ESE 531 Recitation 6

5.4, 5.7, 5.11

5.4

(a). $H(z) = \dfrac{Y(z)}{X(z)}$

$$X(z) = \mathcal{Z}\{x[n]\} = \underbrace{\frac{1}{1-\frac{1}{2}z^{-1}}}_{|z|>\frac{1}{2}} - \underbrace{\frac{1}{1-2z^{-1}}}_{|z|<2}$$

$$Y(z) = \mathcal{Z}\{y[n]\} = \underbrace{\frac{6}{1-\frac{1}{2}z^{-1}}}_{|z|>\frac{1}{2}} - \underbrace{\frac{6}{1-\frac{3}{4}z^{-1}}}_{|z|>\frac{3}{4}}$$

$$H(z) = \frac{Y(z)}{X(z)} = 6\,\frac{1-\frac{3}{4}z^{-1} - 1 + \frac{1}{2}z^{-1}}{1-2z^{-1} - 1 + \frac{1}{2}z^{-1}} \times \frac{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}{(1-\frac{1}{2}z^{-1})(1-\frac{3}{4}z^{-1})}$$

$$= 6 \times \frac{-\frac{1}{4}z^{-1}}{-\frac{3}{2}z^{-1}} \times \frac{1+2z^{-1}}{1-\frac{3}{4}z^{-1}}$$

$$= \frac{1-2z^{-1}}{1-\frac{3}{4}z^{-1}}$$

zeroes: $z = 2$, poles: $z = \frac{3}{4}$

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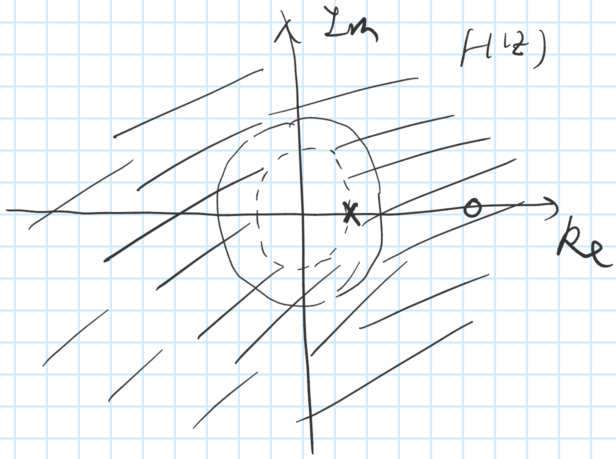
ROC of $H(z)$: $|z| > \frac{3}{4}$



(b). From part $H(z) = \dfrac{1 - 2z^{-1}}{1 - \frac{3}{4} z^{-1}}$, $|z| > \frac{3}{4}$

Rewrite $H(z)$:

$$H(z) = \frac{1}{1 - \frac{3}{4} z^{-1}} - 2 \frac{z^{-1}}{1 - \frac{3}{4} z^{-1}}$$

$\Downarrow$ From table 3.1

$$h[n] = Z^{-1}\left\{ H(z) \right\} = \left(\frac{3}{4}\right)^n u[n] - 2 \times \left(\frac{3}{4}\right)^{n-1} u[n-1]$$

(c).

$$H(z) = \frac{\sum_{k=0}^{M} \underline{b_k} \, z^{-k}}{\sum_{k=0}^{N} \underline{a_k} \, z^{-k}}$$

$\Rightarrow \sum^{N} \ldots = \sum^{M} \ldots [n-k] \ldots$

$$\Rightarrow \sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k] \quad \cdots \quad ①$$

From part (a.) $H(z) = \dfrac{1-2z^{-1}}{1-\frac{3}{4}z^{-1}}$

$$\Rightarrow \begin{cases} a_0 = 1 \\ a_1 = -\frac{3}{4} \end{cases} \qquad \begin{cases} b_0 = 1 \\ b_1 = -2 \end{cases}$$

