ESE 531 Recitation 3

4.11, 4.12, 4.27, 4.29

4.11

(a). $x_c(nT) = x[n]$

$x_c(t) = \sin(10\pi t)$

$\Rightarrow \underline{x[n]} = x_c(nT) = \underline{\sin(10\pi \times nT)}$

$= \underline{\sin(\frac{\pi n}{4})}$

$10\pi nT = \frac{\pi n}{4} + 2m\pi \quad , m \in \mathbb{Z}$

$\Rightarrow \underline{T} = \frac{1}{40} + \frac{m}{5n} \quad , n \in \mathbb{Z}$

$= \frac{1}{40} \pm \frac{1}{5}$

T is not unique.

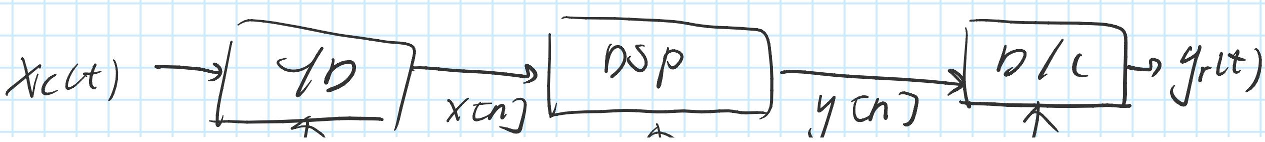
(b). $x_c(t) = \sin(10\pi t)/(10\pi t)$

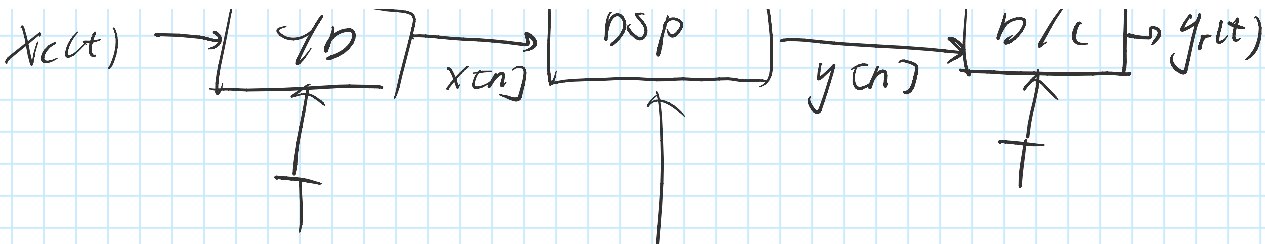
$x[n] = x_c(nT) = \sin(10\pi nT)/10\pi nT$

$= \sin(\pi n/2)/(\pi n/2)$

$\frac{\pi n}{2} = 10\pi nT \Rightarrow T = \frac{1}{20}$

4.12





$$H(e^{j\omega}) = j\omega/\underset{1/10}{T}, \quad \omega < |\pi|$$

(a). Check aliasing

(i). $x_c(t) = \cos(6\pi t)$

$$\Omega_N = 6\pi \qquad \Omega_s = \frac{2\pi}{T} = 20\pi$$

$2\Omega_N < \Omega_s \Rightarrow$ no aliasing

$$H_{eff}(e^{j\omega}) = \begin{cases} H(e^{j\omega}) & \text{for } |\Omega| < \frac{\pi}{T} \\ 0, & \text{otherwise} \end{cases}$$

$$Y_c(j\Omega) = H_{eff}(e^{j\Omega T}) \cdot X_c(j\Omega)$$

$$= H(e^{j\Omega T}) \cdot X_c(j\Omega)$$

$$= j \cdot \Omega T / T \cdot \underbrace{X_c(j\Omega)}_{\substack{\shortparallel \\ \text{F.T. of } (x_c(t))}}$$

$$= j\Omega X_c(j\Omega)$$

$$y_c(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} Y_c(j\Omega) \cdot e^{j\Omega t} d\Omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} j\Omega \cdot \left[\pi \delta(\Omega - \Omega_0) + \pi \delta(\Omega + \Omega_0)\right] \cdot e^{j\Omega t} d\Omega$$

$$e^{j\Omega t}\, d\Omega$$

$$= \frac{\pi}{2\pi}\left[j\Omega_0 (1)\cdot e^{j\Omega_0 t} + (-j\Omega_0)\cdot e^{-j\Omega_0 t} \right]$$

$$= \frac{j\Omega_0}{2}\left[e^{j\Omega_0 t} - e^{-j\Omega_0 t} \right]$$

$$= \frac{j\Omega_0}{2} \cdot 2j\sin(\Omega_0 t)$$

$$= -\Omega_0 \sin(\Omega_0 t)$$

$$= -6\pi \sin(6\pi t)$$

(ii). $\underline{x_c(t)} = \cos(14\pi t)$

$$\Omega_N = 14\pi \qquad \Omega_s = 20\pi$$

$$2\Omega_N > \Omega_s \Rightarrow \text{aliasing}$$

$$x[n] = x_c(nT) = \cos\left(14\pi n \frac{1}{10}\right)$$

$$= \cos\left(\frac{6\pi}{10} n\right)$$

$$= \cos\left(\frac{3}{5}\pi n\right)$$

$x[n]$ in (i):

$$x[n] = \cos(6\pi nT) = \cos\left(6\pi n \frac{1}{10}\right) = \cos\left(\frac{3}{5}\pi n\right)$$

