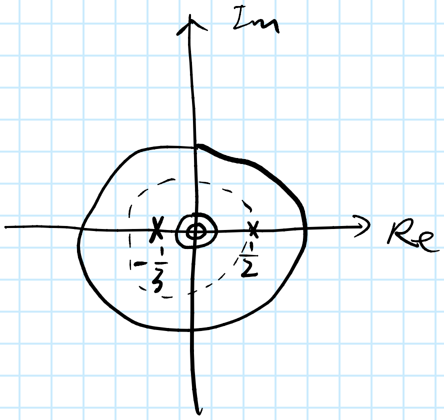
ESE 531 Recitation 13: Final (2020) Review

Q1 (ver. 4), Q2 (ver. 3)

Q3, Q4

Q1 (ver 4).



$H(z) = 6$ at $z = 1$

ROC: $|z| > \frac{1}{2}$

(a). ROC: $|z| > \frac{1}{2}$

$$H(z) = \frac{A}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{3}z^{-1})}$$

$A \rightarrow$ constant

$$H(z)\big|_{z=1} = H(1) = \frac{A}{(1-\frac{1}{2})(1+\frac{1}{3})} = 6$$

$$\Rightarrow A = 4$$

$$H(z) = \frac{4}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{3}z^{-1})} \qquad \times \frac{4z^2}{(z-\frac{1}{2})(z+\frac{1}{3})}$$

(b). $h[n] = Z^{-1}\{H(z)\}$

$H(z) = \frac{12}{5} \qquad \frac{8}{5}$

$$H(z) = \frac{\frac{12}{5}}{1-\frac{1}{2}z^{-1}} + \frac{\frac{8}{5}}{1+\frac{1}{3}z^{-1}}$$

$\downarrow$ inverse z-transform, ROC $|z| > \frac{1}{2}$

$$h[n] = \frac{12}{5} \times \left(\frac{1}{2}\right)^n u[n] + \frac{8}{5}\left(-\frac{1}{3}\right)^n u[n]$$

(c). Yes.

(d). First find out $x[n]$.

$$x[n] = x(nT) \qquad T = \frac{1}{f}$$

$$\underset{t}{\overset{\parallel}{nT}} \qquad \Omega_s = 2\pi f$$

$$\Rightarrow \quad t = \frac{2\pi}{\Omega_s} \cdot n = \frac{2\pi}{2\pi \times 40} \times n$$

$$= \frac{n}{40}$$

$$\Rightarrow x[n] = 50 + 30\cos\left(40\pi \times \frac{n}{40}\right)$$

$$= 50 + 30\underline{\cos(\pi n)}$$

Next:

$$y[n] = \underline{x[n]} * \underline{h[n]}$$

$$Y(e^{jw}) = \underline{X(e^{jw})}\,\underline{H(e^{jw})}$$

$$X(e^{jw}) = \mathrm{DTFT}\{x[n]\}$$

$$\textcircled{1} \Rightarrow \; = 30 \sum_{k=-\infty}^{\infty} \Big[\pi\, \delta(w - \pi + 2\pi k) + \pi\, \delta(w + \pi + 2\pi k) \Big]$$

↳ from cos

$$\textcircled{2} \Rightarrow \; + \; 50 \sum_{k=-\infty}^{\infty} 2\pi\, \delta(w + 2\pi k)$$

$$Y(e^{jw}) = \underline{\qquad} \times H(e^{jw})$$

$$= \delta(\quad) \times \frac{4}{(1 - \frac{1}{2} e^{-jw})(1 + \frac{1}{3} e^{-jw})}$$

$Y(e^{jw})$ is non-zero only at w', where w' is in $\delta(w')$

$$Y(e^{jw}) = $$

from ① $w = \pm\pi$ $\quad \dfrac{4}{(1 + \frac{1}{2})(1 - \frac{1}{3})}$

$$\frac{4}{\frac{3}{2} \times \frac{2}{3}} \times \textcircled{1} + \frac{4}{\frac{1}{2} \times \frac{4}{3}} \times \textcircled{2}$$

from ② $w = 0$ $\quad \dfrac{4}{(1 - \frac{1}{2})(1 + \frac{1}{3})}$

$$= 4 \times 30 \sum_{k=-\infty}^{\infty} \Big[\pi\, \delta(w - \pi + 2\pi k) + \pi\, \delta(w + \pi + 2\pi k) \Big]$$