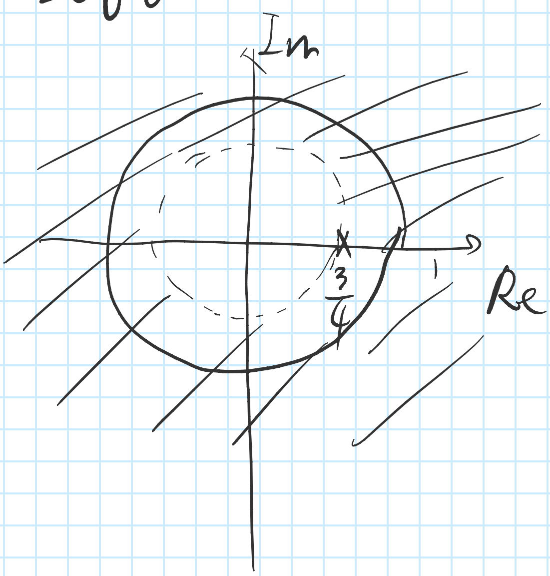
plug into Eqn. ①:

$$y[n] - \frac{3}{4} y[n-1] = x[n] - 2x[n-1]$$

(d).

For a stable system: unit circle is included in ROC

For a causal system: h[n] right-sided sequence



$\Rightarrow$ Stable ✓

causal ✓

3.7

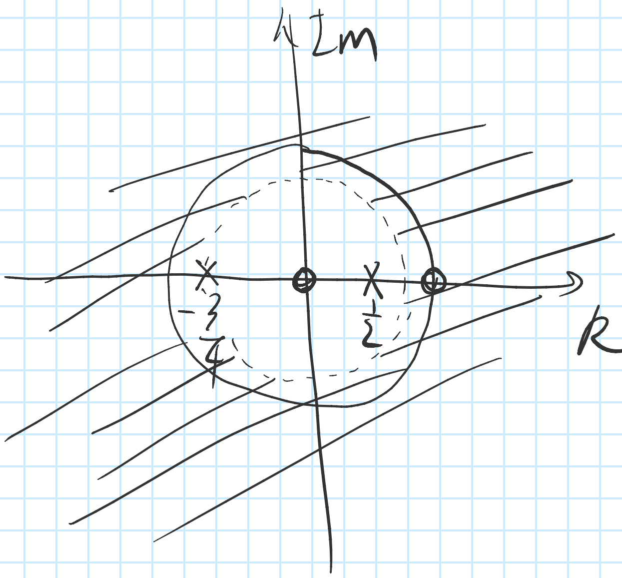
(a). $H(z) = \dfrac{Y(z)}{X(z)}$

$$X(z) = \mathcal{Z}\{x[n]\} = 5 \times \frac{1}{1-z^{-1}}, \quad |z|>1$$

$$Y(z) = \mathcal{Z}\{y[n]\} = 2 \times \frac{1}{1-\frac{1}{2}z^{-1}} + 3 \times \frac{1}{1+\frac{3}{4}z^{-1}}, \quad |z|>\frac{3}{4}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2+\frac{3}{2}z^{-1}+3-\frac{3}{2}z^{-1}}{5} \times \frac{1-z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{3}{4}z^{-1}\right)}$$

$$= \frac{1-z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{3}{4}z^{-1}\right)}, \quad \text{ROC: } |z|>\frac{3}{4}$$



There should be an additional zero

Check $z=0$.

$$\lim_{z\to 0} H(z) = 0$$

(b).

$$H(z) = \frac{1 - z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 + \frac{3}{4}z^{-1})}, \quad |z| > \frac{3}{4}$$

$$H(z) = \frac{A}{(1 - \frac{1}{2}z^{-1})} + \frac{B}{(1 + \frac{3}{4}z^{-1})}$$

$$\Rightarrow \quad A + B + \left(\frac{3}{4}A - \frac{1}{2}B\right) = 1 - z^{-1}$$

$$\begin{cases} A + B = 1 \\ \frac{3}{4}A - \frac{1}{2}B = -1 \end{cases} \Rightarrow \begin{cases} A = -\frac{2}{5} \\ B = \frac{7}{5} \end{cases}$$

$$H(z) = -\frac{\frac{2}{5}}{1 - \frac{1}{2}z^{-1}} + \frac{\frac{7}{5}}{1 + \frac{3}{4}z^{-1}}, \quad |z| > \frac{3}{4}$$

$$h[n] = Z^{-1}\left\{H(z)\right\} = -\frac{2}{5}\left(\frac{1}{2}\right)^n u[n] + \frac{7}{5} \times \left(-\frac{3}{4}\right)^n u[n]$$

(c).

