

University of Pennsylvania
Department of Electrical and System Engineering
Digital Signal Processing

HW6: All Pass, Min Phase, and GLP Systems

Sunday, Feb. 26

Due: Sunday, Mar. 12th, 11:59PM

- **Homework Problems:** All problems must be turned in and are not optional for full credit

1. Homework problems from the book: 5.24, 5.34, 5.39, 5.42
2. Matlab problem: Frequency response for difference equations.
For the following difference equation

$$y[n] - 1.8\cos\left(\frac{\pi}{16}\right)y[n-1] + 0.81y[n-2] = x[n] + 0.5x[n-1] \quad (1)$$

do the following frequency-domain computations:

- (a) Make plots for the magnitude and phase responses with 512 frequency samples around the entire unit circle (i.e from 0 to 2π). Use `freqz` to generate your plots.

HINT: The command `[H, W] = freqz(b, a, N, 'whole')` will evaluate the frequency response of a filter at `N`, equally spaced in radian frequency around the unit circle. If you do not use the `'whole'` option, `freqz` will use only the upper half of the unit circle (from 0 to π), which is sufficient for filter with real coefficients. The output vectors `H` and `W` will return `N` frequency response samples (`H`) and `N` equally spaced values of ω (`W`).

- (b) Now redo the frequency response using only the upper half of the unit circle (for $0 \leq \omega \leq \pi$). Submit your plot and explain why this is sufficient?
- (c) Specify the type of filter defined by this difference equation: high-pass, low-pass, all-pass, band-pass, or bandstop.
- (d) Redo a-c for the difference equation:

$$y[n] + 0.13y[n-1] + 0.52y[n-2] + 0.3y[n-3] = 0.16x[n] - 0.48x[n-1] + 0.48x[n-2] - 0.16x[n-3] \quad (2)$$

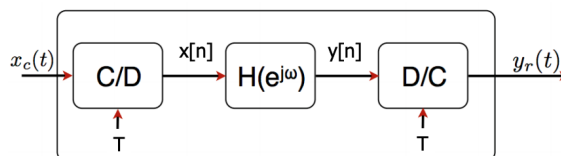
- (e) Redo a-c for the difference equation:

$$y[n] - 0.268y[n-2] = 0.634x[n] - 0.634x[n-2] \quad (3)$$

- (f) Experiment with your own difference equations! (nothing to turn in)

3. Matlab problem: Frequency response of a notch filter.

A notch filter attempts to remove one particular frequency. Suppose that a bandlimited continuous-time signal is known to contain a 60-Hz interference component, which we want to remove by processing with the standard system below for filtering a continuous-time signal with a discrete-time filter.



- Assume that the value of the sampling period is $T_S = 1\text{ms}$. What is the highest frequency that the analog signal can contain if aliasing is to be avoided?
- The discrete-time system to be used has a frequency response

$$H(e^{j\omega}) = \frac{[1 - e^{-j(\omega - \omega_0)}][1 - e^{-j(\omega + \omega_0)}]}{[1 - 0.9e^{-j(\omega - \omega_0)}][1 - 0.9e^{-j(\omega + \omega_0)}]} \quad (4)$$

Plot (in Matlab) the magnitude and phase of $H(e^{j\omega})$. Pick a trial value of $\omega_0 = 2\pi/5$. Submit your plots.

- What value should be chosen for ω_0 to eliminate the 60-Hz component? Will the gain at the other frequencies be equal to 1?
- Make a Matlab plot of the frequency response magnitude only using the value from part (c). Submit your plot.
- Generate 150 samples of a 60-Hz sine wave sampled at $f_S = 1/T_S = 1000\text{Hz}$. Use the function `filter` to process this input signals with the system from (b) and the value of ω_0 from (c). Display the output signal to illustrate that the filter actually removes the 60-Hz sinusoid. Submit the input and output signals.
- EXTRA (not to turn in): Generate 150 samples of a 60-Hz + 20-Hz sine waves of different amplitudes. Use the function `filter` to process this input signals with the system from (b) and the value of ω_0 from (c). Display the output signal to illustrate that the filter actually removes the 60-Hz sinusoid.
- Since the DTFT is a frequency response, it describes the steady-state behaviour of the filter. Thus you should observe a "transient" response before the zero of the filter at 60Hz rejects the input completely. measure the duration of this transient (in milliseconds) from the beginning of the signal until a point where the output is less than 1% of the input signal amplitude.

- **Recommended Problems for Practice:** From the book: 5.10, 5.13, 5.36, 5.43, 5.51