

ESE 531 Recitation 4

4.41, 4.46, 4.59, 4.62

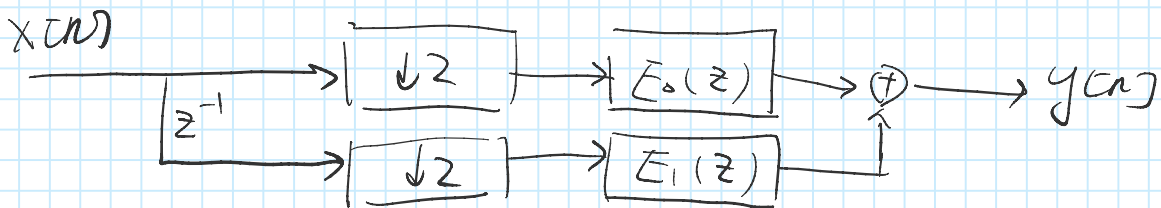
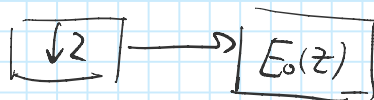
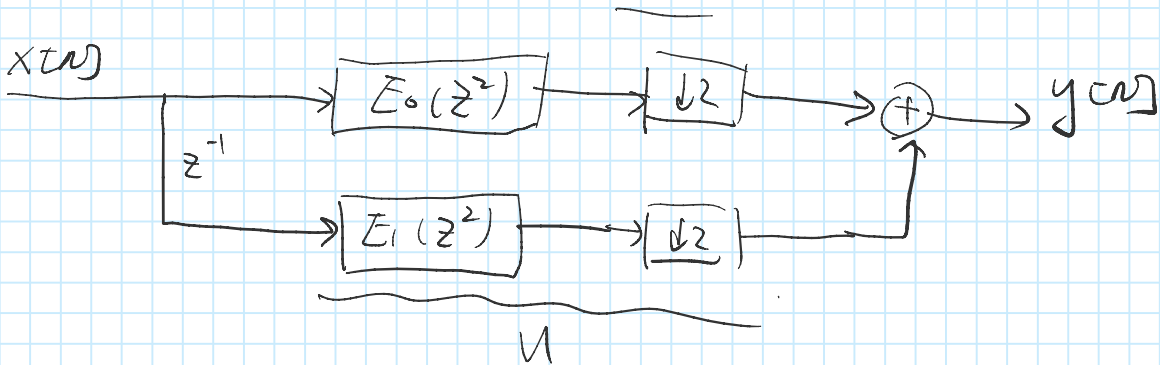
4.41

h[n]

(a). From eq. (4.107), $e_k = \underline{h[nM+k]}$ k is from $0, 1, \dots, M-1$

$$M=2 \Rightarrow \begin{cases} e_0[n] = h[2n], & 0 \leq n \leq 5 \\ e_1[n] = h[2n+1], & 0 \leq n \leq 5 \end{cases}$$

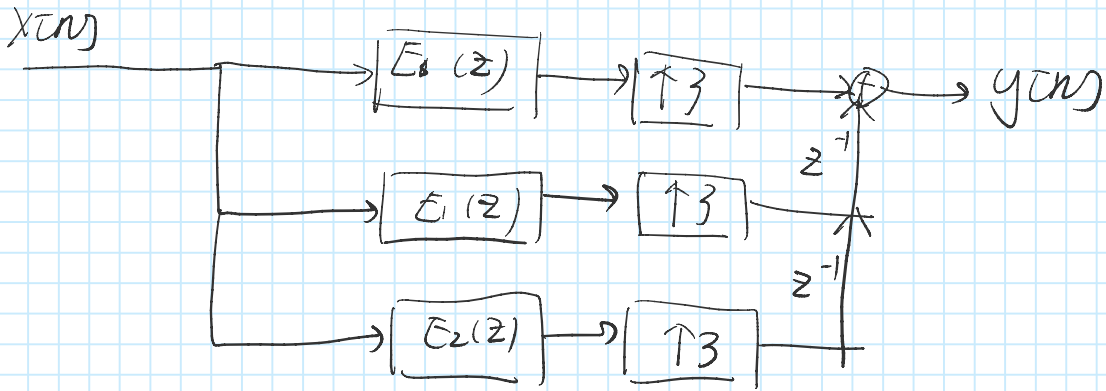
$$H(z) = \sum_{k=0}^1 E_k(z^2) z^{-k} \\ = E_0(z^2) + E_1(z^2) \cdot \underline{z^{-1}}$$



