ESE 531 Review Session

Midterm 2020, Problem 2, 3, 4

2.

(a). Prove linearity:

For a linear system:

$$a y_1[n] + b y_2[n] = T\{a x_1[n] + b x_2[n]\}$$

Assume $x'[n] = a x_1[n] + b x_2[n] \quad \cdots ①$

$$\begin{aligned} y'[n] &= x'[n] - \tfrac{1}{2}x'[n-1] \\ &= a x_1[n] + b x_2[n] - \tfrac{1}{2}a x_1[n-1] - \tfrac{1}{2}b x_2[n-1] \\ &= a\{x_1[n] - \tfrac{1}{2}x_1[n-1]\} + b\{x_2[n] - \tfrac{1}{2}x_2[n-1]\} \\ &= a y_1[n] + b y_2[n] \quad \cdots ② \end{aligned}$$

Compare ① & ② $\Rightarrow$ this system is linear.

Prove time-invariance:

$$x'[n] = x[n-n_0] \quad \cdots ③$$

$$\begin{aligned} y'[n] &= x'[n] - \tfrac{1}{2}x'[n-1] \\ &= x[n-n_0] - \tfrac{1}{2}x[n-n_0-1] \\ &= y[n-n_0] \quad \cdots ④ \end{aligned}$$

Compare ③ & ④ $\Rightarrow$ this system is TI.

(b).

$y[n] = x[n] - \frac{1}{2}x[n-1]$, which is in the form of a difference eqn. --- 5

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{m=0}^{k} b_m x[n-m] \quad \text{---} \; 6$$

Compere 5 & 6 :

$$\Rightarrow \begin{cases} a_0 = 1 \end{cases} \qquad \begin{cases} b_0 = 1 \\ b_1 = -\frac{1}{2} \end{cases}$$

$$\Rightarrow H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}} = \frac{1 - \frac{1}{2}z^{-1}}{1}$$

$$= 1 - \frac{1}{2}z^{-1}$$

$$\Rightarrow Z^{-1}\{H(z)\} = h[n]$$

$$\begin{cases} 1 \xrightarrow{Z^{-1}} \delta[n] \\ z^{-m} \xrightarrow{Z^{-1}} \delta[n-m] \end{cases} \Rightarrow h[n] = \delta[n] - \frac{1}{2}\delta[n-1]$$

(c). From part (b)

$$H(z) = 1 - \frac{1}{2}z^{-1}$$

$$\Rightarrow H(e^{j\omega}) = 1 - \frac{1}{2}e^{-j\omega}$$

(d).

$$x[n] \longrightarrow \boxed{h[n]} \xrightarrow{y[n]} \boxed{hi[n]} \longrightarrow x[n]$$

In freq. domain:

$$Y(e^{jw}) = X(e^{jw}) \times H(e^{jw})$$

$$X(e^{jw}) = Y(e^{jw}) \times Hi(e^{jw})$$

$$X(e^{jw}) = X(e^{jw}) \times H(e^{jw}) \times Hi(e^{jw})$$

$$\Rightarrow \quad H(e^{jw}) \times Hi(e^{jw}) = 1$$

$$\Rightarrow \quad H(e^{jw}) = 1 - \frac{1}{2} e^{-jw}$$

$$Hi(e^{jw}) = \frac{1}{1 - \frac{1}{2} e^{-jw}}$$

$$\Rightarrow \quad \frac{1}{1 - az^{-1}} \Rightarrow \begin{cases} \text{left-sided} & u[-n] \\ \text{right-sided} & u[n] \end{cases}$$

