

↓ linear convolution

$x_3[n], 0 \leq n \leq N-1$



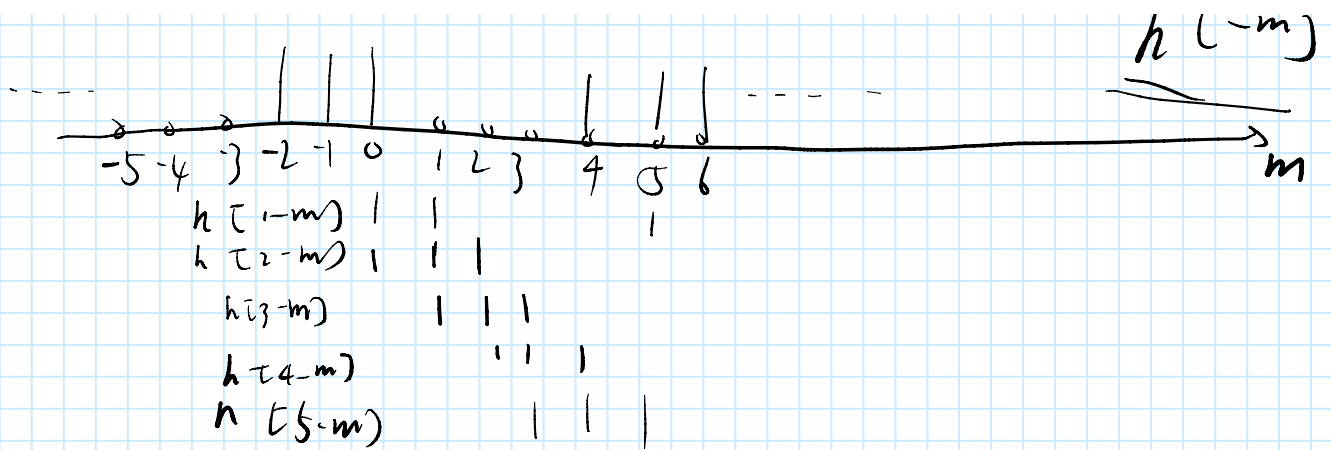
↓ ⑥



↪ $\tilde{h}[n-m]$

$\tilde{h}[n-m]$





$$v[n] = x[n] \circledast h[n]$$

$$\Rightarrow v[n] = \sum_{m=0}^5 x[m] \tilde{h}[n-m]$$

$$\Rightarrow v[0] = 6 + 2 + 1 = 9$$

$$v[1] = 6 + 5 + 1 = 12$$

$$v[2] = 6 + 5 + 4 = 15$$

$$v[3] = 12$$

$$v[4] = 9$$

$$v[5] = 6$$



(d).

$$x[n] \circledast h[n] = x[n] * h[n]$$

Length 6 3

$$N \geq L + p - 1 \Rightarrow \text{no aliasing during } N\text{-point circular}$$

N -point circular convolution

(e).

$$\underline{x_1[n]} \circledast h[n] = x[n] * h[n]$$

Make $x[n]$ periodic.

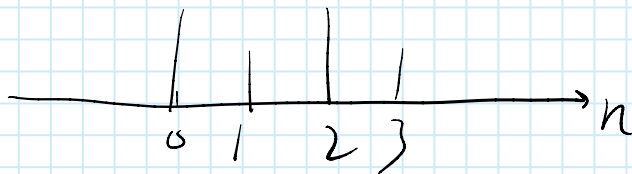
$$x_1[n] = \sum_{k=-\infty}^{\infty} x[n - Lk], \quad 0 \leq n \leq L-1$$

(f).

$$\begin{array}{cc} h[n] & \text{length } M \\ x[n] & \text{length } L \end{array}$$

$$x_1[n] = \sum_{k=-\infty}^{\infty} x[n - Lk], \quad 0 \leq n \leq L-1$$

q.28



$\Rightarrow$ 4-point DFT

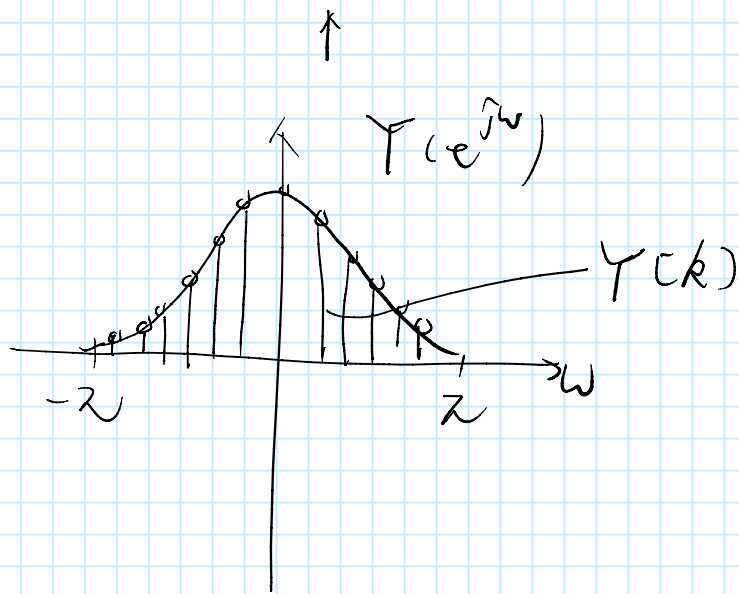
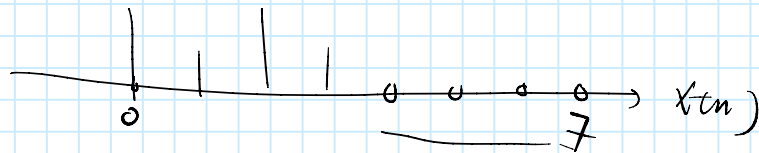
$\Rightarrow$	8-point DFT	1

	2-point DFT	1
	$\oplus$	10
	$\otimes$	100

$$X_1[k] = \sum_{n=0}^3 x[n] W_4^{nk}, \quad 0 \leq k \leq 3$$

$$X_1[k] = \sum_{n=0}^7 x[n] W_4^{nk}, \quad 0 \leq k \leq 3$$

$$X_2[k] = \sum_{n=0}^7 x[n] W_8^{nk}, \quad 0 \leq k \leq 7$$



$$\Rightarrow X_1[k] = X_2[2k]$$

$\Rightarrow$ Only need Module 1, 8-point DFT