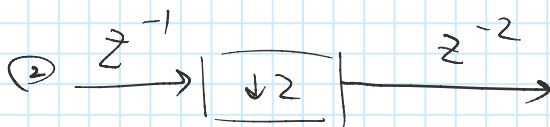
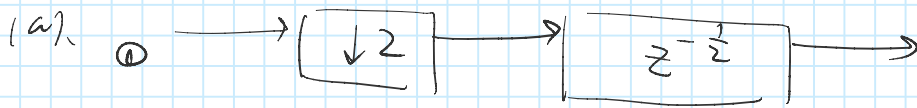
(b). Similarly, from $e_k[n] = h[nM+k]$

$$\Rightarrow \begin{cases} e_0[n] = h[3n] \\ e_1[n] = h[3n+1], & 0 \leq n \leq 3 \\ e_2[n] = h[3n+2] \end{cases}$$

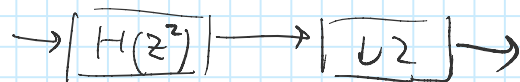
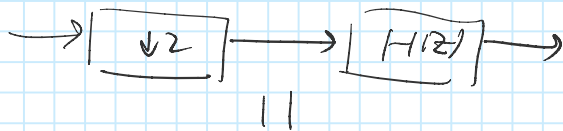
$$| e_2(n) = n \lfloor 5n+2 \rfloor$$



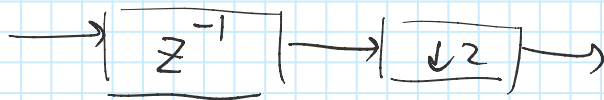
4.46



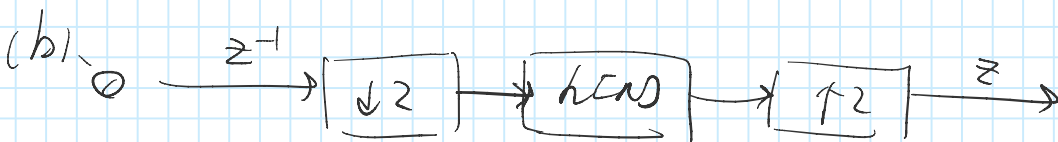
①: from interchangeable identity:

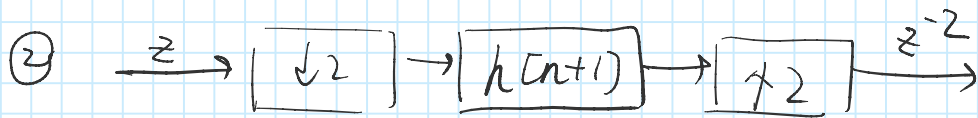
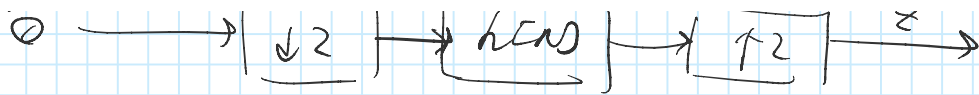


② becomes:



① & ③ are not identical.





