ESE 5310: Digital Signal Processing

Lecture 20: April 6, 2023

Fast Fourier Transform

Penn

Fast Fourier Transform Algorithms

- We are interested in efficient computing methods for the DFT and inverse DFT:

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn}, \quad k = 0, \ldots, N-1$$

$$x[n] = \sum_{k=0}^{N-1} X[k] W_N^{-kn}, \quad n = 0, \ldots, N-1$$

$$W_N = e^{-j\left(\frac{2\pi}{N}\right)}.$$

Computation Order

- *Fast Fourier transform algorithms* enable computation of an N-point DFT (or inverse DFT) with the order of just $N \cdot \log_2 N$ complex multiplications.
  - This can represent a huge reduction in computational load, especially for large N.

N	N^2	$N \cdot \log_2 N$	$\frac{N^2}{N \cdot \log_2 N}$
16	256	64	4.0
128	16,384	896	18.3
1,024	1,048,576	10,240	102.4
8,192	67,108,864	106,496	630.2
6×10^6	36×10^{12}	135×10^6	2.67×10^5

* 6Mp image size

Eigenfunction Properties

- Most FFT algorithms exploit the following properties of W_N^{kn}:

$$W_N^0 = W_N^N = W_N^{2N} = \cdots = 1$$
$$W_N^{k+N} = W_N^k$$

  - Conjugate Symmetry

$$W_N^{k(N-n)} = W_N^{-kn} = (W_N^{kn})^*$$

  - Periodicity in n and k

$$W_N^{kn} = W_N^{k(n+N)} = W_N^{(k+N)n}$$

  - Power

$$W_N^2 = W_{N/2}$$

FFT Algorithms via Decimation

- Most FFT algorithms decompose the computation of a DFT into successively smaller DFT computations.
  - Decimation-in-time algorithms decompose x[n] into successively smaller subsequences.
  - Decimation-in-frequency algorithms decompose X[k] into successively smaller subsequences.

- Note: Assume length of x[n] is power of 2 ($N = 2^v$). If not, zero-pad to closest power of 2.

Decimation-in-Time FFT

- We start with the DFT

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn}, \qquad k = 0, \ldots, N-1$$

Decimation-in-Time FFT

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$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn}, \qquad k = 0, \ldots, N-1$$

- Separate the sum into even and odd terms:

$$X[k] = \sum_{n \text{ even}} x[n] W_N^{kn} + \sum_{n \text{ odd}} x[n] W_N^{kn}$$

  - These are two DFTs, each with half the number of samples (N/2)

Decimation-in-Time FFT

$$X[k] = \sum_{n \text{ even}} x[n] W_N^{kn} + \sum_{n \text{ odd}} x[n] W_N^{kn}$$

Let $n = 2r$ (n even) and $n = 2r + 1$ (n odd):

Decimation-in-Time FFT

$$X[k] = \sum_{n \text{ even}} x[n] W_N^{kn} + \sum_{n \text{ odd}} x[n] W_N^{kn}$$

Let $n = 2r$ (n even) and $n = 2r + 1$ (n odd):

$$X[k] = \sum_{r=0}^{(N/2)-1} x[2r] W_N^{2rk} + \sum_{r=0}^{(N/2)-1} x[2r+1] W_N^{(2r+1)k}$$

Decimation-in-Time FFT

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$$= \sum_{r=0}^{(N/2)-1} x[2r] W_N^{2rk} + W_N^k \sum_{r=0}^{(N/2)-1} x[2r+1] W_N^{2rk}$$

Decimation-in-Time FFT

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$$= \sum_{r=0}^{(N/2)-1} x[2r] W_N^{2rk} + W_N^k \sum_{r=0}^{(N/2)-1} x[2r+1] W_N^{2rk}$$

$$X[k] = \sum_{r=0}^{(N/2)-1} x[2r] W_{N/2}^{rk} + W_N^k \sum_{r=0}^{(N/2)-1} x[2r+1] W_{N/2}^{rk}$$

Decimation-in-Time FFT

Hence:

$$X[k] = \sum_{r=0}^{(N/2)-1} x[2r] W_{N/2}^{rk} + W_N^k \sum_{r=0}^{(N/2)-1} x[2r+1] W_{N/2}^{rk}$$

$$\triangleq G[k] + W_N^k H[k], \quad k = 0, \ldots, N-1$$

where we have defined:

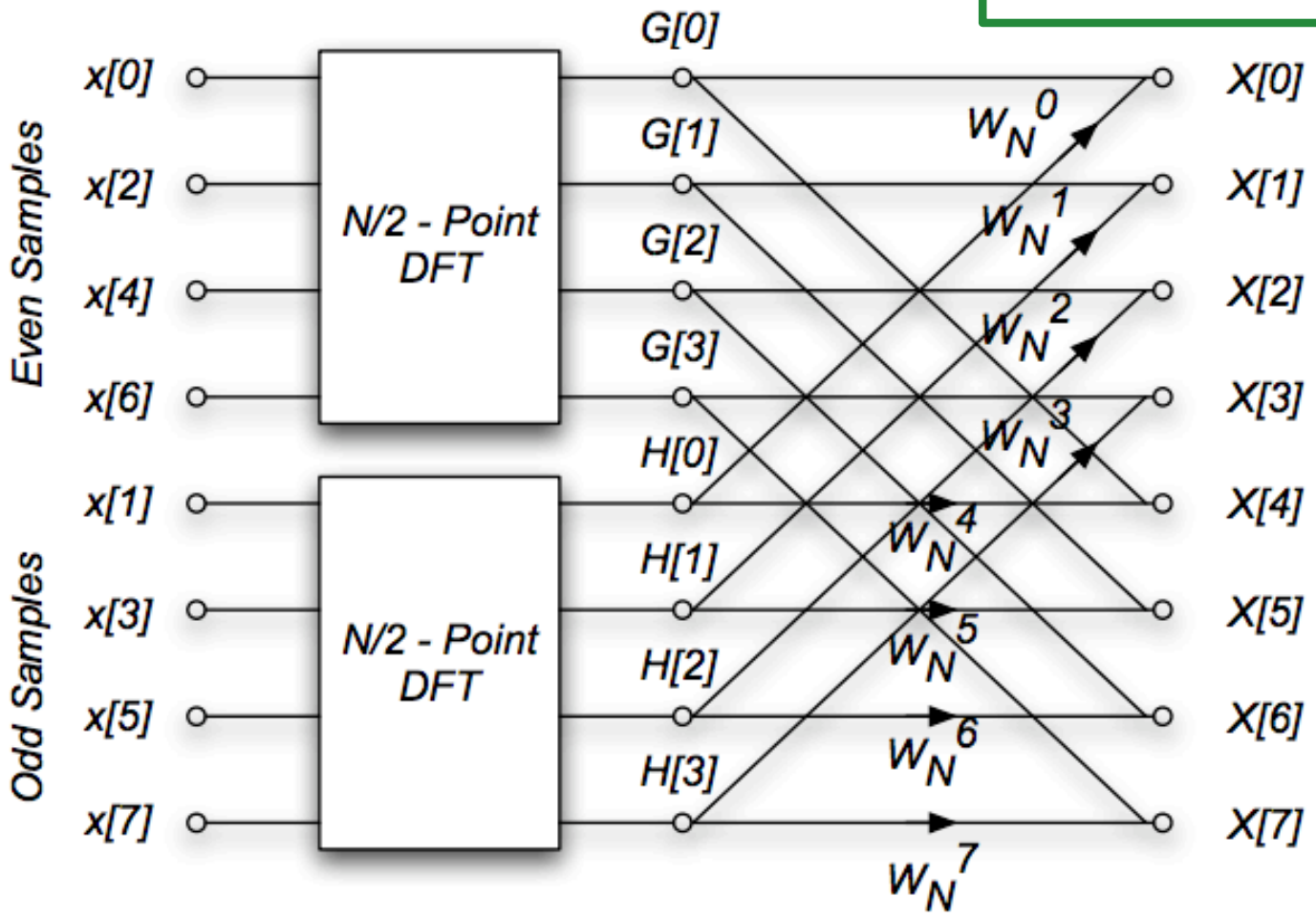
$$G[k] \triangleq \sum_{r=0}^{(N/2)-1} x[2r] W_{N/2}^{rk} \quad \Rightarrow \text{DFT of even samples}$$