$$+ \; 6 \times 50 \sum^{\infty} 2\pi\, \delta(w + 2\pi k)$$

$+ \; 6 \times 50 \sum_{k=-\infty}^{\infty} 2\pi \, \delta(\omega + 2\pi k)$

$y[n] = IDTFT\left\{ Y(e^{j\omega}) \right\}$

$= 120 \cos(\pi n) + 300$

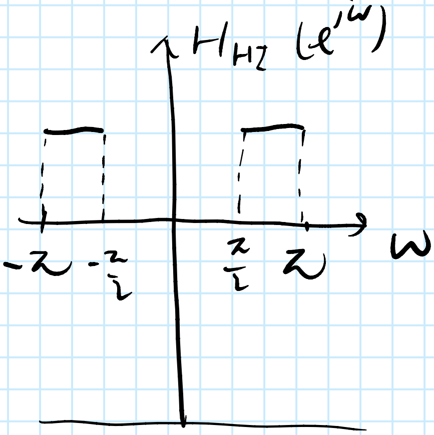
Q2 (ver. 3)

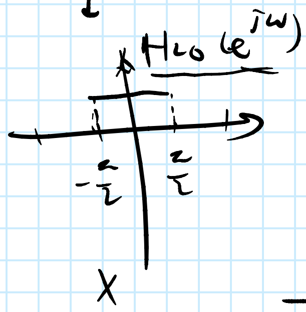
(a). Need to determine $Y(e^{j\omega})$

$x[n] \longrightarrow \boxed{H_{HI}(z)} \longrightarrow \boxed{\downarrow 2} \longrightarrow \boxed{H_{LO}(z)} \longrightarrow \boxed{\downarrow 2} \longrightarrow y[n]$

↓ from interchangable relation

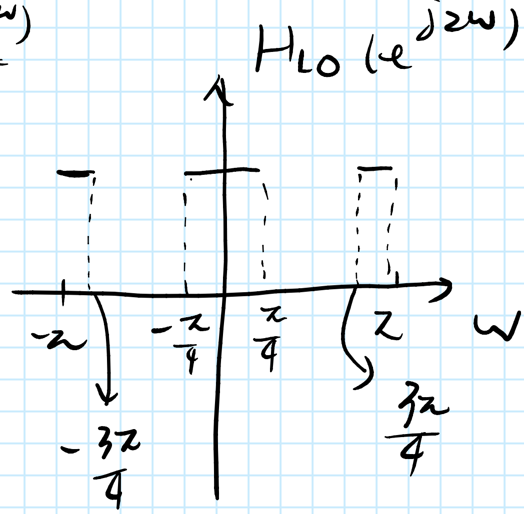
$\boxed{H_{HI}(z)} \longrightarrow \boxed{H_{LO}(z^2)} \longrightarrow \boxed{\downarrow 4}$