$\Rightarrow$ ROC includes unit circle

$\Rightarrow$ pole at $z = \frac{1}{2}$

$\Rightarrow$ right-sided

$h_i[n] = (\frac{1}{2})^n u[n]$

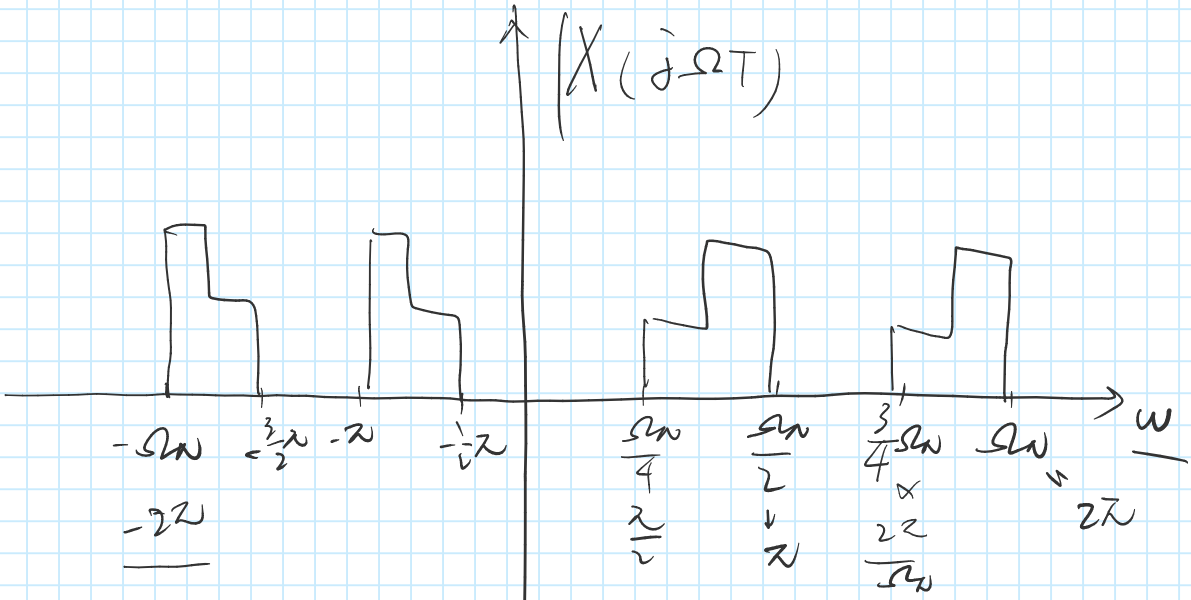
3. Check aliasing:

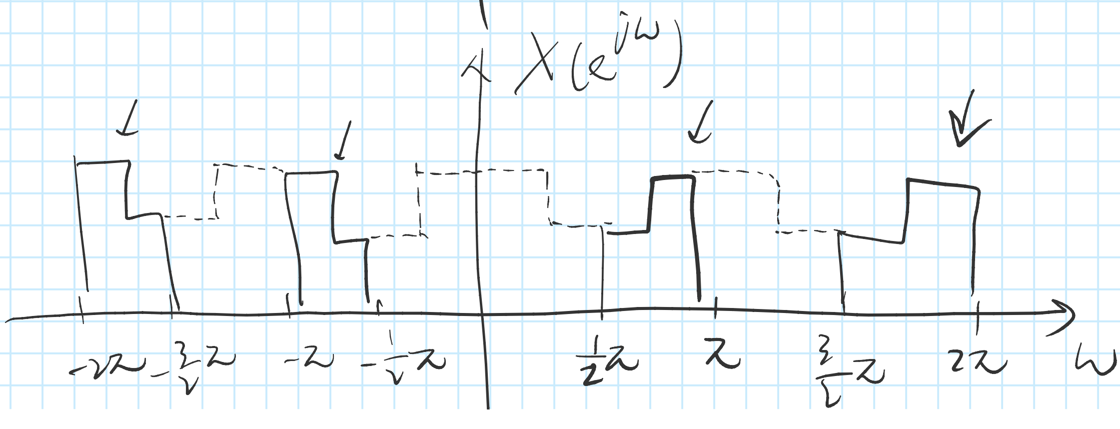
$\Omega_N \to$ sample at $2\Omega_N$

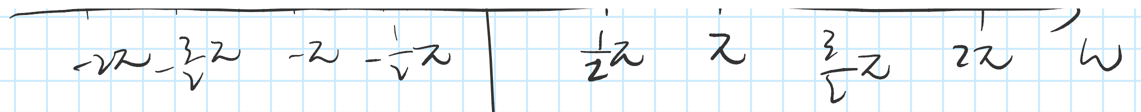
now: only sample at Ω_N

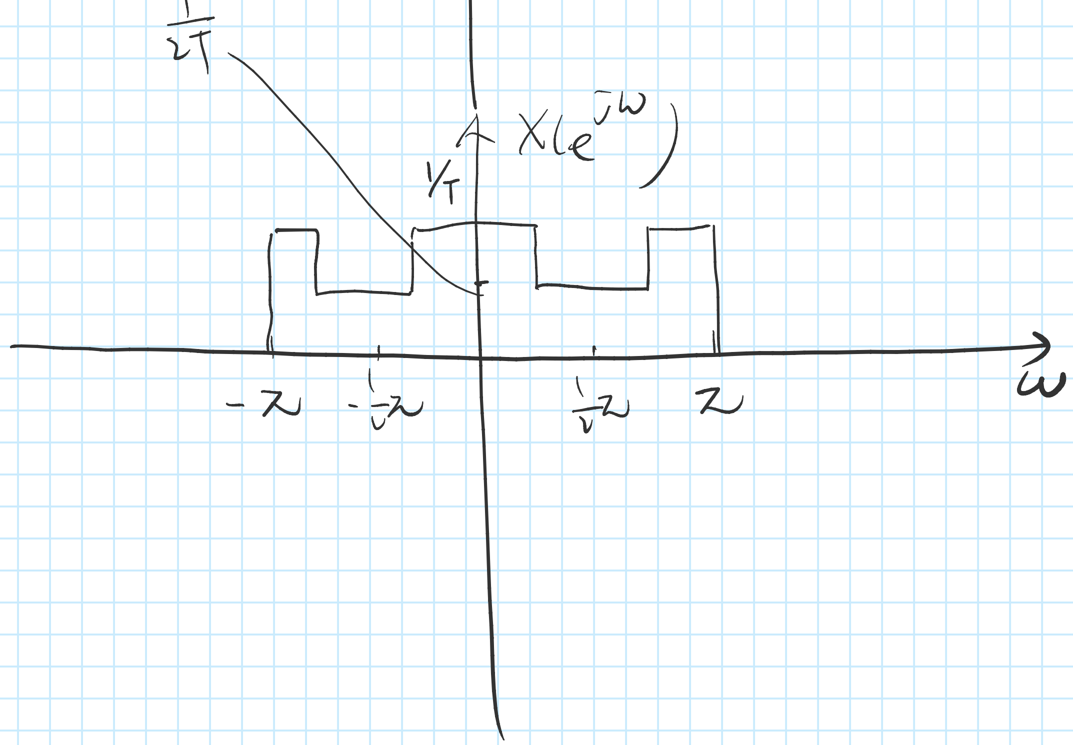
(a). Know $|X(j\Omega)|$

$\Rightarrow \underline{w} = \Omega T = \Omega \times \frac{2\pi}{\Omega_s} = \underline{\Omega \times \frac{2\pi}{\Omega_N}}$