$$H[k] \triangleq \sum_{r=0}^{(N/2)-1} x[2r+1] W_{N/2}^{rk} \quad \Rightarrow \text{DFT of odd samples}$$

Decimation-in-Time FFT

An 8 sample DFT can then be diagrammed as

$$X[k] = G[k] + W_N^k H[k]$$



Decimation-in-Time FFT

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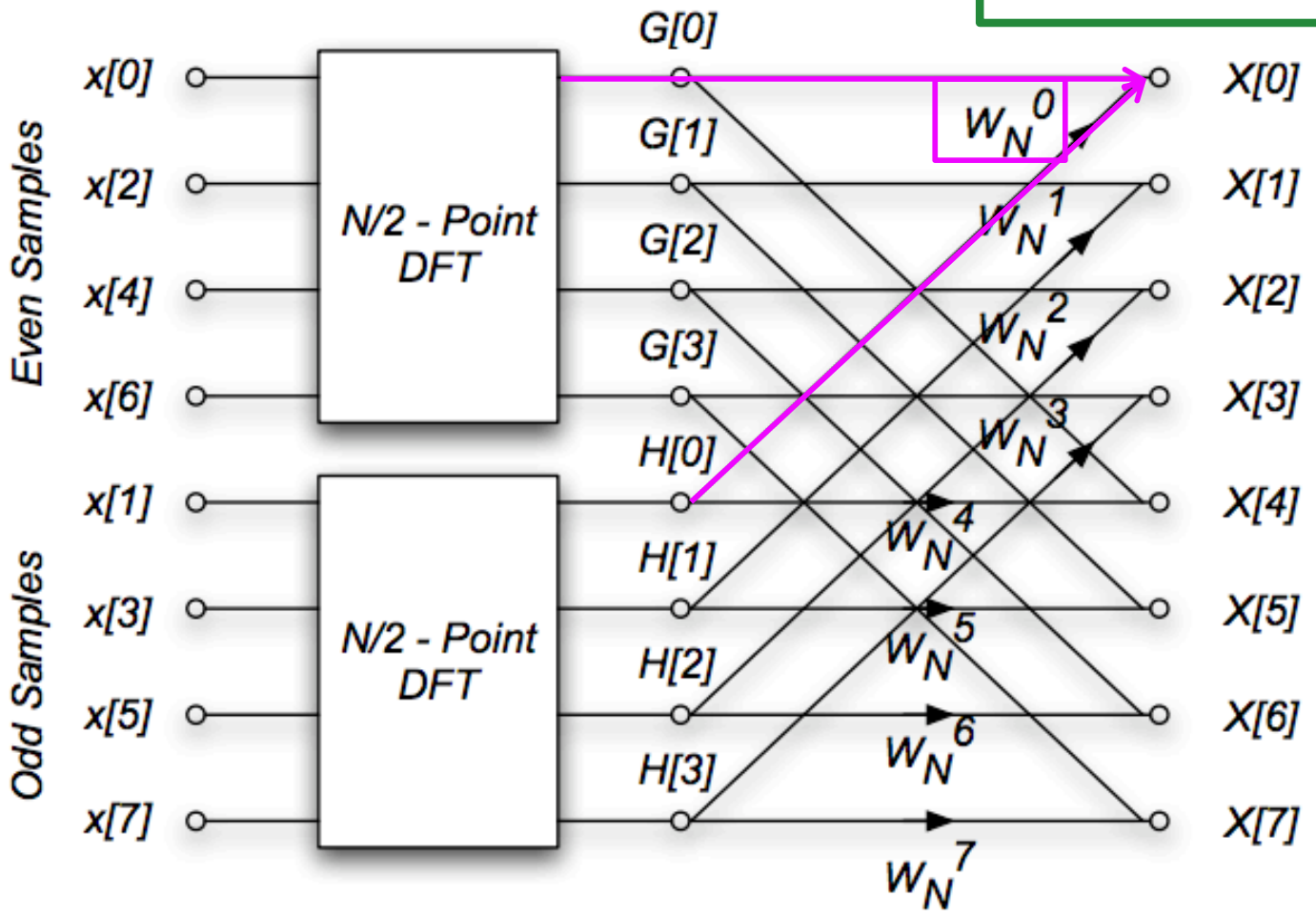

$X[0] = G[0] + W_N^0 H[0]$

Decimation-in-Time FFT

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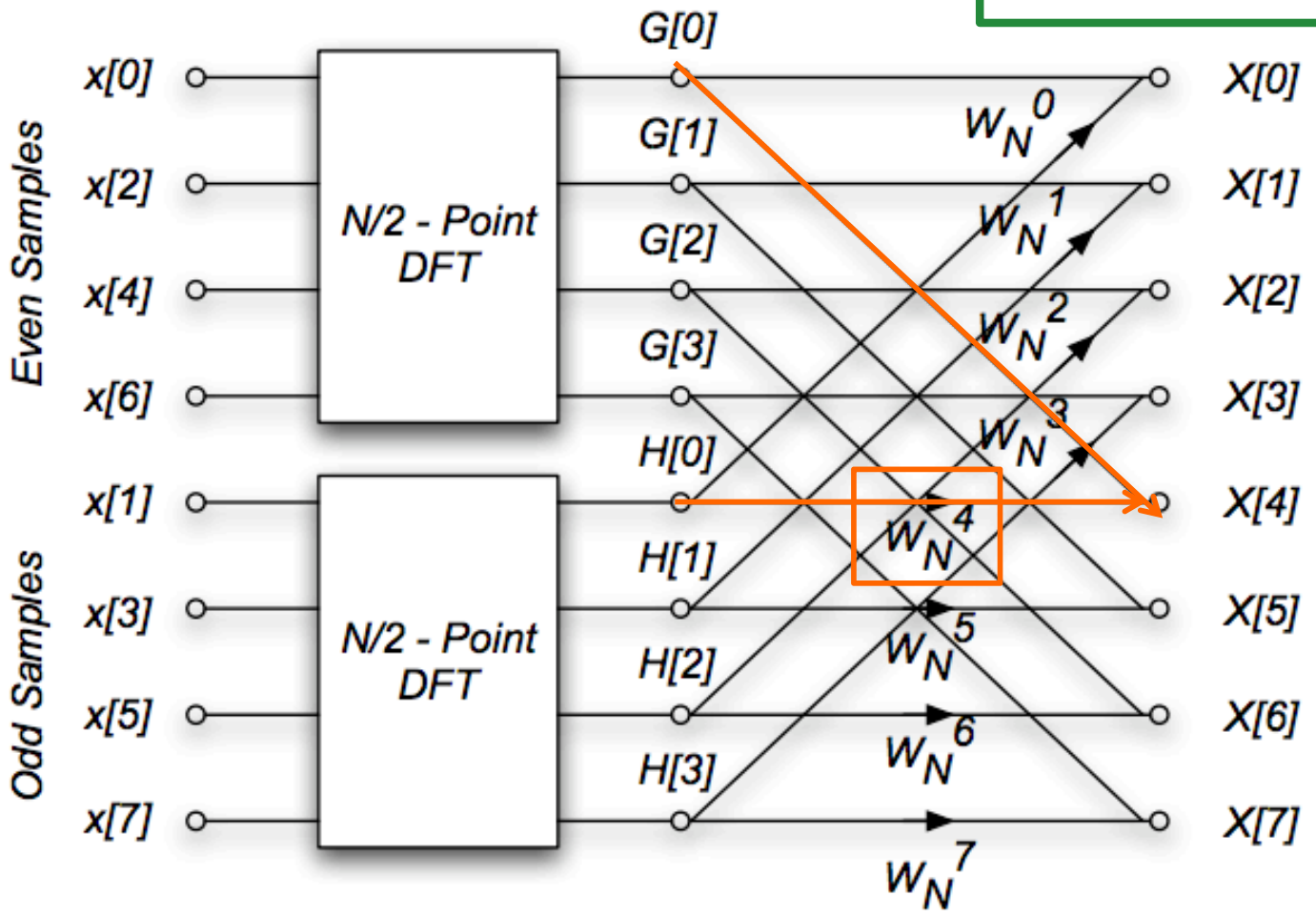