From (c) in Problem 5.4

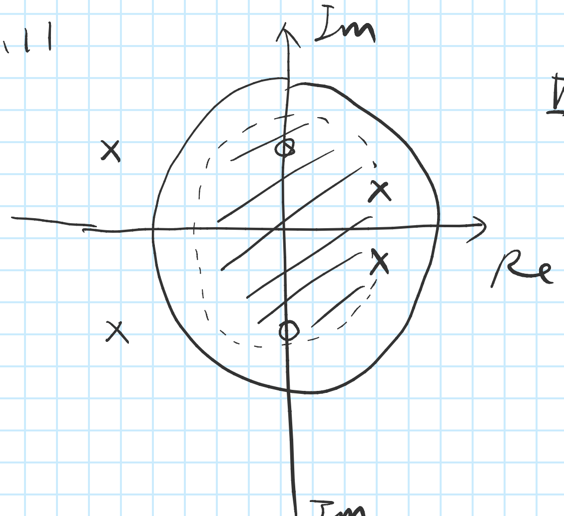
$$\begin{cases} b_0 = 1 \\ b_1 = -1 \end{cases} \qquad \begin{cases} a_0 = 1 \\ a_1 = \frac{1}{4} \end{cases} \quad 3$$

$$\left\{ b_1 = -1 \quad \middle| \quad \begin{array}{l} a_1 = 4 \\ a_2 = -\dfrac{3}{8} \end{array} \right.$$

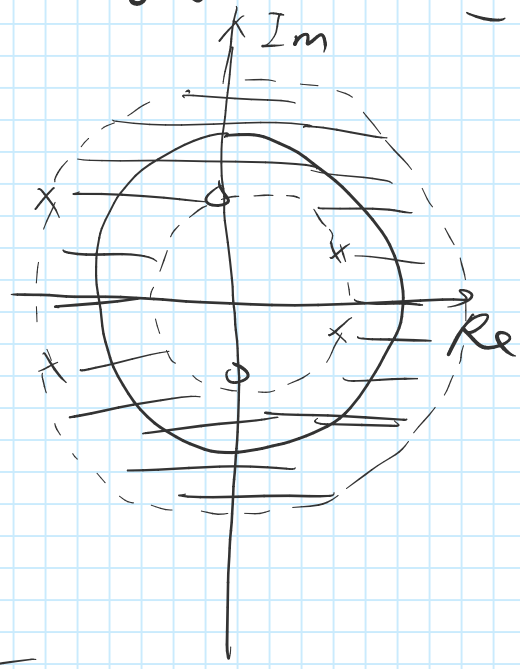
$$\Rightarrow y[n] + \frac{1}{4} y[n-1] - \frac{3}{8} y[n-2] = x[n] - x[n-1]$$

5.11

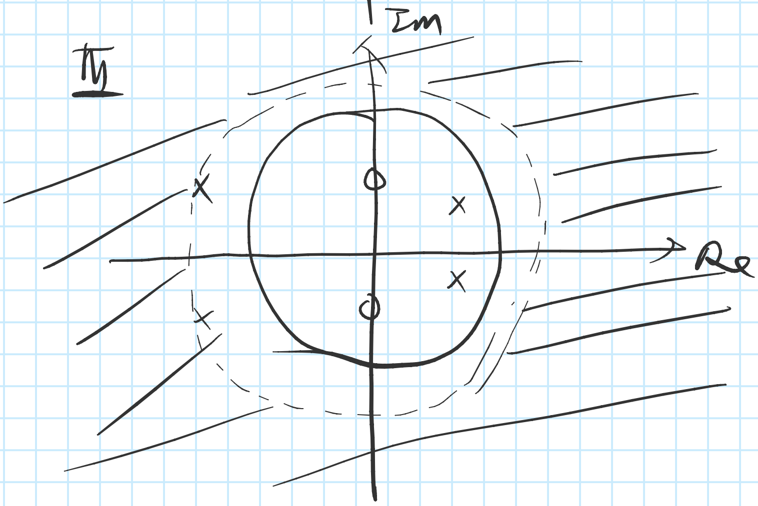
I



II



III



(a), (b), Can't be determined

(c). Recall causal $\Rightarrow$ $h[n]$ is right-sided

$\Rightarrow$ III: unit circle is not included in ROC

$\Rightarrow$ not stable $\Rightarrow$ False

(d). Stable $\Rightarrow$ u.c. is included in ROC

$\Rightarrow$ II $\Rightarrow$ ROC is a ring $\Rightarrow$ $h[n]$ is two-sided

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