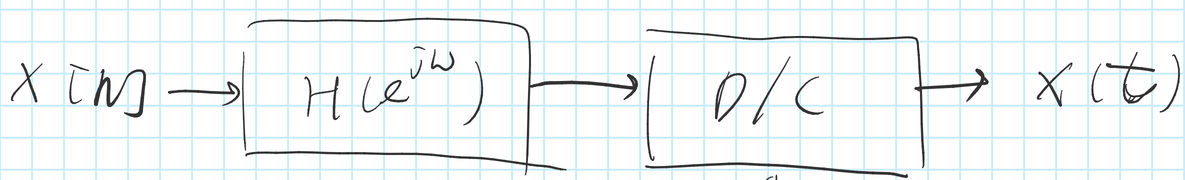


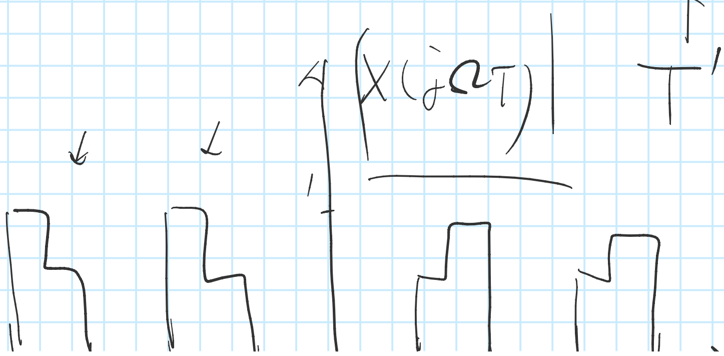


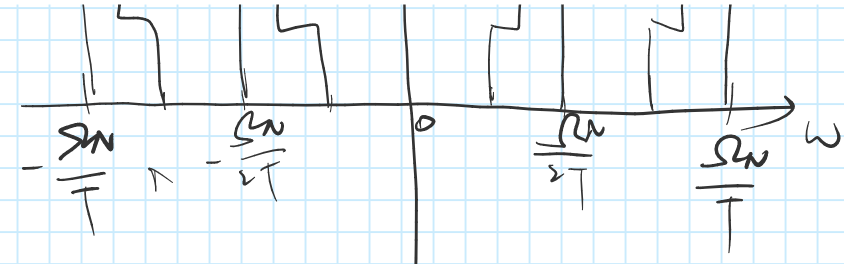
(b). From part (a): No.

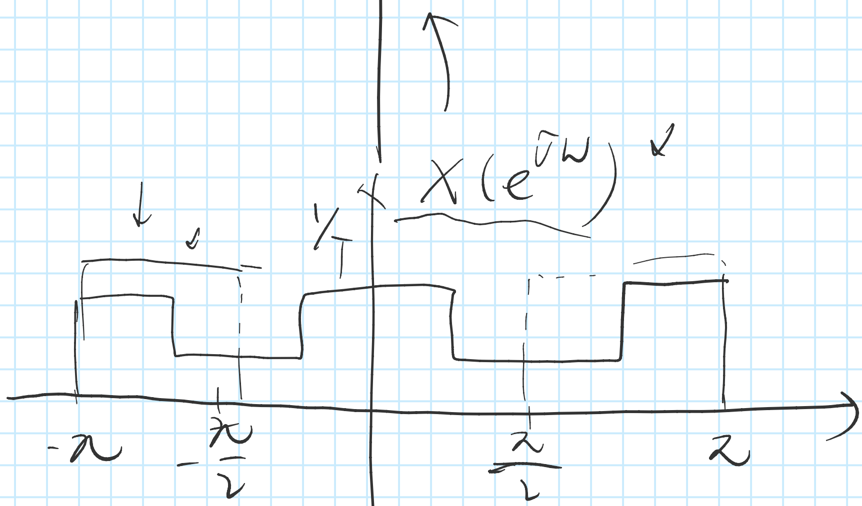
Need to sample at $2\,\Omega_N$, now we only sample at Ω_N

(c)

