ESE 531 Recitation 7

ESE_531_Recitation_7_Matlab.m

```matlab
% The purpose of this problem is to
% 1. design a low-pass filter from its poles and zeros,
% 2. find out (draw) the magnitude and phase of its frequency response,
% 3. perform filtering to an input signal and see the filtering effect
%% Define zeroes and poles

j = sqrt(-1);
Z = [1.1*(-0.8+j) 1.1*(-0.8-j) -1]'; % Zeroes
P = [0.6*(0.5+j) 0.6*(0.5-j)]; % Poles
K = 1/11; % Scaling factor
[b, a] = zp2tf(Z,P,K); % Coefficients for the trasnfer function

%% Define frequency response

[h, w] = freqz(b,a,'whole', 30000); %% h: frequency response, calculated over 30000 points

%% Plot magnitude response

figure
% plot(w, 10*log(abs(h))); % w plots from 0 to 2pi

Combined = plot(w(1:15000)/pi,20*log10(abs(h(1:15000))),... % First half remains the same
    (w(15001:30000)-2*(pi-pi/30000))/pi, 20*log10(abs(h(15001:30000)))); % Shift the frequency from (pi 2pi) to (-pi 0)

% format plot
Combined(1).Color = 'b';
Combined(1).LineWidth = 2;
Combined(2).Color = 'b';
Combined(2).LineWidth = 2;
grid;

% label axes
xlabel(['' char(969) '(' char(0215) '' char(960) ')']);
ylabel([' |H(e^{j' char(969) '})| (dB)']);

%% plot phase response
figure

Combined = plot(w(1:15000)/pi,unwrap(angle(h(1:15000))),... % First half remains the same
(w(15002:30000)-2*(pi-pi/30000))/pi, unwrap(angle(h(15002:30000)))); % Shift the frequency from (pi 2pi) to (-pi 0)

% formatting plot
Combined(1).Color = 'r';
Combined(1).LineWidth = 2;
Combined(2).Color = 'r';
Combined(2).LineWidth = 2;
grid;

% labeling axes
xlabel(['' char(969) '(' char(0215) '' char(960) ')']);
ylabel(['' char(934) '\{H(e^{j' char(969) '})\} (' char(0215) '' ...
char(960) ')']);

%% Define a input signal

n = 0:200;
x = cos(0.1*pi*n) + cos(0.8*pi*n); % a low-freq component and a high-freq component

figure
stem(x)
xlabel('n')
title('Input')

%% Filter the input x[n] using the filter designed above

y = filter(b,a,x);
figure
stem(y(1:end)) % high-freq component is filtered out
xlabel('n')
title('Output after filtering')
```

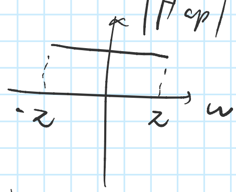
5.13, 5.51

5.13

Recall $H_{ap}(z) = \dfrac{z^{-1} - a^*}{1 - az^{-1}}$

$$H_{ap}(e^{-j\omega}) = \frac{e^{-j\omega} - a^*}{1 - ae^{-j\omega}}$$

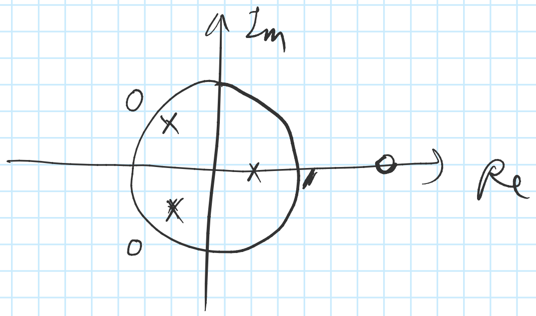
$|H_{ap}(e^{j\omega})| = \underline{1}$ for all ω.



$\underset{\sim\sim\sim}{= a,\ a \text{ is a constant}}$

for a real $h[n]$, poles & zeroes of a all-pass system has the following characteristics:

	real	complex	
poles	d_k	e_k	e_k^*
		$\updownarrow$	$\updownarrow$
zeroes	$\frac{1}{d_k}$	$(e_k^*)^{-1}$	$(e_k)^{-1}$



Now examine $H_1(z) \to H_4(z)$

$H_1(z)$ ✓

$H_2(z)$ ✗

$H_3(z)$ ✓

$H_4(z)$ ✓

5.51

$$H(z) = \frac{(1+iz^{-1})(1-2z^{-1})(1+0.9z^{-1})}{(\ \ )(\ \ )}$$

5.51 $H(z) = \frac{(\quad)(\quad)}{(\quad)(\quad)(\quad)}$

(a).

