University of Pennsylvania
Department of Electrical and System Engineering
Digital Signal Processing

HW8: DFT, FFT

Tuesday, Apr. 4

Due: Tuesday, Apr 11, 11:59PM

- **Homework Problems:** All problems must be turned in and are not optional for full credit

  1. Homework problems from the book: 8.23, 8.43, 9.26

  2. A system has been built for computing the 8-point DFT, $Y[0], Y[1], ..., Y[7]$, of a discrete-time sequence, $y[0], y[1], ..., y[7]$. However, the system is not working properly and only the even DFT samples, $Y[0], Y[2], Y[4], Y[6]$, are being computed correctly. To help you solve the problem, the data you can access are:
     - the (correct) even DFT samples, $Y[0], Y[2], Y[4], Y[6]$ and
     - the first 4 input values $y[0], y[1], y[2], y[3]$ (the other inputs are unavailable).

     (a) If $y[0] = 1$ and $y[1] = y[2] = y[3] = 0$, and $Y[0] = Y[2] = Y[4] = Y[6] = 2$, what are the odd DFT values, $Y[1], Y[3], Y[5], Y[7]$? Explain how you got them.

     (b) You need to build an efficient system that computes the odd DFT values, $Y[1], Y[3], Y[5], Y[7]$, for any set of inputs. The computational modules you have available are one 4-point DFT and one 4-point IDFT. Both are free. You can purchase adders, subtracters, or multipliers for \$10 each. Multiplying by 1 or 0 is also free. Design a system of the lowest possible cost that takes as inputs $y[0], y[1], y[2], y[3]$ and $Y[0], Y[2], Y[4], Y[6]$ and produces the output $Y[1], Y[3], Y[5], Y[7]$. Draw or describe the associated block diagram and indicate the total cost.

3. Matlab problem: Let $x[n]$ be a discrete time sequence:

$$x[n] = \begin{cases} (0.7)^n 0 \leq n \leq 7 \\ 0 \text{else} \end{cases} \tag{1}$$

   a) Determine the analytical expression for the DTFT of $x[n]$ and plot the magnitude and phase of the DTFT.
   b) Compute in MATLAB the 8-point DFT of $x[n], 0 \leq n \leq 7$ using the `fft` function. Plot the magnitude and phase. Use the `stem, abs, angle` commands.
   c) Compute, in MATLAB, the 16-point DFT of $x[n], 0 \leq n \leq 15$ and stem plot its magnitude and phase. Comment on the effect of zero-padding the signal on its DFT.
   d) Compute, in MATLAB, the 128-point DFT of $x[n], 0 \leq n \leq 127$ and plot its magnitude and phase. Note: the plot command is used instead of stem when many dense points exist to avoid appearance of a black blob.
   e) Compare the results from part (d) to the plots of part (a). How does this relate to the relationship between digital frequency ω and DFT index k?

4. Matlab problem: Download the file tones.mat from the course calendar. The file contains the a signal which has multiple tones in it. Load the signal using the following commands,

```
s = load(tones.mat);
x = s.y1;
```

   The variable x should now contain the signal.

   a) Compute the DFT of x using a transform length N = 25. Plot the magnitude of the DFT using the plot command. How many distinct frequencies do you see?
   b) Experiment with the sequence length (by adding a different number of zeros). Compute the DFT for these different sequence lengths obtained by zero-padding. Can you find more frequencies? How many tones can you distinguish and what are their values?

- **Recommended Problems for Practice:** From the book: 8.28, 8.40 (there is a typo and there should only be one k in the exponent), 9.28, 9.32, 9.33ab