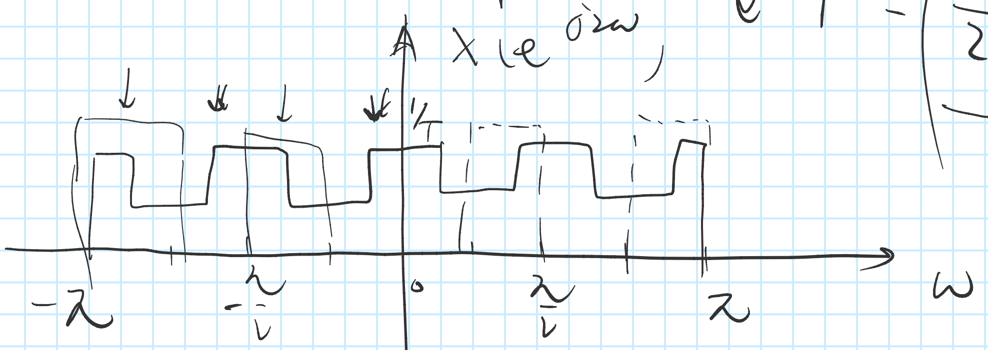




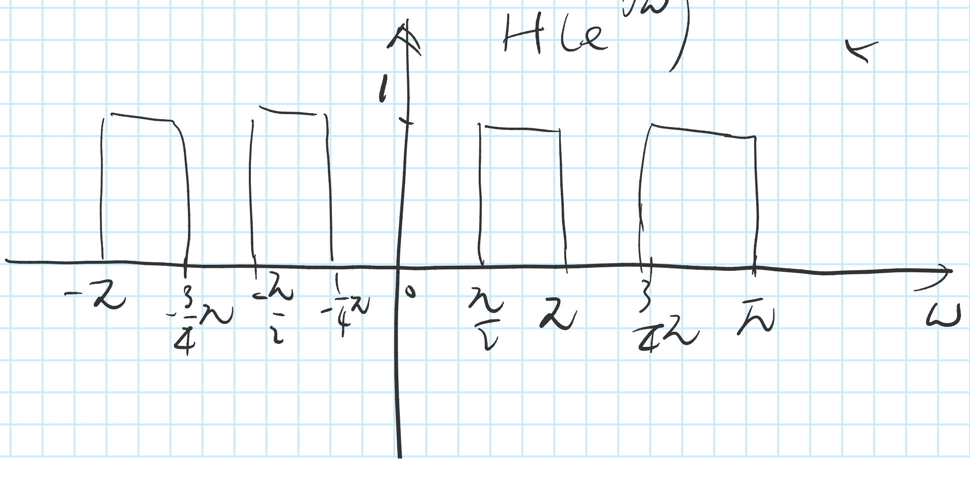


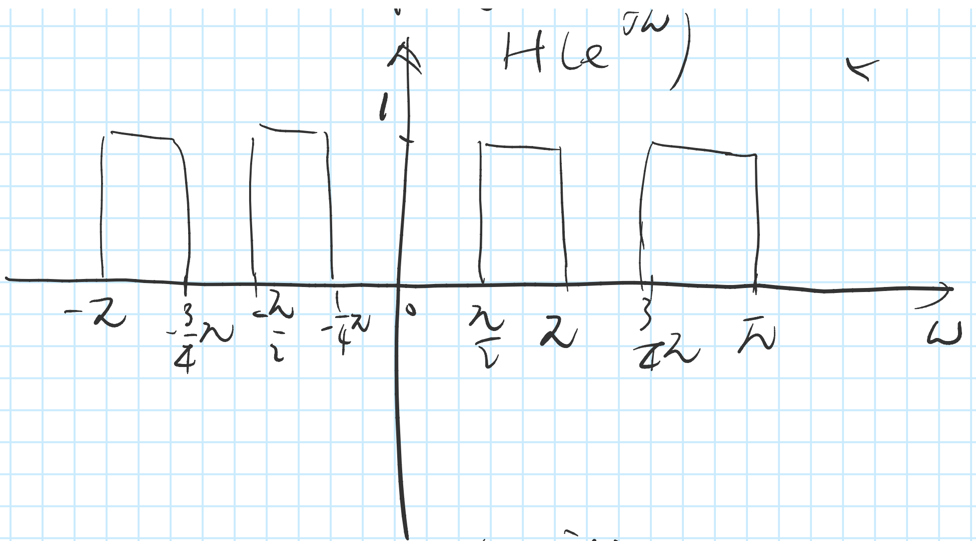


$\uparrow 2$ $\quad\downarrow$ $T' = \frac{T}{2}$

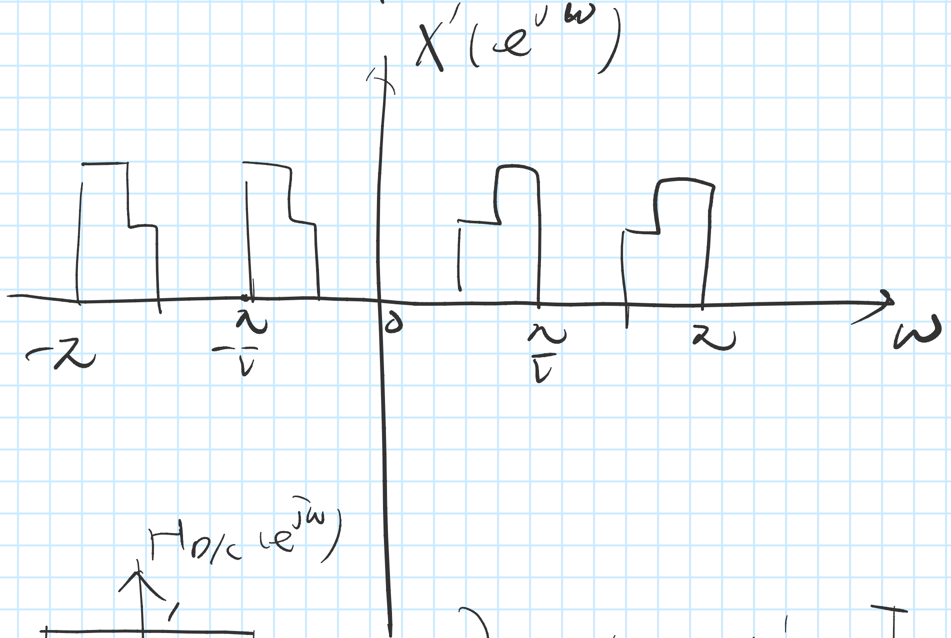


$\downarrow$ band-pass-filtering



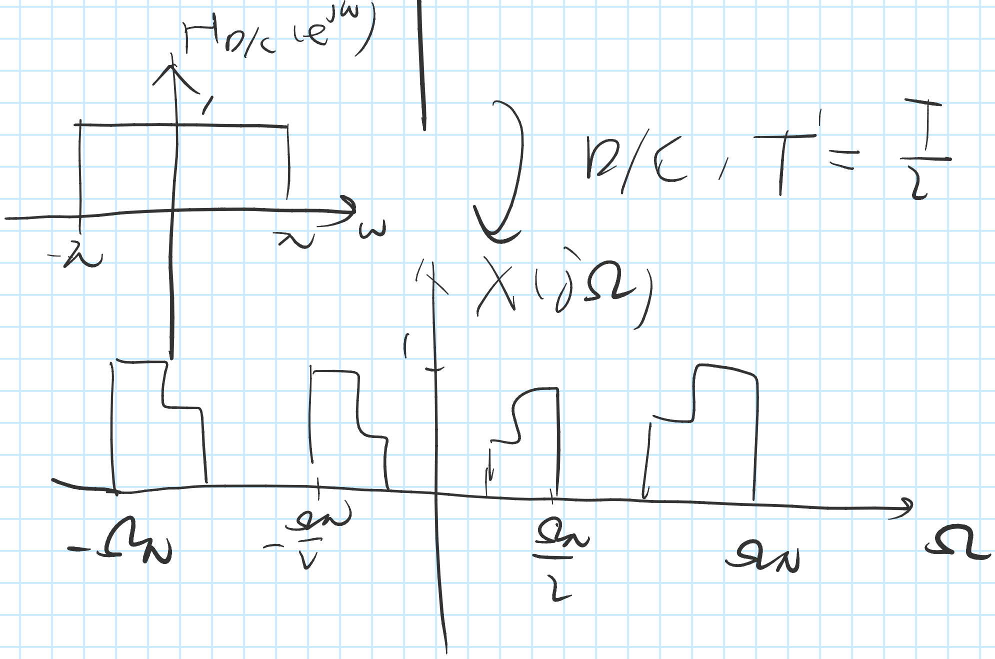


H(e^jω)
1
−π
−3/4 π
−π/2
−1/4 π
0
π/2
π
3/4 π
π
ω

