ESE 531 Recitation 2

3.57, 4.4, 4.8

3.57

From the definition of z-transform:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \quad \cdots\cdots \text{①}$$

Also: $x[n] = 0$, for $n < 0$, i.e. $x[n]$ is causal.

① becomes: $X(z) = \sum_{n=0}^{\infty} x[n] z^{-n}$

$$\lim_{z\to\infty} X(z) = \lim_{z\to\infty} \underline{\sum_{n=0}^{\infty} x[n] z^{-n}}$$

$$= \lim_{z\to\infty} \left(x[0] z^{-0} + \underbrace{x[1] z^{-1} + x[2] z^{-2} \cdots}_{\Downarrow \; 0} \right)$$

$$= x[0]$$

If $x[n] = 0$, for $n > 0$, then:

$$X(z) = \sum_{n=-\infty}^{0} x[n] z^{-n}$$

$$\lim_{z\to 0} X(z) = \lim_{z\to 0} \underline{\sum_{n=-\infty}^{0} x[n] z^{-n}}$$

$$= \lim_{z\to 0} \left(\underbrace{\cdots + x[-2] z^{2} + x[-1] z^{1}}_{= 0} + x[0] \right)$$

$$= x[0]$$

4.4

(a). Recall $x[n] = x_c(nT) \cdots$ ①

4.7

(a). Recall $x[n] = x_c(nT)$ ... ①

From ①, $x_c(t)$, $x[n]$:

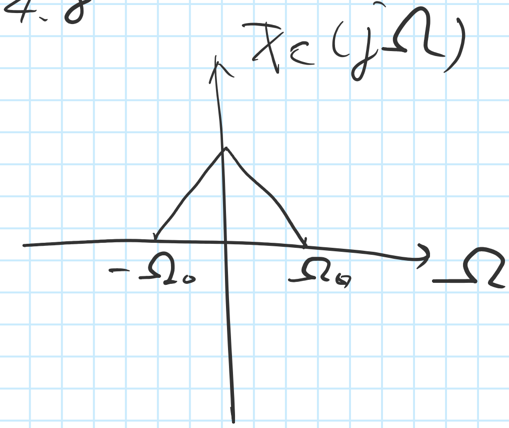
$$\sin(20\pi \times nT) + \cos(40\pi \times nT)$$
$$\|$$
$$\sin\left(\frac{2\pi n}{5}\right) + \cos\left(\frac{2\pi n}{5}\right)$$

$$\begin{cases} 20\pi \times nT = \frac{2\pi n}{5} + 2\pi p & (p \in \mathbb{Z}) \\ 40\pi \times nT = \frac{2\pi n}{5} + 2\pi p \end{cases}$$

$$\Rightarrow \quad T = \frac{1}{100} + \frac{p}{10}, \quad p \in \mathbb{Z}$$

(b). No.

4.8



(a).

$$\Omega_s \geq 2\Omega_0 \Rightarrow \text{no aliasing}$$

$$\frac{2\pi}{T} \geq 2 \times 2\pi \times 10^4$$

$$\Rightarrow \quad T \leq \frac{1}{2 \times 10^4}$$

(b) $y[n] = \left[T \sum^{n} x[k] \right]$

(b).
$$y[n] = \boxed{T \sum_{k=-\infty}^{n} x[k]}$$
$$= x[n] * h[n]$$
$$= \boxed{\sum_{k=-\infty}^{\infty} x[k]\, h[n-k]}$$

$$T \sum_{k=-\infty}^{n} x[k] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$
$$\sum_{k=-\infty}^{n} x[k] \times T = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

When $k \le n$

$$n-k \ge 0$$

Recall that $u[n] = 1$ when $n \ge 0$
$$h[n] = T\, u[n]$$
$$y[n] = T \sum_{k=-\infty}^{\infty} x[k]\, u[n-k]$$
$$= T \sum_{k=-\infty}^{n} x[k] \times 1$$
$$= T \sum_{k=-\infty}^{n} x[k]$$
$$\Rightarrow h[n] = T\, u[n]$$

(c). $X(e^{jw}) = \boxed{\sum^{\infty} x[n] e^{-jwn}}$

(c). $\underline{X(e^{j\omega})} = \boxed{\sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}}$

$$y[n] = T\sum_{k=-\infty}^{n} x[k]$$

$$\lim_{n\to\infty} y[n] = \boxed{T\sum_{k=-\infty}^{\infty} x[k]}$$

Notice $e^{-j\omega n} \to 1$ once $\omega \to 0$

$$\lim_{n\to\infty} y[n] = T X(e^{j\cdot 0}) = T X(e^{j\omega})\Big|_{\omega=0}$$

(d). $y[n]\Big|_{n=\infty} = \int_{-\infty}^{\infty} x_c(t)\,dt$ ··· (I)