??

$$X[4] = G[4] + W_N^4 H[4]$$

Decimation-in-Time FFT

Both $G[k]$ and $H[k]$ are periodic, with period $N/2$. For example

$$G[k] \triangleq \sum_{r=0}^{(N/2)-1} x[2r] W_{N/2}^{rk}$$

Decimation-in-Time FFT

Both $G[k]$ and $H[k]$ are periodic, with period $N/2$. For example

$$G[k] \triangleq \sum_{r=0}^{(N/2)-1} x[2r] W_{N/2}^{rk}$$

$$G[k+N/2] = \sum_{r=0}^{(N/2)-1} x[2r] W_{N/2}^{r(k+N/2)}$$

Decimation-in-Time FFT

Both $G[k]$ and $H[k]$ are periodic, with period $N/2$. For example

$$G[k] \triangleq \sum_{r=0}^{(N/2)-1} x[2r] W_{N/2}^{rk}$$

$$\begin{aligned} G[k+N/2] &= \sum_{r=0}^{(N/2)-1} x[2r] W_{N/2}^{r(k+N/2)} \\ &= \sum_{r=0}^{(N/2)-1} x[2r] W_{N/2}^{rk} \underbrace{W_{N/2}^{r(N/2)}}_{=1} \\ &= \sum_{r=0}^{(N/2)-1} x[2r] W_{N/2}^{rk} \\ &= G[k] \end{aligned}$$

Decimation-in-Time FFT

- So,

$$\begin{aligned} G[k+(N/2)] &= G[k] \\ H[k+(N/2)] &= H[k] \end{aligned}$$

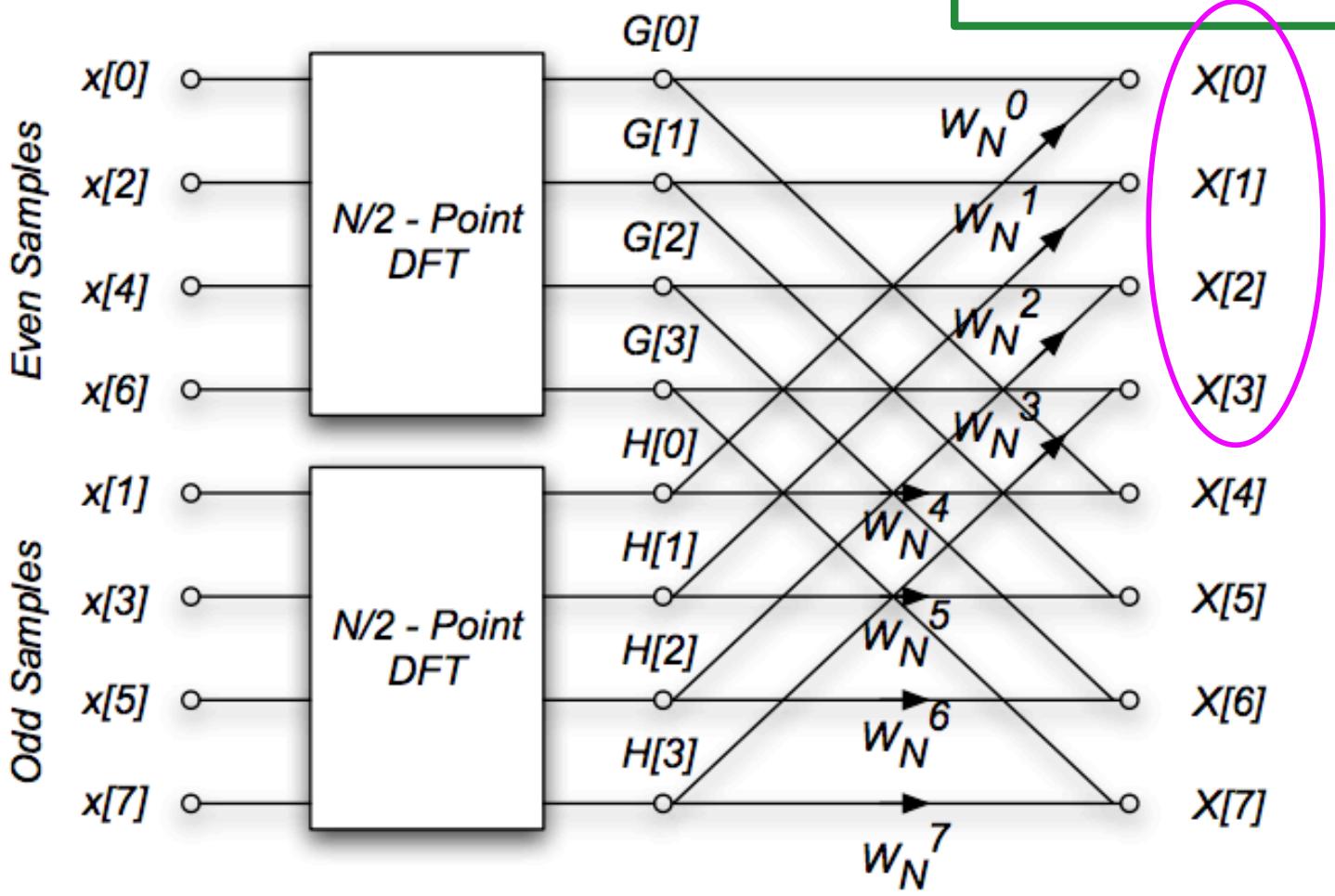
- The periodicity of G[k] and H[k] allows us to further simplify. For the first N/2 points we calculate G[k] and $W_N^k H[k]$, and then compute the sum

$$X[k] = G[k] + W_N^k H[k] \qquad \forall\{k : 0 \leq k < \frac{N}{2}\}.$$

Decimation-in-Time FFT

An 8 sample DFT can then be diagrammed as

$$X[k] = G[k] + W_N^k H[k]$$



First N/2 DFTs
$0 \le k < N/2$

Decimation-in-Time FFT

How does periodicity help for $\frac{N}{2} \leq k < N$?

