University of Pennsylvania
Department of Electrical and System Engineering
Digital Signal Processing

HW6: All Pass and Min Phase Systems

Sunday, Feb. 27

Due: Sunday, Mar. 6th, 11:59PM

- **Homework Problems:** All problems must be turned in and are not optional for full credit

1. Homework problems from the book: 5.24, 5.36, 5.42
2. Matlab problem 1: Frequency response for difference equations
For the following difference equation

$$y[n] - 1.8cos\left(\frac{\pi}{16}\right)y[n-1] + 0.81y[n-2] = x[n] + 0.5x[n-1] \tag{1}$$

do the following frequency-domain computations:

(a) Make plots for the magnitude and phase responses with 512 frequency samples around the entire unit circle (i.e from 0 to 2π). Use `freqz` to generate your plots.
HINT: The command `[H, W] = freqz(b, a, N, 'whole')` will evaluate the frequency response of a filter at `N`, equally spaced in radian frequency around the unit circle. If you do not use the `'whole'` option, `freqz` will use only the upper half of the unit circle (from 0 to π), which is sufficient for filter with real coefficients. The output vectors `H` and `W` will return `N` frequency response samples (`H`) and `N` equally spaced values of ω (`W`).

(b) Now redo the frequency response using only the upper half of the unit circle. Submit your plot and explain why this is sufficient?

(c) Specify the type of filter defined by this difference equation: high-pass, low-pass, all-pass, band-pass, or bandstop.

(d) Redo a-c for the difference equation:

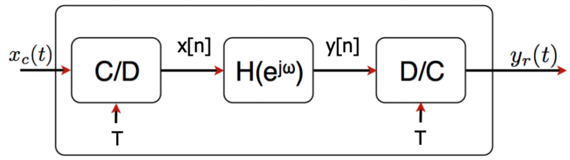
$$y[n]+0.13y[n-1]+0.52y[n-2]+0.3y[n-3] = 0.16x[n]-0.48x[n-1]+0.48x[n-2]-0.16x[n-3] \tag{2}$$

(e) Redo a-c for the difference equation:

$$y[n] - 0.268y[n-2] = 0.634x[n] - 0.634x[n-2] \tag{3}$$

(f) Experiment with your own difference equations! (nothing to turn in)

3. Matlab problem 2: Frequency response of a notch filter A notch filter attempts to remove one particular frequency. Suppose that a bandlimited continuous-time signal is known to contain a 60-Hz interference component, which we want to remove by processing with the standard system below for filtering a continuous-time signal with a discrete-time filter.



   (a) Assume that the value of the sampling period is $T_S = 1$ms. What is the highest frequency that the analog signal can contain if aliasing is to be avoided?

   (b) The discrete-time system to be used has a frequency response

$$H(e^{j\omega}) = \frac{[1 - e^{-j(\omega-\omega_0)}][1 - e^{-j(\omega+\omega_0)}]}{[1 - 0.9e^{-j(\omega-\omega_0)}][1 - 0.9e^{-j(\omega+\omega_0)}]} \tag{4}$$

   Plot (in Matlab) the magnitude and phase of $H(e^{j\omega})$. Pick a trial value of $\omega_0 = 2\pi/5$. Submit your plots.

   (c) What value should be chosen for ω_0 to eliminate the 60-Hz component? Will the gain at the other frequencies be equal to 1?

   (d) Make a Matlab plot of the frequency response magnitude only using the value from part (c). Submit your plot.

   (e) Generate 150 samples of a 60-Hz sine wave sampled at $f_S = 1/T_S = 1000Hz$. Use the function `filter` to process this input signals with the system from (b) and the value of ω_0 from (c). Display the output signal to illustrate that the filter actually removes the 60-Hz sinusoid. Submit the input and output signals.

   (f) Since the DTFT is a frequency response, it describes the steady-state behaviour of the filter. Thus you should observe a "transient" response before the zero of the filter at 60Hz rejects the input completely. measure the duration of this transient (in milliseconds) from the beginning of the signal until a point where the output is less than 1% of the input signal amplitude.

- **Recommended Problems for Practice:** From the book: 5.13, 5.43, 5.51