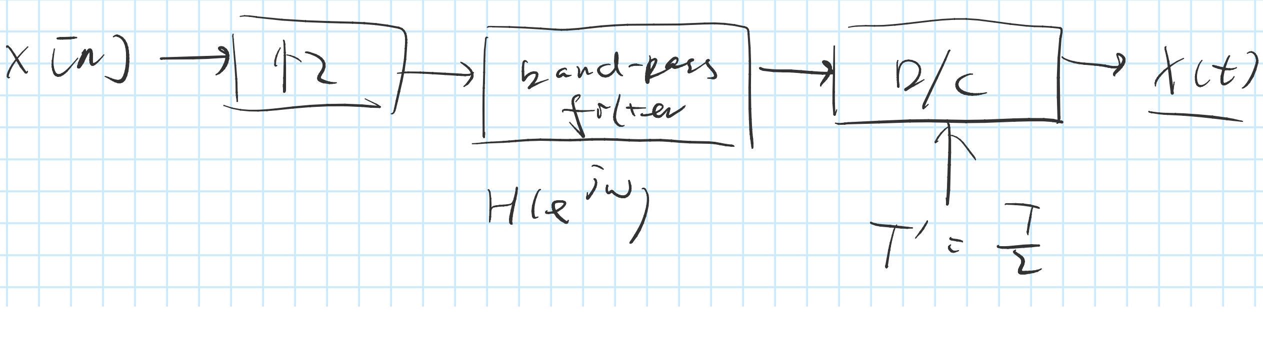


X'(e^jω)
−π
−π/2
0
π/2
π
ω

D/C, $T' = \frac{T}{2}$

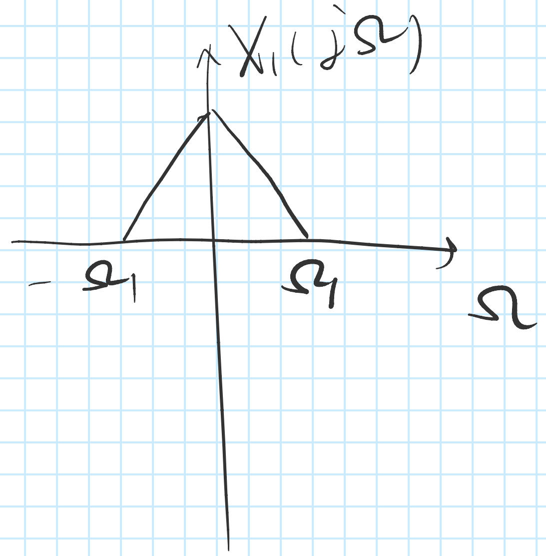


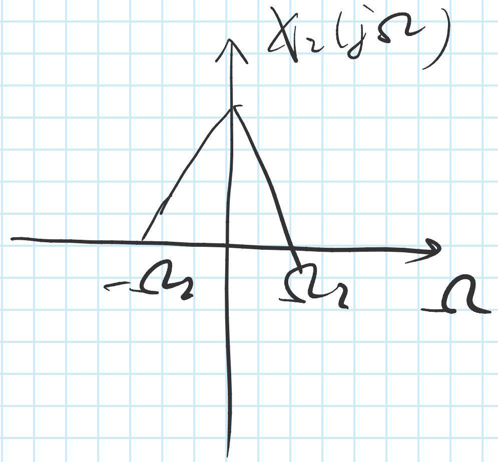
H_D/C(e^jω)
1
−π
π
ω
X(jΩ)
1
−Ω_N
−Ω_N/2
Ω_N/2
Ω_N
Ω



x[n]
↑2
band-pass filter
H(e^jω)
D/C
T' = T/2
x(t)

4.





$\Omega_1 = 3\times10^4\pi$ $\qquad$ $\Omega_2 = 0.5\times10^4\Omega$