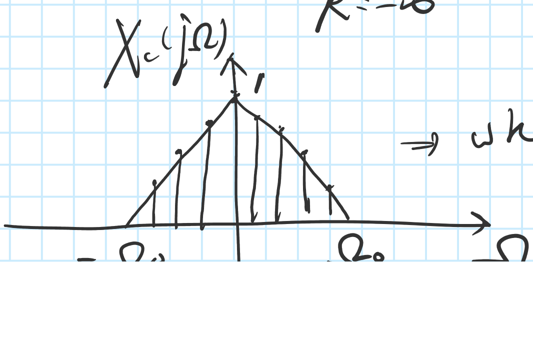
From (c): $y[n]\Big|_{n=\infty} = T X(e^{j\omega})\Big|_{\omega=0}$ ··· ②

$$X(e^{j\omega}) = \frac{1}{T}\cdot\sum_{k=-\infty}^{\infty} X_c\left[j\left(\frac{\omega}{T} - \frac{2\pi k}{T}\right)\right] \quad \cdots ③$$

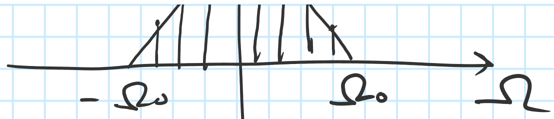
plug ③ into ②:

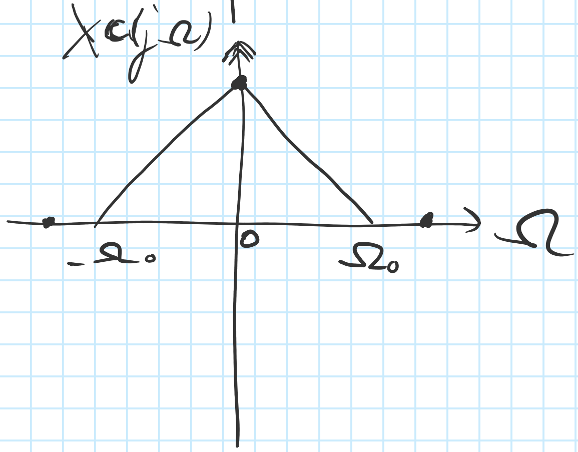
$$y[n]\Big|_{n=\infty} = T\cdot\frac{1}{T}\cdot\sum_{k=-\infty}^{\infty} X_c\left(j\frac{2\pi k}{T}\right)$$

$$= \sum_{k=-\infty}^{\infty} X_c\left(j\frac{2\pi k}{T}\right) : \text{LH of (I)}$$



→ when T is small





$\Rightarrow$ when T is large

RH of (I): $\displaystyle\int_{-\infty}^{\infty} x_c(t)\,dt$

Recall F.T. of a continuous-time signal $x(t)$

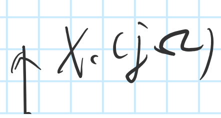
$$X(j\Omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega t}\, dt \quad \cdots\cdots \text{④}$$

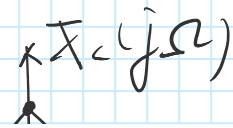
Compare ④ with RH of (I)

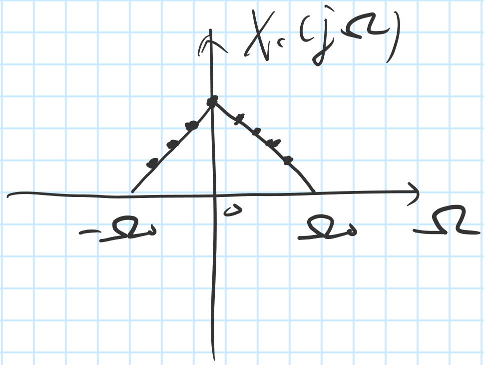
④ = RH of (I) as long as $\Omega = 0$

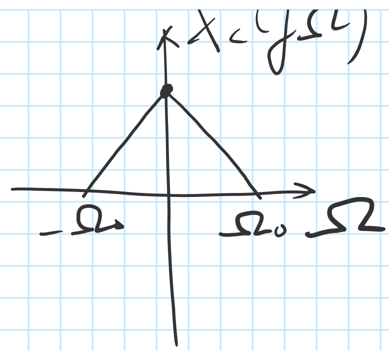
Combine the LH of (I) & RH of (I):

$$\sum_{k=-\infty}^{\infty} X_c\left(j\frac{2\pi k}{T}\right) = X_c(j\Omega)\Big|_{\Omega=0}$$

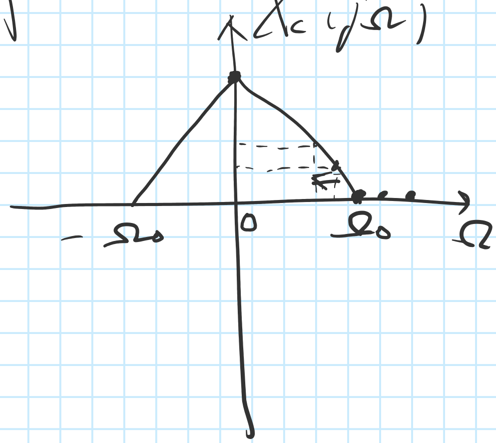








$\Rightarrow$ Need T to be as large as possible for (I) to be valid. Then T_{min}:



$$T_{min} = \frac{2\pi}{\Omega_0} = \frac{2\pi}{2\pi \times 10^4} = \frac{1}{1 \times 10^4} = 10^{-4}$$

$$T \geq 10^{-4}.$$