$$X[k] = G[k] + W_N^k H[k] \qquad \forall \{k : 0 \leq k < \frac{N}{2}\}.$$

for $\frac{N}{2} \leq k < N$:

$$W_N^{k+(N/2)} = ?$$

$$X[k + (N/2)] = ?$$

Decimation-in-Time FFT

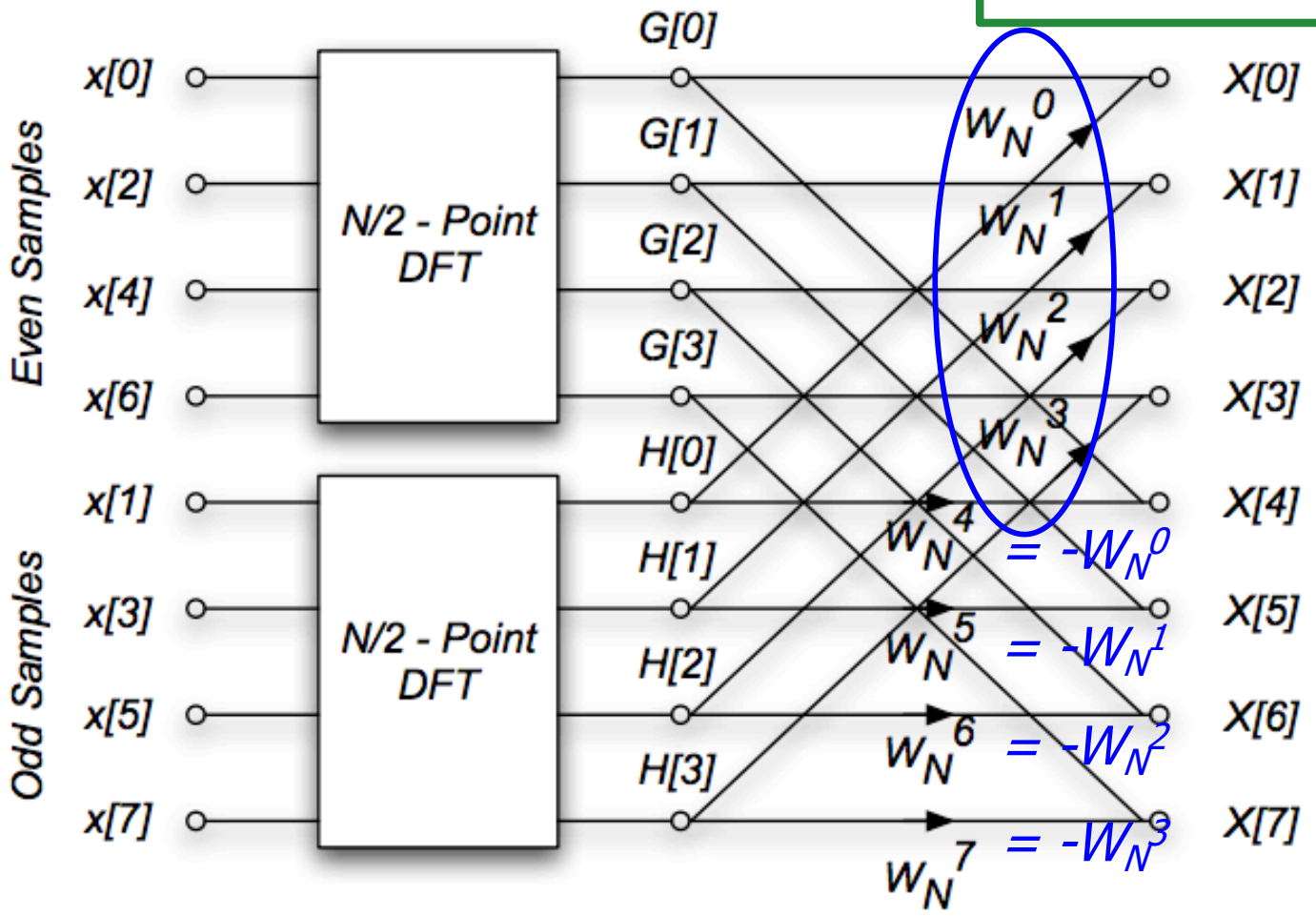
$$X[k + (N/2)] = G[k] - W_N^k H[k]$$

- We previously calculated G[k] and $W_N^k H[k]$.
- Now we only have to compute their difference to obtain the second half of the spectrum. No additional multiplies are required.

Decimation-in-Time FFT

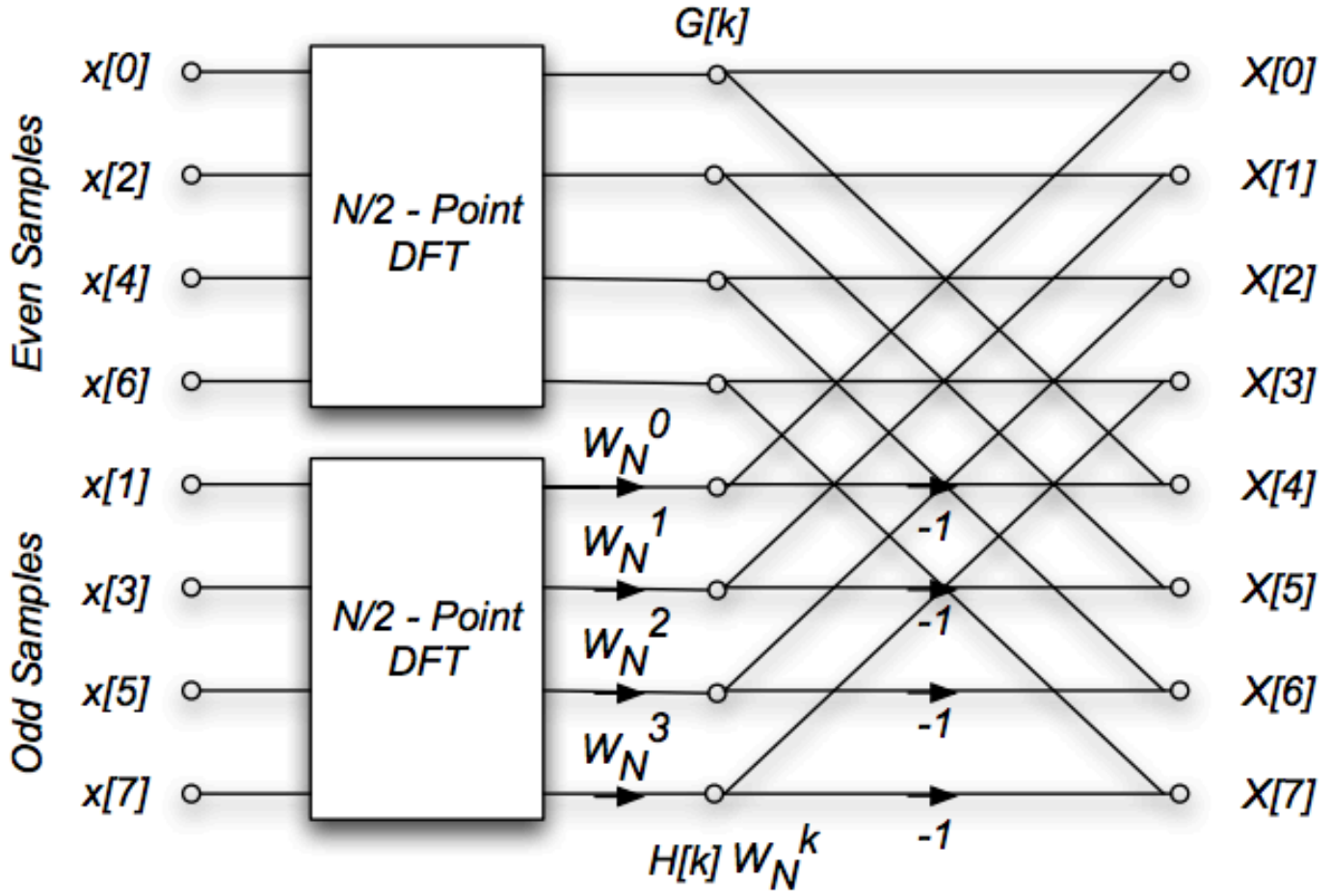
An 8 sample DFT can then be diagrammed as