$$y_c(t) = x_1(t)\times x_2(t)$$

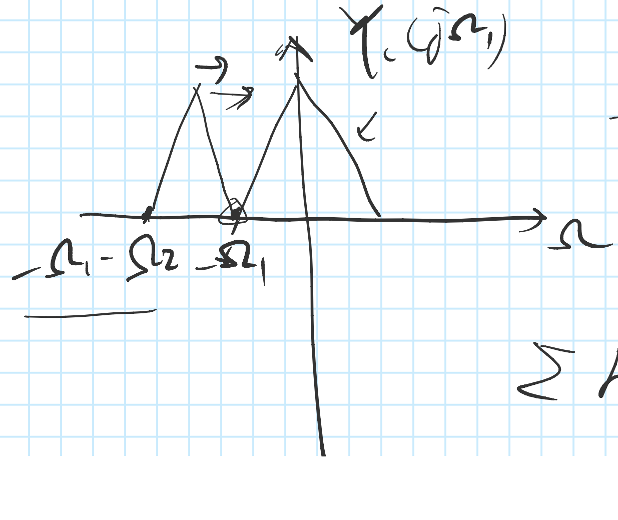
$$\underline{y_1[n]} = y_c(nT)$$

First find out what's $y_c(t)$, what Ω_{max} in $y_c(t)$?

$$y_c(t) = x_1(t)\times x_2(t)$$

$$\Rightarrow Y_c(j\Omega) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(e^{j\theta})X_2(e^{j(\omega-\theta)})\,d\theta$$

<u>Convolution</u> in freq. domain.