$\Rightarrow$ $y_c(t)$ in (ii) is identical to $y_c(t)$ in (i). Which is $y_c(t) = -6\pi \sin(6\pi t)$

$y_c(t)$ in (i). which is $y(t)$ [illegible]

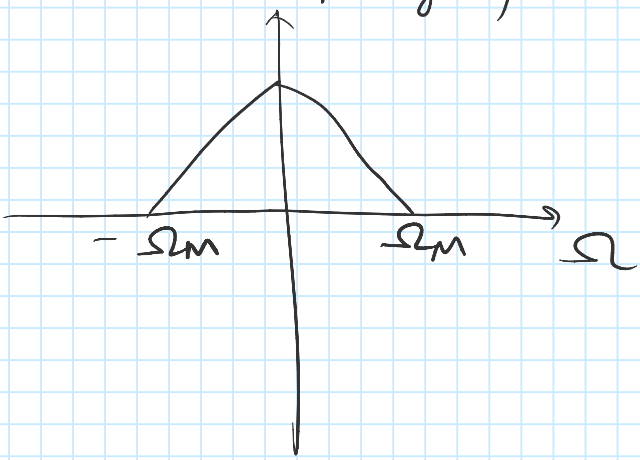
(b).
$$Y(e^{j\omega}) = X(e^{j\omega}) \cdot H(e^{j\omega})$$

$$\Rightarrow \quad y(t) = \frac{d}{dt} x(t)$$

$\Rightarrow$ True for (i)
Not true for (ii).

4.27.

$X_c(j\Omega)$



$$\Omega_s = \frac{2\pi}{T} = \frac{2\pi}{\pi} \times \Omega_M$$
$$= 2\Omega_M$$

$\Rightarrow$ no aliasing in sampling $x_c(t)$

$x_d[n] = x_c(nT)$

(a). Since there's no aliasing in the system

$$Y_c(j\Omega) = \underline{H_d(e^{j\Omega T})} \cdot X_c(j\Omega) \quad \text{when } \underline{\Omega < \frac{\pi}{T}}$$

$$\Rightarrow H_c(j\Omega) = H_d(e^{j\Omega T}) \quad \text{for } \Omega < \frac{\pi}{T}$$
$$= \frac{e^{j\Omega T/2} - e^{-j\Omega T/2}}{T}$$

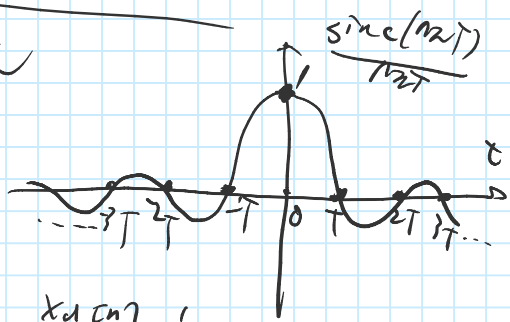
$$\overline{}$$
$$T$$

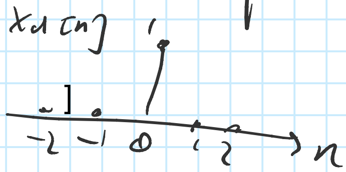
$$= \frac{2j\sin(\Omega T/2)}{T}$$

(b).

$$x_d[n] = x_c(nT) = \frac{\sin\left(\Omega_M \cdot n \cdot \frac{\pi}{\Omega_M}\right)}{n\pi}$$

$$= \frac{\sin(n\pi)}{n\pi} \Rightarrow$$





$$\Rightarrow \quad x_d[n] = \delta[n]$$

$$y_d[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} Y_d(e^{j\omega})e^{j\omega n}\,d\omega$$

$$Y_d(e^{j\omega}) = X_d(e^{j\omega})\cdot H_d(e^{j\omega})$$

$$= 1 \times \frac{2j \times \sin\left(\frac{\omega}{2}\right)}{T}$$

$$\Rightarrow \quad y_d[n] = \frac{2j}{2\pi T}\int_{-\pi}^{\pi} \sin\left(\frac{\omega}{2}\right)e^{j\omega n}\,d\omega$$

$$\vdots$$

$$= \frac{2\times(-1)^n \times n}{\pi T\left(n^2 - \frac{1}{4}\right)}$$

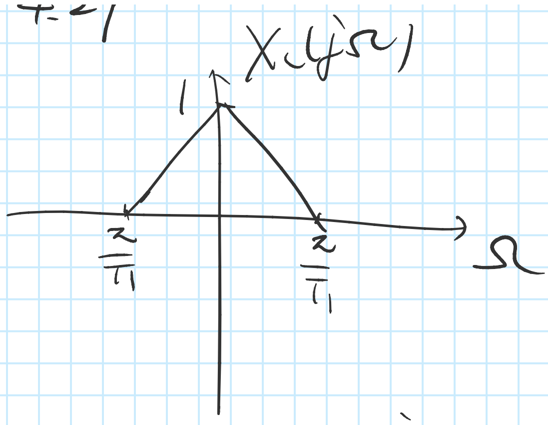
$$y_c(t) = y_d\left[\frac{t}{T}\right] = \frac{2\times(-1)^{\frac{t}{T}} \times t}{\pi T^2\left(\frac{t^2}{T^2} - \frac{1}{4}\right)}$$