$$X[k] = G[k] + W_N^k H[k]$$



Decimation-in-Time FFT

The N-point DFT has been reduced two $N/2$-point DFTs, plus $N/2$ complex multiplications. The 8 sample DFT is then:



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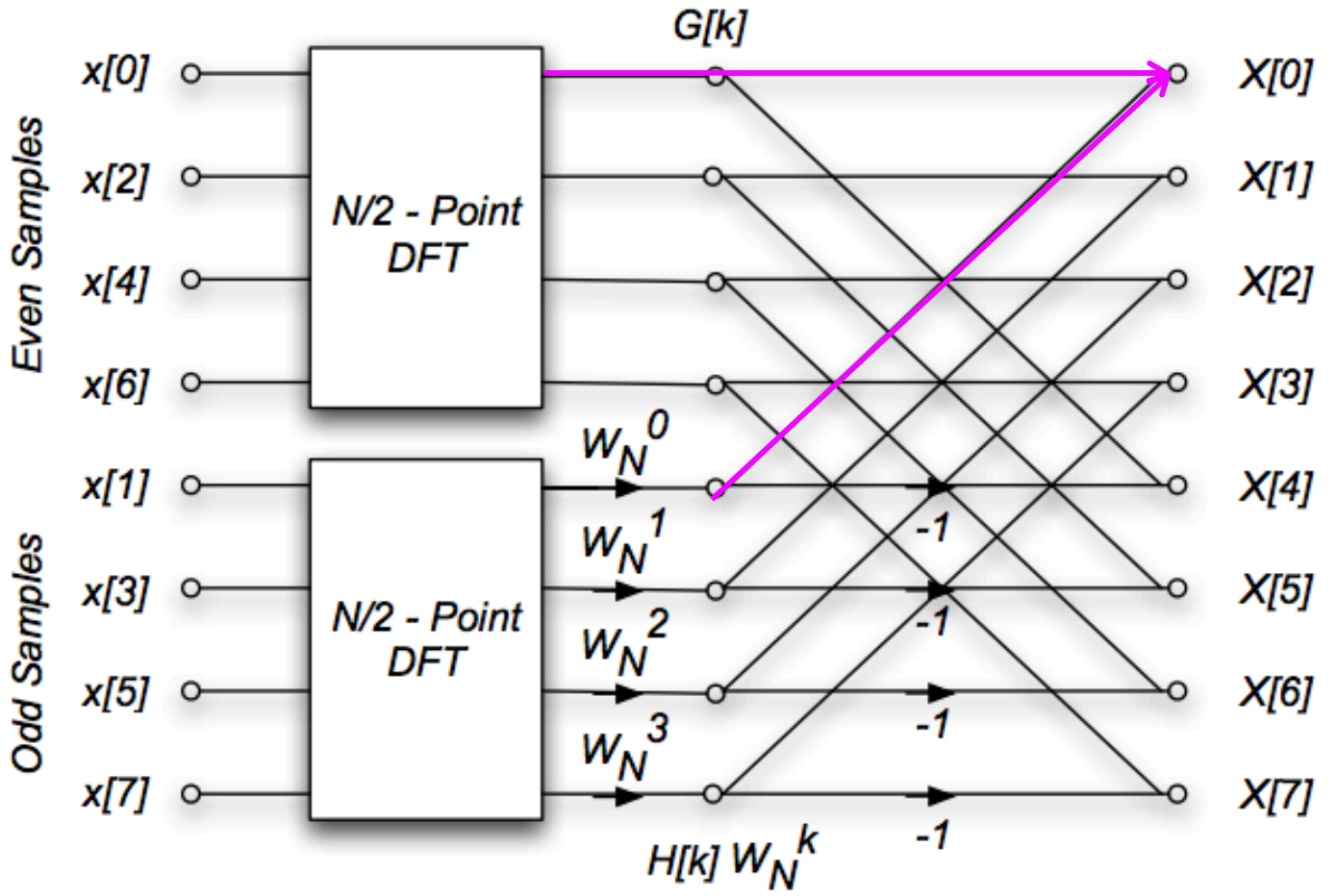