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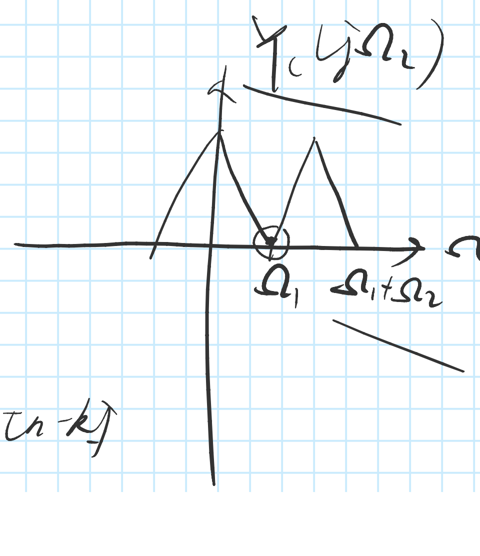

$$\sum h[k]\,x[n-k]$$

$$\left| \sum h[k]\, x[n-k] \right|$$

$\Rightarrow \quad \Omega_{max} = \Omega_1 + \Omega_2$

$\Rightarrow$ Sample at Nyquist.

$$\Omega_s = 2(\Omega_{max}) = 2(\Omega_1 + \Omega_2)$$

$$= 2(5\pi \times 10^4 + 0.5\pi \times 10^4)$$

$$\Rightarrow \quad T = \frac{2\pi}{\Omega_s} = \frac{2\pi}{2\pi(5.5 \times 10^4)} = \frac{1}{5.5 \times 10^4} \text{ sec}$$

(b)

$X_1[n] = X_1(nT_1) \xrightarrow{A} k_1 X_1(nT)$

$X_2[n] = X_2(nT_2) \xrightarrow{B} k_2 X_2(nT)$

C_1

$y_2[n] = k_1 k_2 X_1(nT) X_2(nT)$

$\boxed{= k\, X_1(nT)\, X_2(nT)}$ C_2

$\Rightarrow$ Sampling period in block C_1 & C_2 are different.

$\Rightarrow$ In blocks A and B, need to resample so that $T_1 \to T$, $T_2 \to T$

Recall resampling:

Recall resampling:

$$\underset{T}{x[n]} \xrightarrow[\frac{T}{L}]{\uparrow L} \boxed{\begin{matrix}\text{LPF,}\\ \omega_c = \max\left\{\frac{\pi}{L}, \frac{\pi}{M}\right\}\end{matrix}} \xrightarrow[\frac{T}{L}]{} \boxed{\downarrow M} \xrightarrow[\frac{TM}{L}]{} x'[n]$$

$\Rightarrow$ after resampling $T \rightarrow \frac{TM}{L}$

for $x_1(t)$, $T_1 \rightarrow T$

$$T = \frac{M_1 T_1}{L_1}$$

$$\Rightarrow \frac{M_1}{L_1} = \frac{T}{T_1} = \frac{1/55000}{2\times 10^{-5}} = \underline{\frac{10}{11}}$$

for $x_2(t)$, $T_2 \rightarrow T$

$$\Rightarrow \frac{M_2}{L_2} = \frac{T}{T_2} = \frac{1/55000}{2\times 10^{-4}} = \frac{1}{11}$$

$$x_1[n] \rightarrow \boxed{\uparrow 11} \rightarrow \boxed{\begin{matrix}\text{LPF}\\ \omega_c = \frac{\pi}{11}\end{matrix}} \rightarrow \boxed{\downarrow 10} \rightarrow w_1[n]$$

$$x_2[n] \rightarrow \boxed{\uparrow 11} \rightarrow \boxed{\begin{matrix}\text{LPF}\\ \omega_c = \frac{\pi}{11}\end{matrix}} \rightarrow w_2[n]$$

$$\Rightarrow y_2[n] = w_1[n] \times w_2[n] = k\, x_1[n]\, x_2[n]$$