Input $x[n]$

$$X_d(e^{j\omega}) = \frac{1}{M} \sum_{i=0}^{M-1} X(e^{j(\frac{\omega}{M} - \frac{2\pi i}{M})})$$

①: $X(e^{j\omega}) \xrightarrow{z^{-1}} X(e^{j\omega}) e^{-j\omega} \xrightarrow{\downarrow 2} \Rightarrow$

$$\frac{1}{2} \left\{ X(e^{j\frac{\omega}{2}}) e^{-j\frac{\omega}{2}} + X(e^{j(\frac{\omega}{2} - \pi)}) e^{j(\frac{\omega}{2} - \pi)} \right\}$$

$\downarrow h[n]$

$$e^{-j\frac{\omega}{2}} \times \frac{1}{2} \times H(e^{j\omega}) \times \left\{ X(e^{j\frac{\omega}{2}}) - X(e^{j(\frac{\omega}{2} - \pi)}) \right\}$$

$\downarrow \uparrow 2$

$$e^{-j\omega} \times \frac{1}{2} \times H(e^{j2\omega}) \times \left\{ X(e^{j\omega}) - X(e^{j\omega - j\pi}) \right\}$$

$$e^{j\omega} \times e^{-j\omega} \times \frac{1}{2} \times H(e^{j2\omega}) \times \left\{ \right\}$$

$$\text{①: } \frac{1}{2} H(e^{j2\omega}) \left\{ X(e^{j\omega}) - X(e^{j\omega - j\pi}) \right\}$$

② $X(e^{j\omega}) \xrightarrow{z} X(e^{j\omega}) e^{j\omega}$

$$\textcircled{2} \quad X(e^{j\omega}) \xrightarrow{z} X(e^{j\omega}) e^{j\omega} \quad \textcircled{1}$$

$$\frac{1}{z} \left\{ X(e^{j(\frac{\omega}{z})}) e^{j\frac{\omega}{z}} + X(e^{j(\frac{\omega}{z}-2\pi)}) e^{j(\frac{\omega}{z}-2\pi)} \right\}$$

\downarrow $h[n+1]$ \downarrow A

$$H(e^{j\omega}) e^{j\omega} \times A$$

$$e^{j\omega} \times H(e^{j2\omega}) e^{j2\omega} \times \frac{1}{z} \left\{ X(e^{j\omega}) - X(e^{j\omega-j2\pi}) \right\}$$

\downarrow z^{-2} \downarrow B

$$e^{-j2\omega} \times B$$

\Downarrow

$$\textcircled{2}: e^{j\omega} \times H(e^{j2\omega}) \times \frac{1}{z} \left\{ X(e^{j\omega}) - X(e^{j\omega-j2\pi}) \right\}$$

(II)

Compare (I) with (II)

$$(II) = (I) \times e^{j\omega}$$

① & ② are not identical

(c)



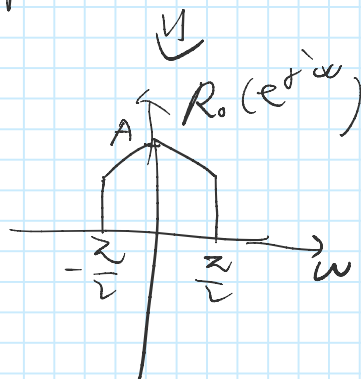
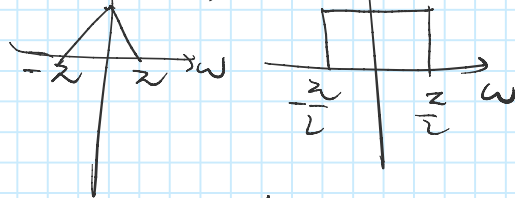
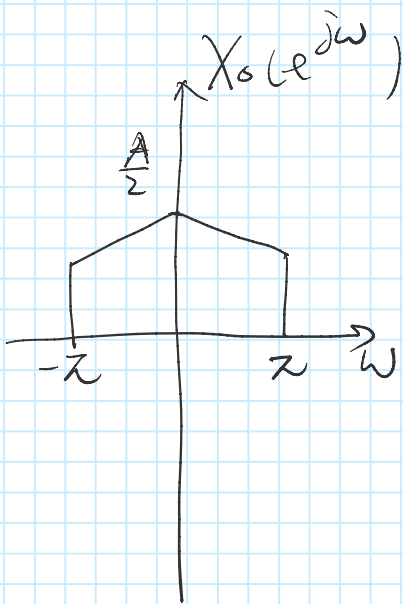
①: $X(e^{j\omega}) \xrightarrow{\uparrow L} X(e^{jL\omega}) \xrightarrow{A} \underline{\underline{[X(e^{jL\omega})]^L}}$