$X[0] = G[0] + W_N^0 H[0]$

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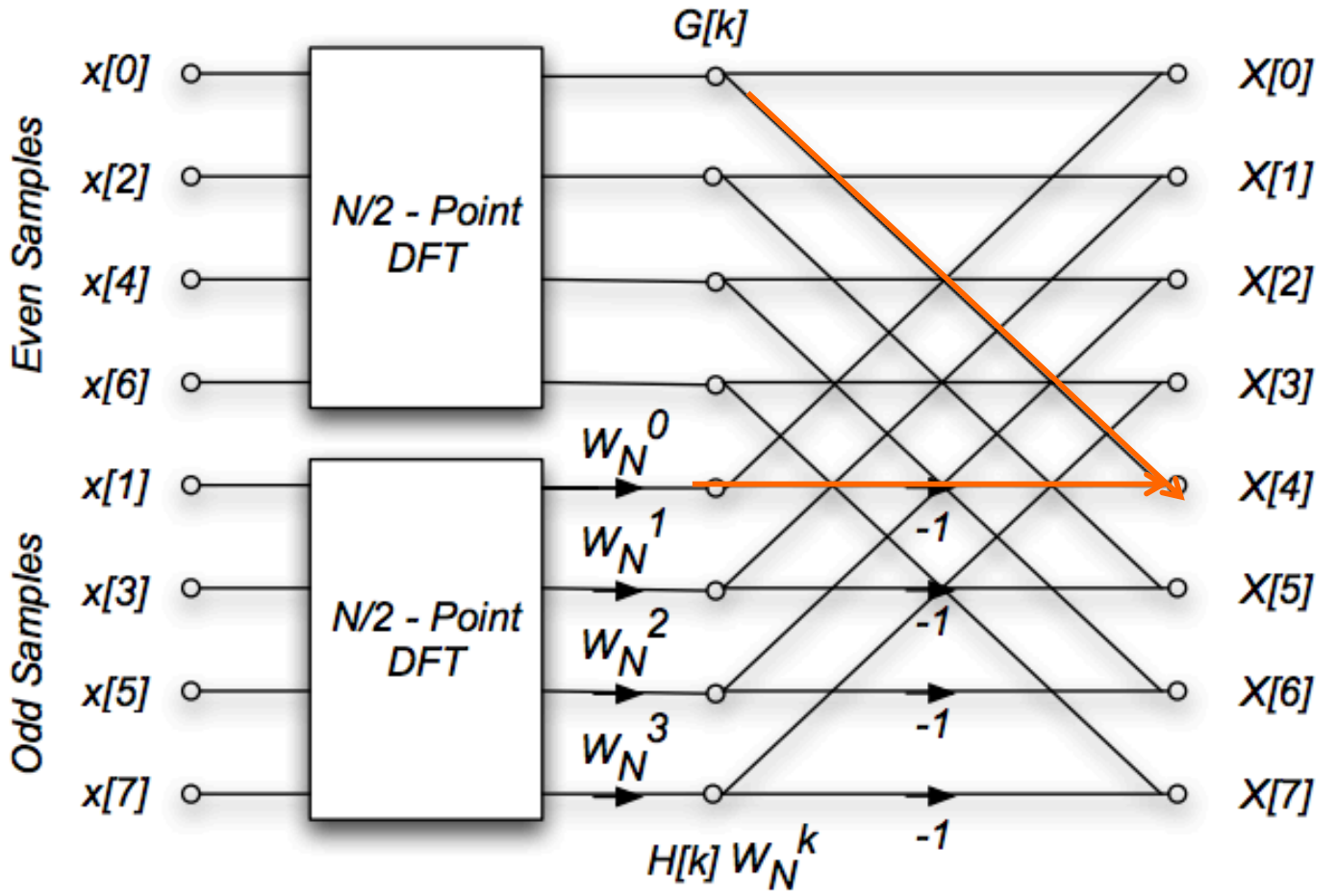
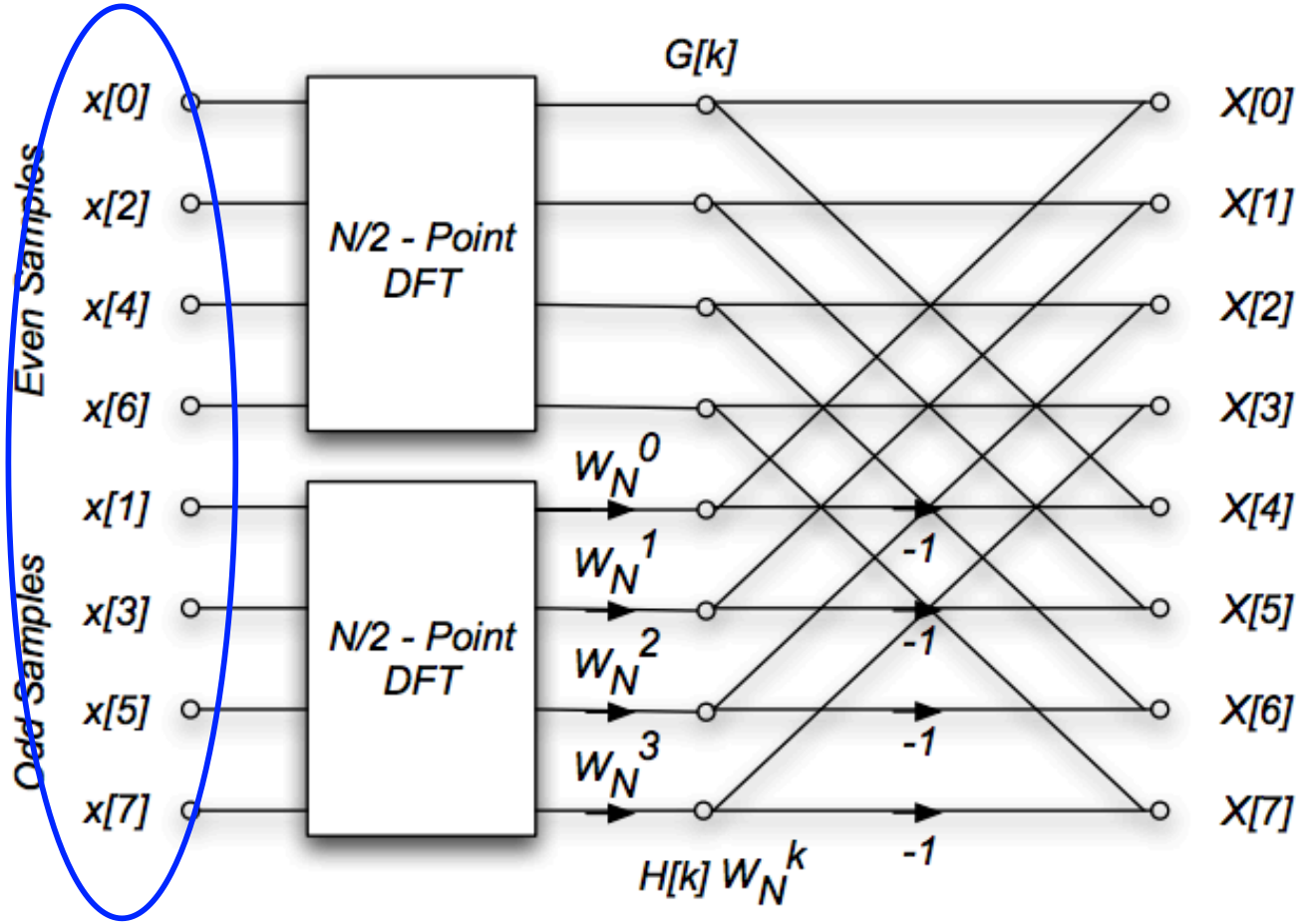


$X[4] = G[0] - W_N^0 H[0]$

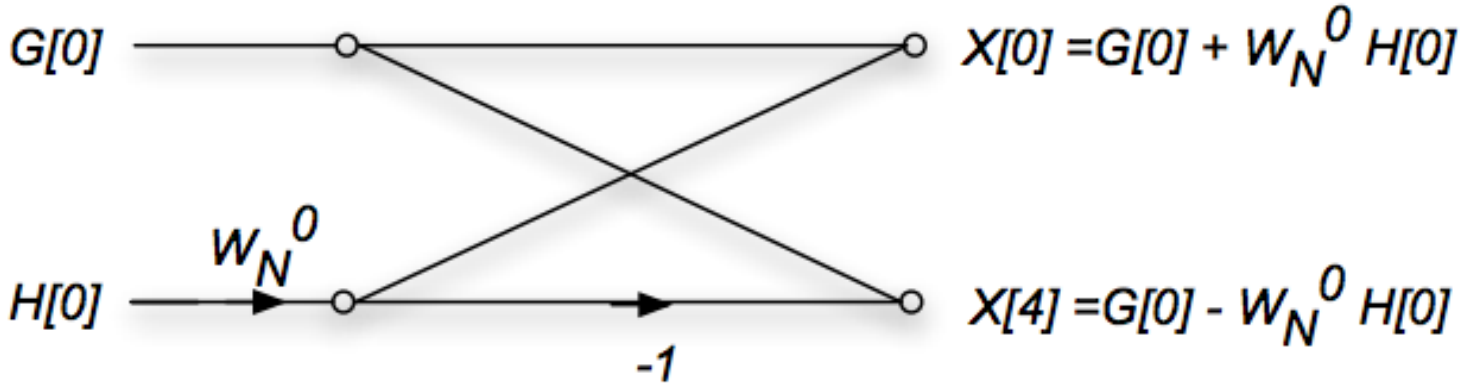
Decimation-in-Time FFT

- Note that the inputs have been reordered so that the outputs come out in their proper sequence.