$$t = nT$$

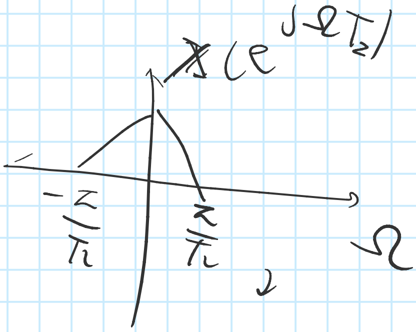
4.29

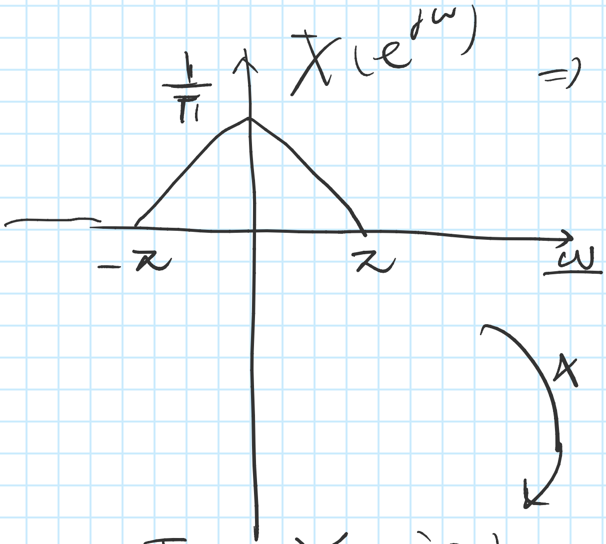
$X_c(j\Omega)$

T=2)



$$\Omega_s = \frac{2\pi}{T_1}$$





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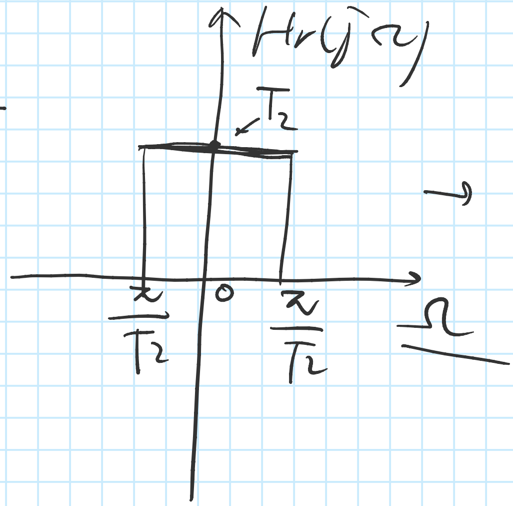
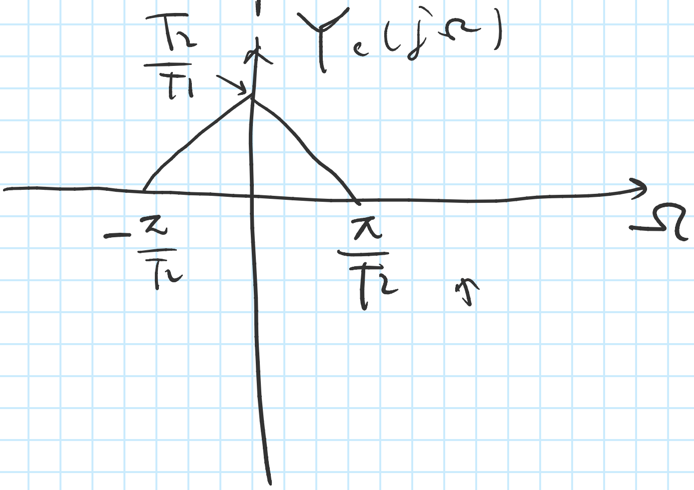


→ t-domain sinc func.



$$Y_c(j\Omega) = \frac{T_2}{T_1} X_c(j \cdot k \cdot \Omega)$$

⇒ find k.

when $k \cdot \Omega = \frac{\pi}{T_1}$. then Ω in $Y_c(j\Omega)$

$$\Omega = \frac{\pi}{T_2}$$

$$\Rightarrow \quad k = \frac{\pi}{T_1} \cdot \frac{T_2}{\pi} = \frac{T_2}{T_1}$$

$$\Rightarrow \quad k = \frac{[illegible]}{T_1} \quad a = \frac{T_2}{T_1}$$

$$\Rightarrow \quad y_c(t) = \frac{T_2}{T_1} x_c\left(\frac{T_2}{T_1} t\right).$$