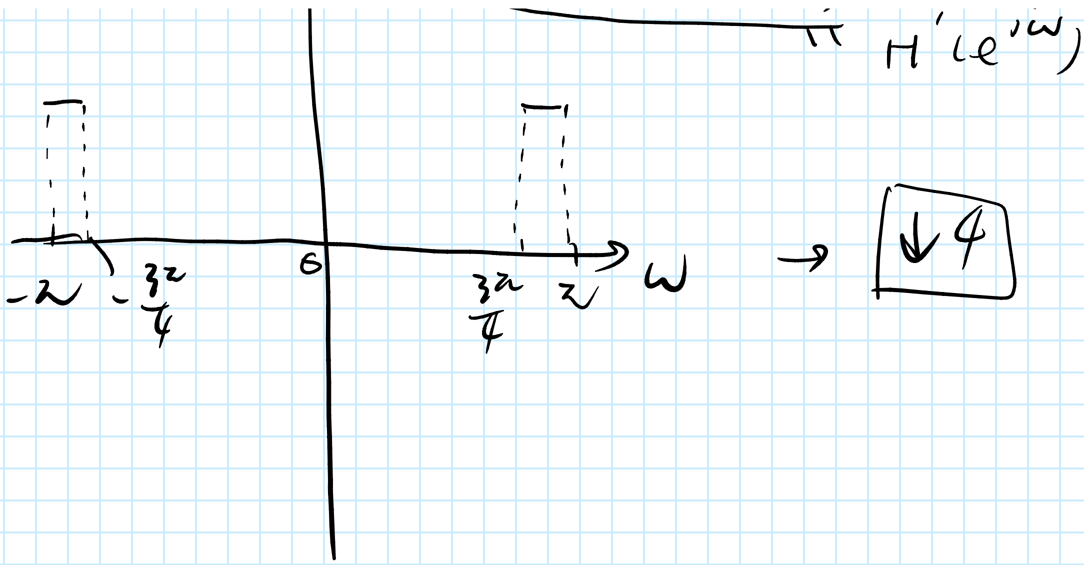


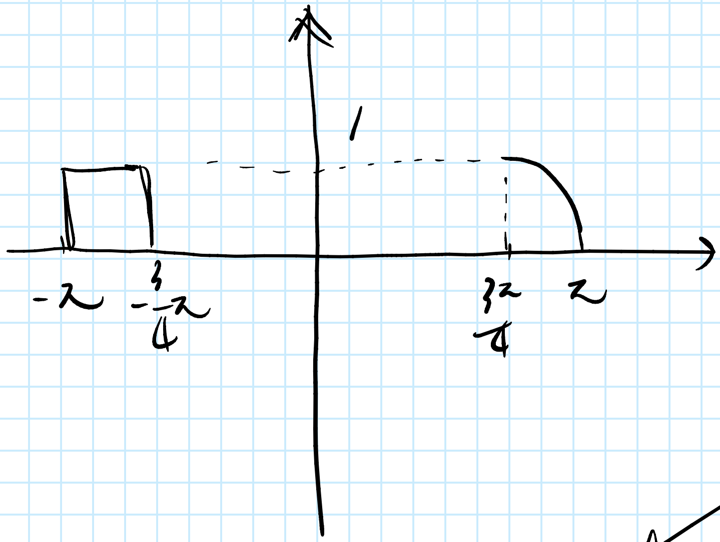
$-\frac{\pi}{2} < 2\omega < \frac{\pi}{2}$

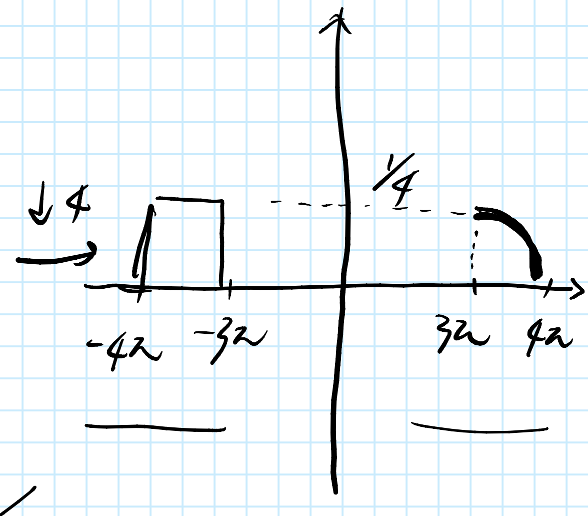
$-\frac{\pi}{4} < \omega < \frac{\pi}{4}$



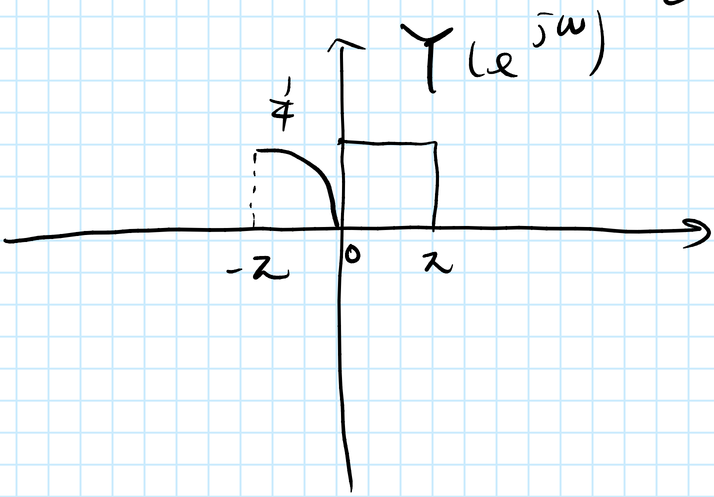
$H_{HI}(e^{j\omega}) \times H_{LO}(e^{j2\omega}) = H'(e^{j\omega})$