②: $X(e^{j\omega}) \xrightarrow{A} \underline{\underline{[X(e^{j\omega})]^L}} \xrightarrow{\uparrow L} \underline{\underline{[X(e^{jL\omega})]^L}}$

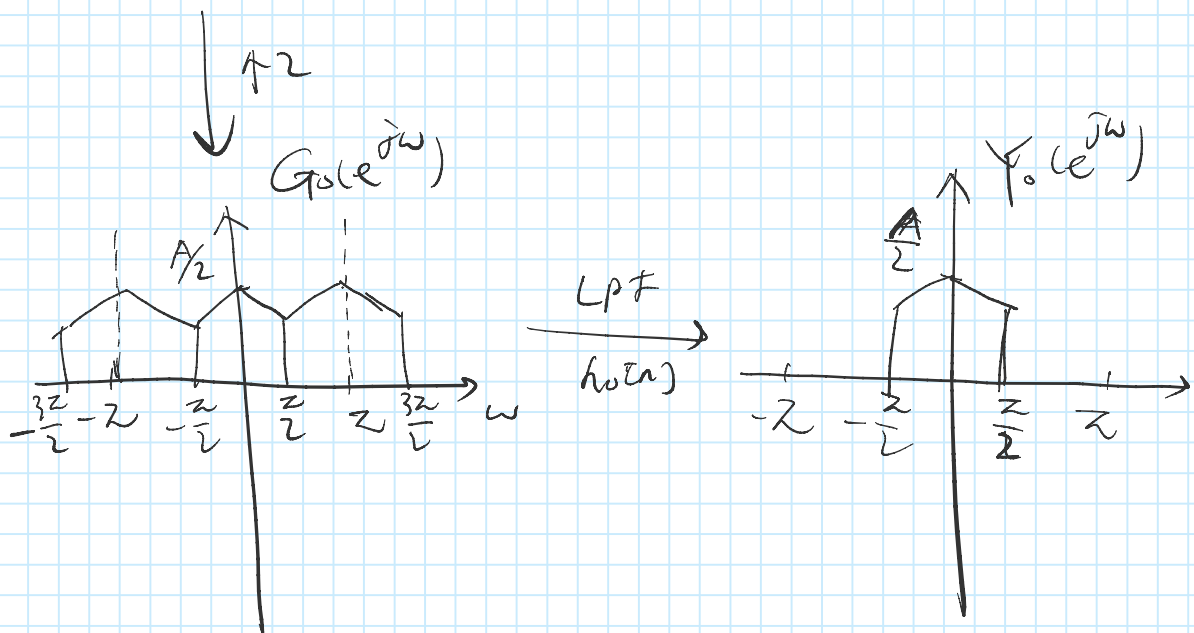
① & ② are identical

4.59

(a) $X_0(e^{j\omega}) = \left[\underbrace{X(e^{j\omega})}_{\substack{\uparrow A \\ \text{triangle}}} \cdot \underbrace{H_0(e^{j\omega})}_{\substack{\uparrow \text{rect}}} \right] \downarrow \times 2$



$\times 2$



(b).

$$G_0(e^{j\omega}) = X(e^{j\omega}) \cdot H_0(e^{j\omega})$$

$$\frac{1}{2} \left[X(e^{j\frac{\omega}{2}}) H_0(e^{j\frac{\omega}{2}}) + X(e^{j(\frac{\omega}{2}-z)}) H_0(e^{j(\frac{\omega}{2}-z)}) \right]$$

$$\frac{1}{2} \left[X(e^{j\omega}) H_0(e^{j\omega}) + X(e^{j\omega-jz}) H_0(e^{j\omega-jz}) \right]$$

$$G_0(e^{j\omega}) = \frac{1}{2} \left[X(e^{j\omega}) H_0(e^{j\omega}) + X(e^{j\omega-jz}) H_1(e^{j\omega}) \right]$$