$$H(z) = \frac{1 - 0.6z^{-1} - 2.35z^{-2} - 0.9z^{-3}}{1 - z^{-1} + 0.49z^{-2} - 0.49z^{-3}}$$

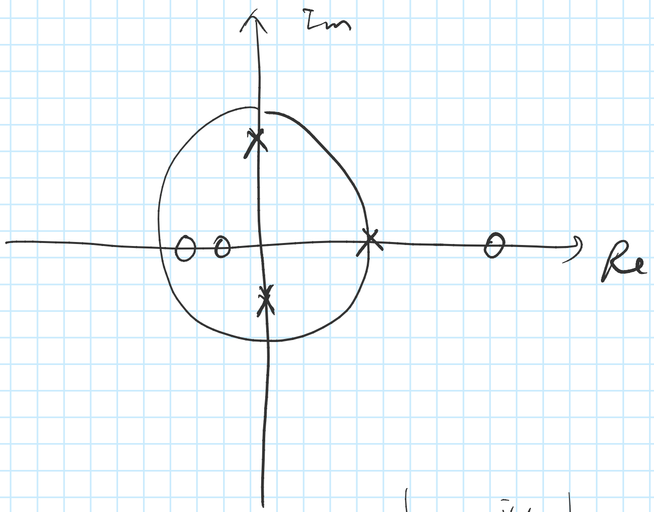
$y[n] - y[n-1] + 0.49y[n-2] - 0.49y[n-3] =$

$x[n] - 0.6x[n-1] - 2.35x[n-2]$
$- 0.9x[n-3]$

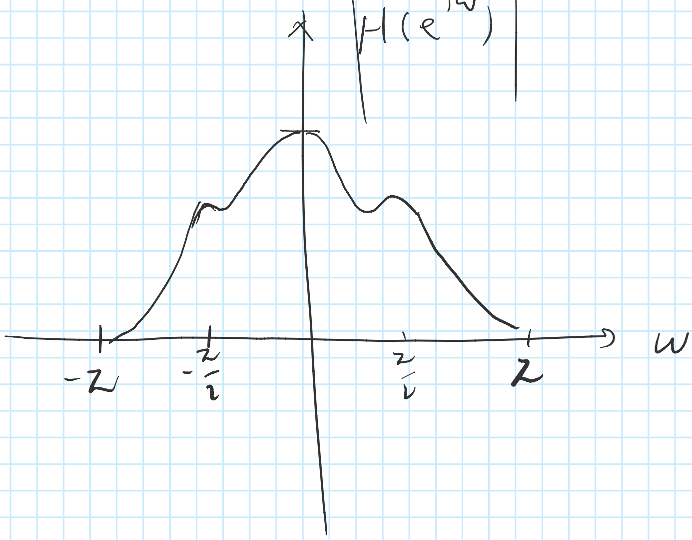
(b).

poles: $1 \quad -0.7j \quad 0.7j$

zeroes: $2 \quad -\frac{1}{2} \quad -0.9$



(c).



(d).

(d).

i. Since $H(z)$ is causal,

$|z| > 1$, $\Rightarrow$ u.c. is not included

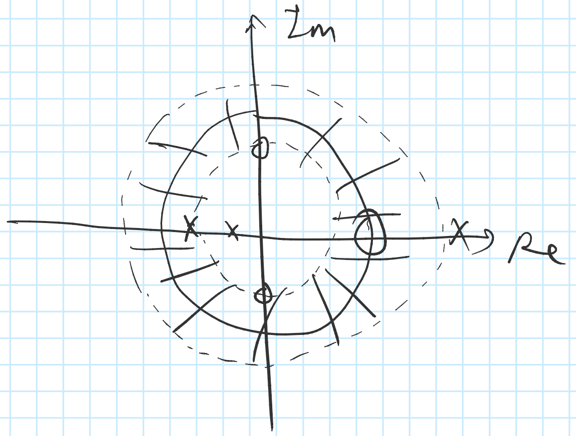
$\Rightarrow$ Not stable

ii. False

iii. From part (c) $\Rightarrow$ False

peaks at $\omega = \pm \pi/2$

iv.



If it's stable, ROC $\to$ ring

but for a causal system, $h[n]$

is right-sided $\Rightarrow$ ROC $\to$ outgoing region

instead of a ring

$\Leftrightarrow$ contradicts.

$\Rightarrow$ False.