periodic 2π

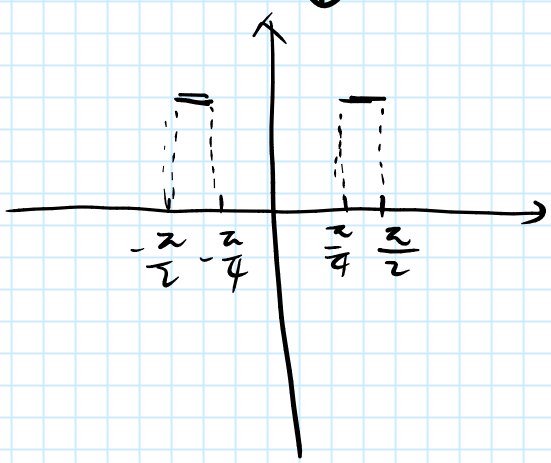


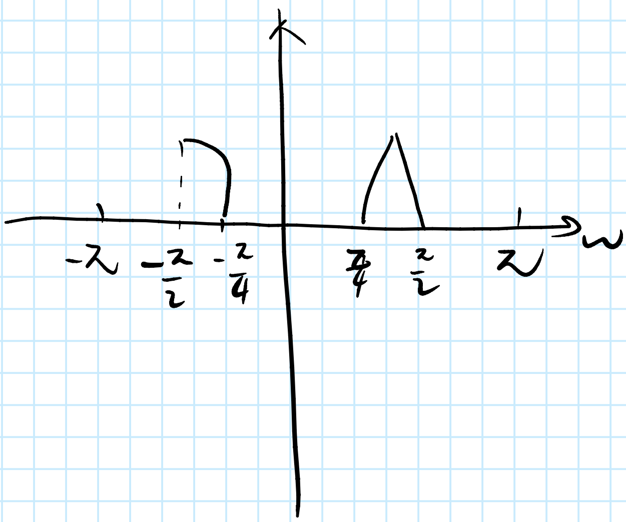
$\Rightarrow Y_F$

(b). Guess

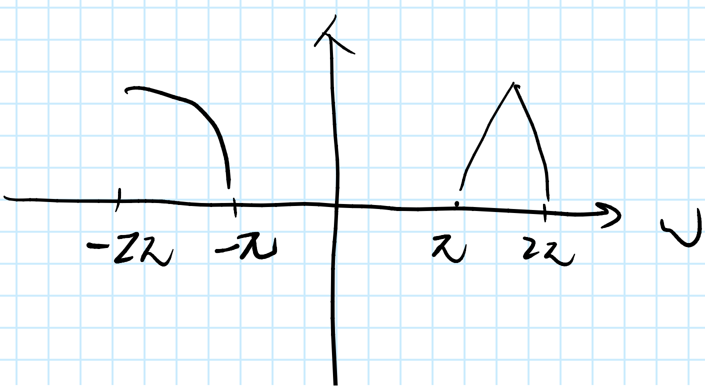
$x[n] \rightarrow \boxed{H_{Lo}(z)} \rightarrow \boxed{\downarrow 2} \rightarrow \boxed{H_{Hi}(z)} \rightarrow \boxed{\downarrow 2}$

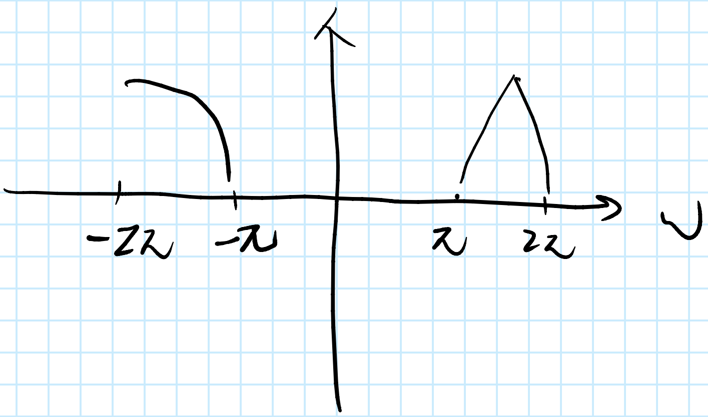
$x[n] \to \boxed{H_{Lo}(z)} \to \boxed{H_{Hz}(z^2)} \to \boxed{\downarrow 4}$



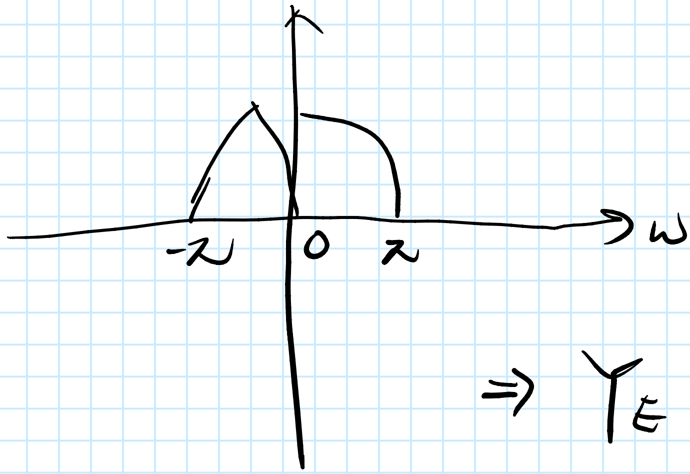


$\Downarrow$ $\downarrow 4$





$\downarrow$ shift to $-\pi$ to π



$\Rightarrow Y_E$

Q3.