(c).

$$Y(e^{j\omega}) = Y_0(e^{j\omega}) - Y_1(e^{j\omega})$$

TW.

$$Y_0(e^{j\omega}) = G_0(e^{j\omega}) \cdot H_0(e^{j\omega})$$

↓ from part (b)

$$= \frac{1}{2} \left\{ X(e^{j\omega}) H_0^2(e^{j\omega}) + X(e^{j\omega-jz}) H_0(e^{j\omega}) H_1(e^{j\omega}) \right\}$$

Similarly:

$$Y_1(e^{j\omega}) = G_1(e^{j\omega}) \cdot H_1(e^{j\omega})$$

$$= \frac{1}{2} \left\{ X(e^{j\omega}) H_1^2(e^{j\omega}) + X(e^{j\omega-jz}) H_0(e^{j\omega}) H_1(e^{j\omega}) \right\}$$

$$Y(e^{j\omega}) = Y_0(e^{j\omega}) - Y_1(e^{j\omega})$$

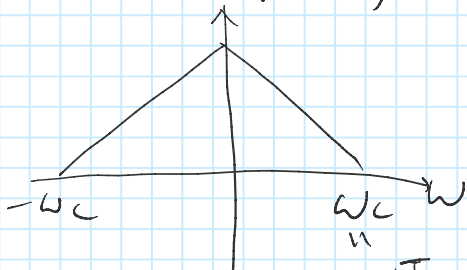
$$= \frac{1}{2} X(e^{j\omega}) \left\{ H_0^2(e^{j\omega}) - H_1^2(e^{j\omega}) \right\}$$

$$H_0^2(e^{j\omega}) - H_1^2(e^{j\omega}) = \text{Constant}$$

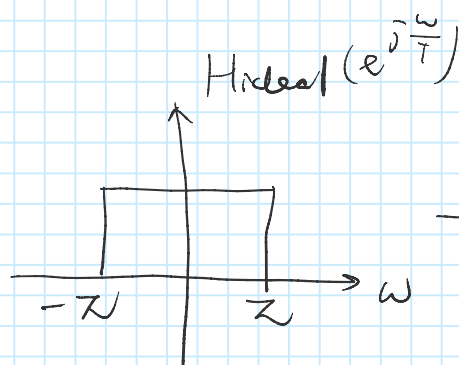
$$|Y(e^{j\omega})| \propto |X(e^{j\omega})|$$