8-DFT of $y[n]$

what we know: $\begin{cases} Y[0],\ Y[2],\ Y[4],\ Y[6] \\ y[0],\ y[1],\ y[2],\ y[3] \end{cases}$

(a). Want to find out $Y[1], Y[3], Y[5], Y[7]$

Do butterfly operation first

then do two 4-point DFT

then do two 4-point DFT

Why? Why this kind structure.

Naturally, from decimation-in-time FFT

$\Rightarrow$ divide input into even & odd groups.

output $\Rightarrow$ $Y[0], Y[1] \cdots Y[7]$

But for this problem, we have only

even samples from Y

$\Rightarrow$ Need a different structure to compute 8-point DFT instead of the structure shown in Fig 9.4 P726 in textbook

i $\Rightarrow$ Need bit-reversed structure in Fig 9.15 P736

Now, from bit-reversed structure.

can compute $g[0] \cdots g[3]$ from

4-point IDFT of $Y[0], Y[2], Y[4], Y[6]$,

which is $g[n] = \text{IDFT}\{Y[\text{even}]\}$

ii. And from $g[n]$ and $y[0], y[1], y[2], y[3]$

↳ can compute $y[4], y[5], y[6], y[7]$

iii. And From $y[0] \cdots y[7]$, can compute $Y[\text{odd}]$.

i. $g[n] = \text{IDFT}\{Y[\text{even}]\}$

$$= \frac{1}{4} \sum_{k=0}^{3} G[k] W_4^{-kn}$$

where
$$\begin{cases} G[0] = Y[0] \\ G[1] = Y[2] \\ G[2] = Y[4] \\ G[3] = Y[6] \end{cases}$$

$$\Rightarrow g[0] = \frac{1}{4} \left\{ G[0] + G[1]W_4^0 + G[2]W_4^0 + G[3]W_4^0 \right\}$$

$$= \frac{1}{4} Y[0] \{1 + 1 + 1 + 1\}$$

$$= Y[0] = 2$$

$$g[1] = \frac{1}{4} \left[G[0] + G[1]W_4^{-1} + G[2]W_4^{-2} + G[3]W_4^{-3} \right]$$

$$\qquad W_4 \quad \Big)$$

$$= \frac{1}{4} Y[0] \left\{ 1 + W_4^{-1} + W_4^{-2} + W_4^{-3} \right]$$

since $W_N = e^{-j\frac{2\pi}{N}}$, is periodic

$$= \frac{1}{4} Y[0] \left\{ 1 + W_4^{-1} + (-1) + -W_4^{-1} \right\}$$

$$= 0$$

$$g[2] = \frac{1}{4} Y[0] \left\{ 1 + \underset{\substack{\shortparallel \\ -1}}{W_4^{-2}} + \underset{\substack{\shortparallel \\ 1}}{W_4^{-4}} + \underset{\substack{\shortparallel \\ -1}}{W_4^{-6}} \right\}$$

$$= 0$$

$$g[3] = \frac{1}{4} Y[0] \left\{ 1 + \underset{-W_4^{-1}}{W_4^{-3}} + \underset{\substack{\shortparallel \\ -1}}{W_4^{-6}} + \underset{W_4^{-1}}{W_4^{-9}} \right\}$$

$$= 0$$

ii. From $g[n]$, $y[0], \cdots y[3]$

$$\begin{aligned} g[0] &= y[0] + y[4] \\ g[1] &= y[1] + y[5] \\ g[2] &= y[2] + y[6] \\ g[3] &= y[3] + y[7] \end{aligned} \Rightarrow \begin{cases} y[4] = 1 \\ y[5] = 0 \\ y[6] = 0 \\ y[7] = 0 \end{cases}$$

$g[3] = y[3] + y[7]$

iii. From $y[0] \cdots y[7]$

can compute $Y[odd]$

⋮

$Y[odd] = 0$.

(b). (b) is part (a).

Express step i. ii. iii. in (a) in a diagram

4 subtraction

7 multiplication

Q4.

Look for the positions of poles and zeroes.

usually: $\boxed{\text{pole}} \Rightarrow$ passband

$\boxed{\text{zero}} \Rightarrow$ stopband