4.62

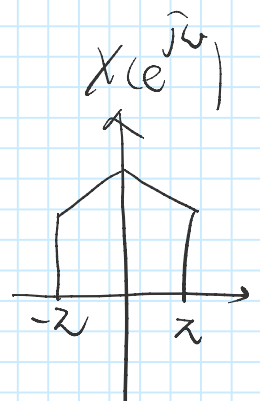
For system 1:
 $X_c(e^{j\omega})$

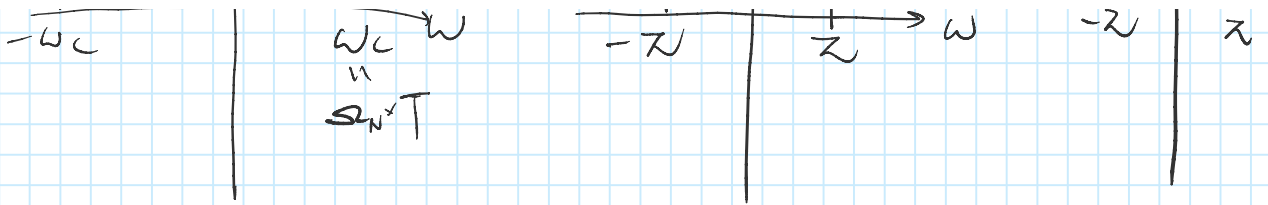


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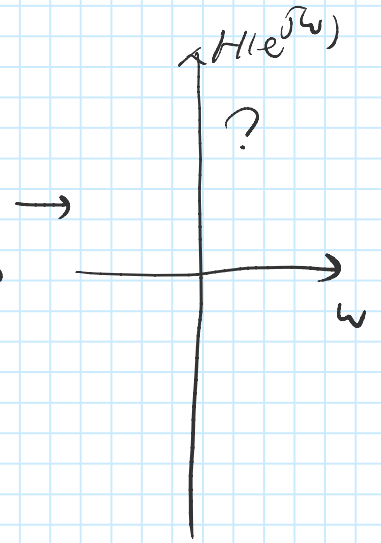
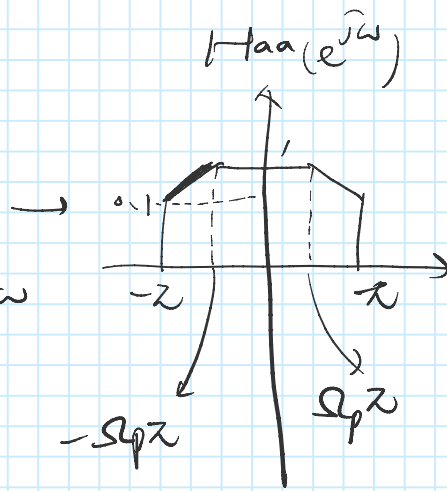
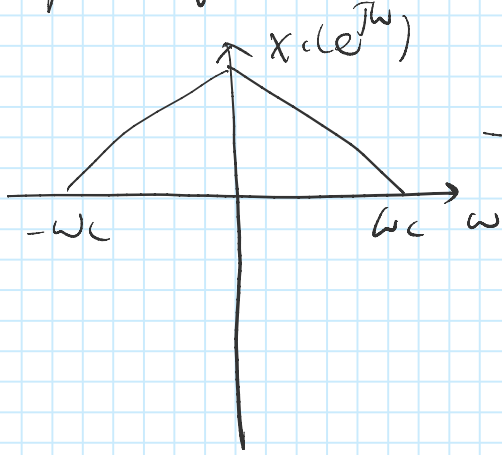


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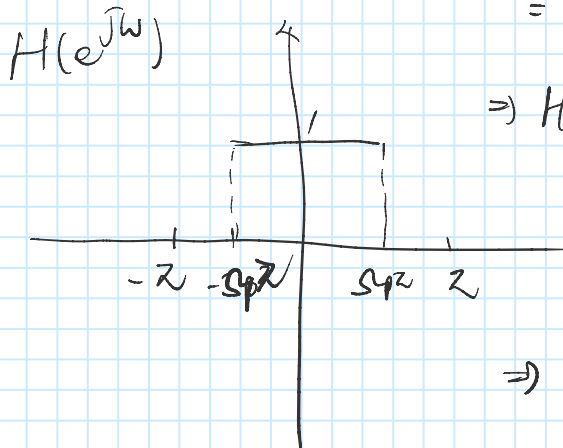


for system z:

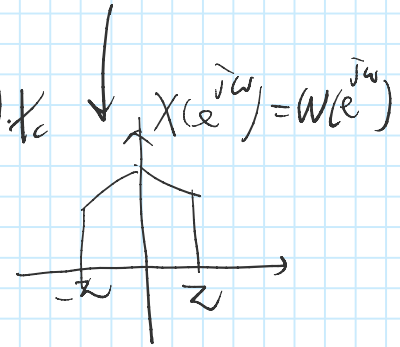


$$X(e^{j\omega}) = H_{ideal}(e^{j\omega}) \cdot H(e^{j\omega}) \cdot X_c \quad \downarrow \quad X(e^{j\omega}) = W(e^{j\omega})$$

$$= H_{ideal} \times X_c$$



$$\Rightarrow H_{ideal} \times H = H_{ideal}$$



$$\Rightarrow H_{aa}(e^{j\omega}) = a\omega + b$$

$$\Rightarrow H(e^{j\omega}) = \frac{1}{a\omega + b}$$

hyperbola,
when $\Omega_p z \leq |\omega| \leq z$