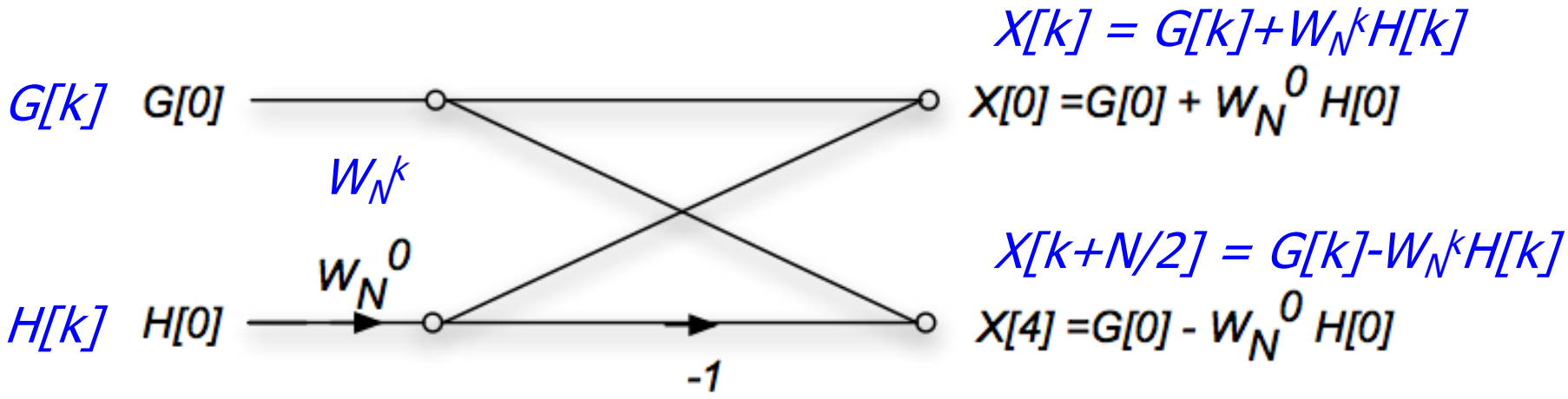
Decimation-in-Time FFT

- Note that the inputs have been reordered so that the outputs come out in their proper sequence.
- We can define a *butterfly operation*, e.g., the computation of X[0] and X[4] from G[0] and H[0]:



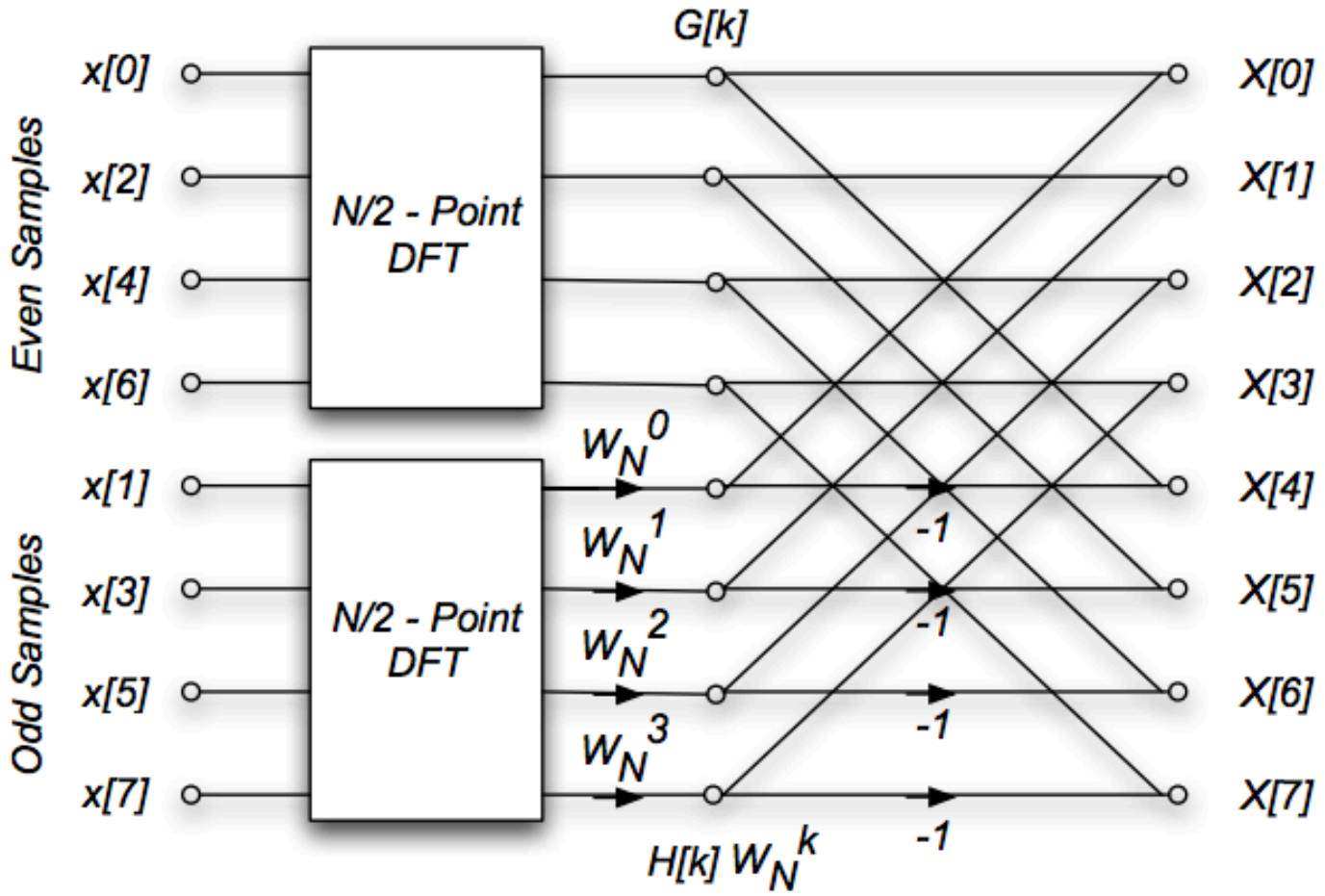
Decimation-in-Time FFT

- Note that the inputs have been reordered so that the outputs come out in their proper sequence.
- We can define a *butterfly operation*, e.g., the computation of X[k] and X[k+N/2] from G[k] and H[k]:



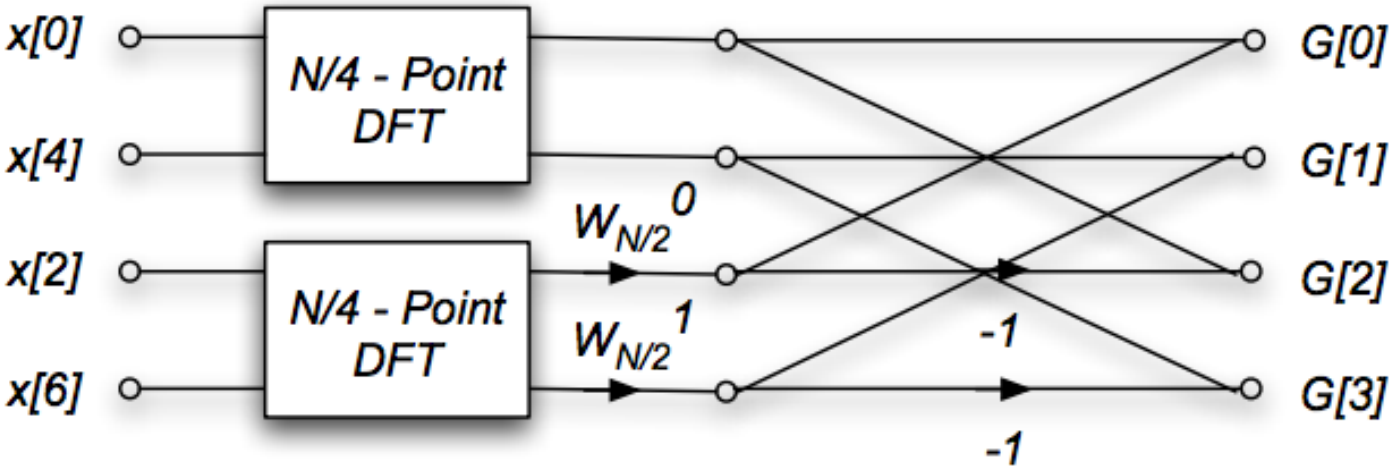
Decimation-in-Time FFT

- Still $O(N^2)$ operations…. What should we do?



Decimation-in-Time FFT

- We can use the same approach for each of the N/2 point DFT's. For the N = 8 case, the N/2 DFTs look like



*Note that the inputs have been reordered again.

Decimation-in-Time FFT

- At this point for the 8 sample DFT, we can replace the N/4 = 2 sample DFT's with a single butterfly. The fundamental eigenfunction is:

$$W_{N/4} = W_{8/4} = W_2 = e^{-j\pi} = -1$$

Decimation-in-Time FFT

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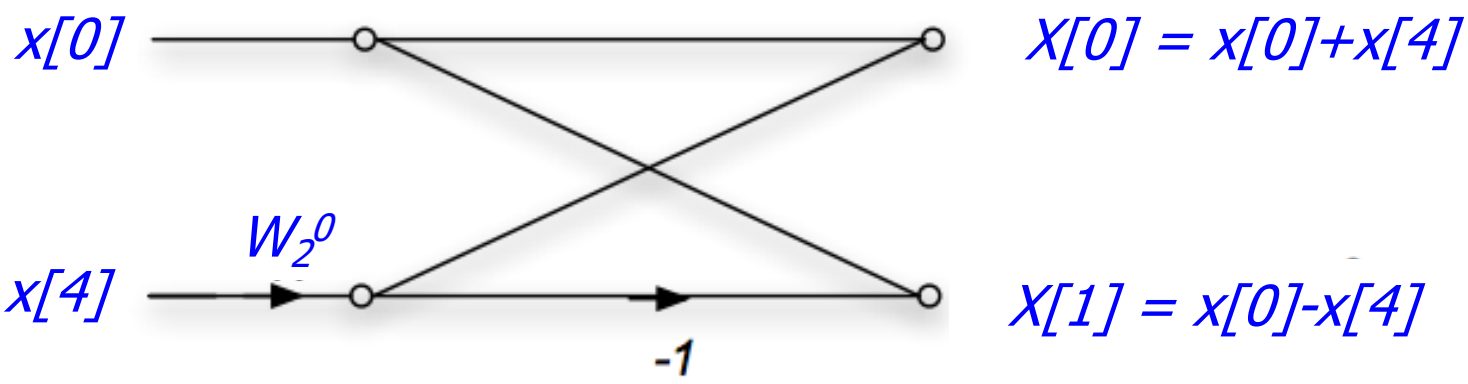
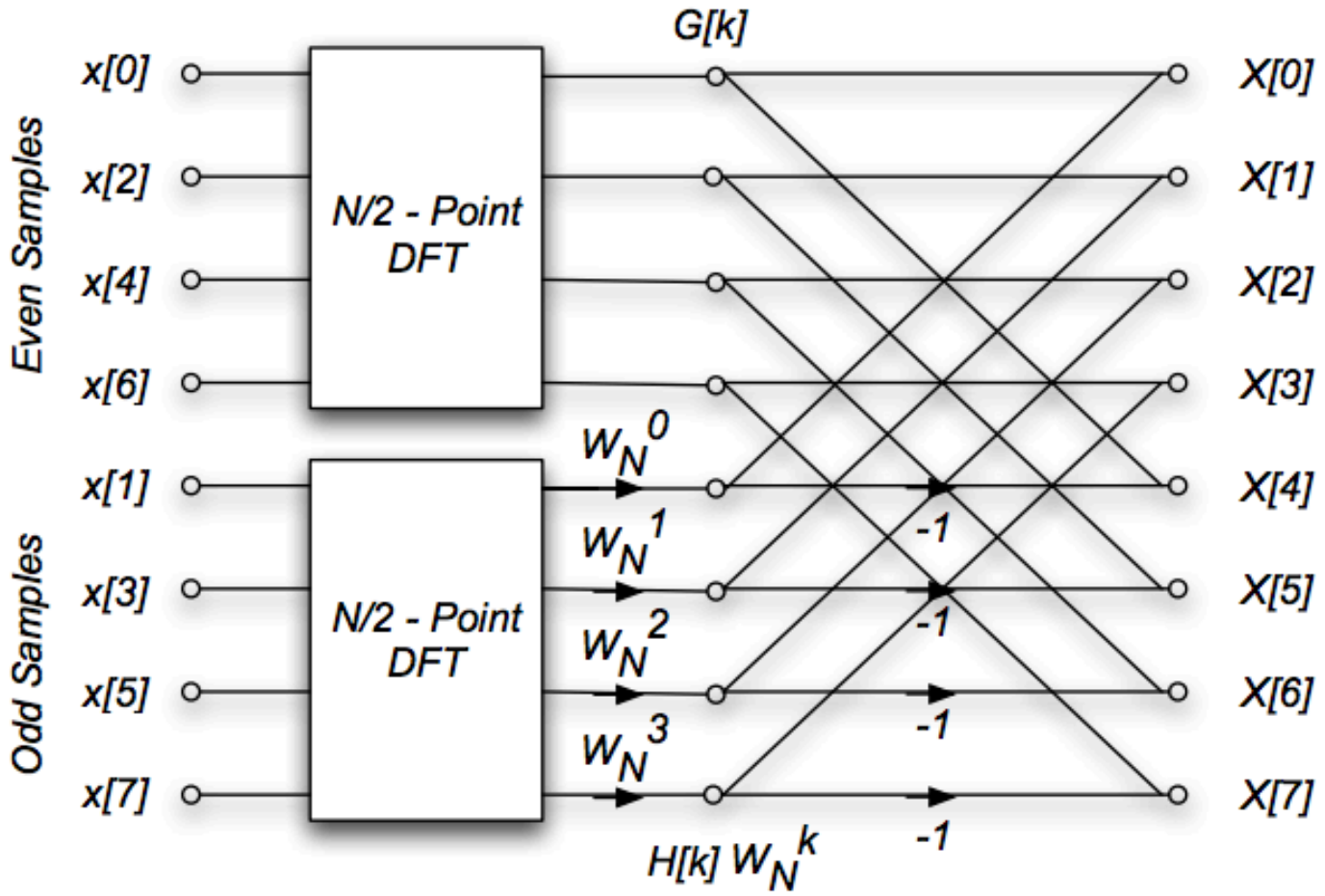


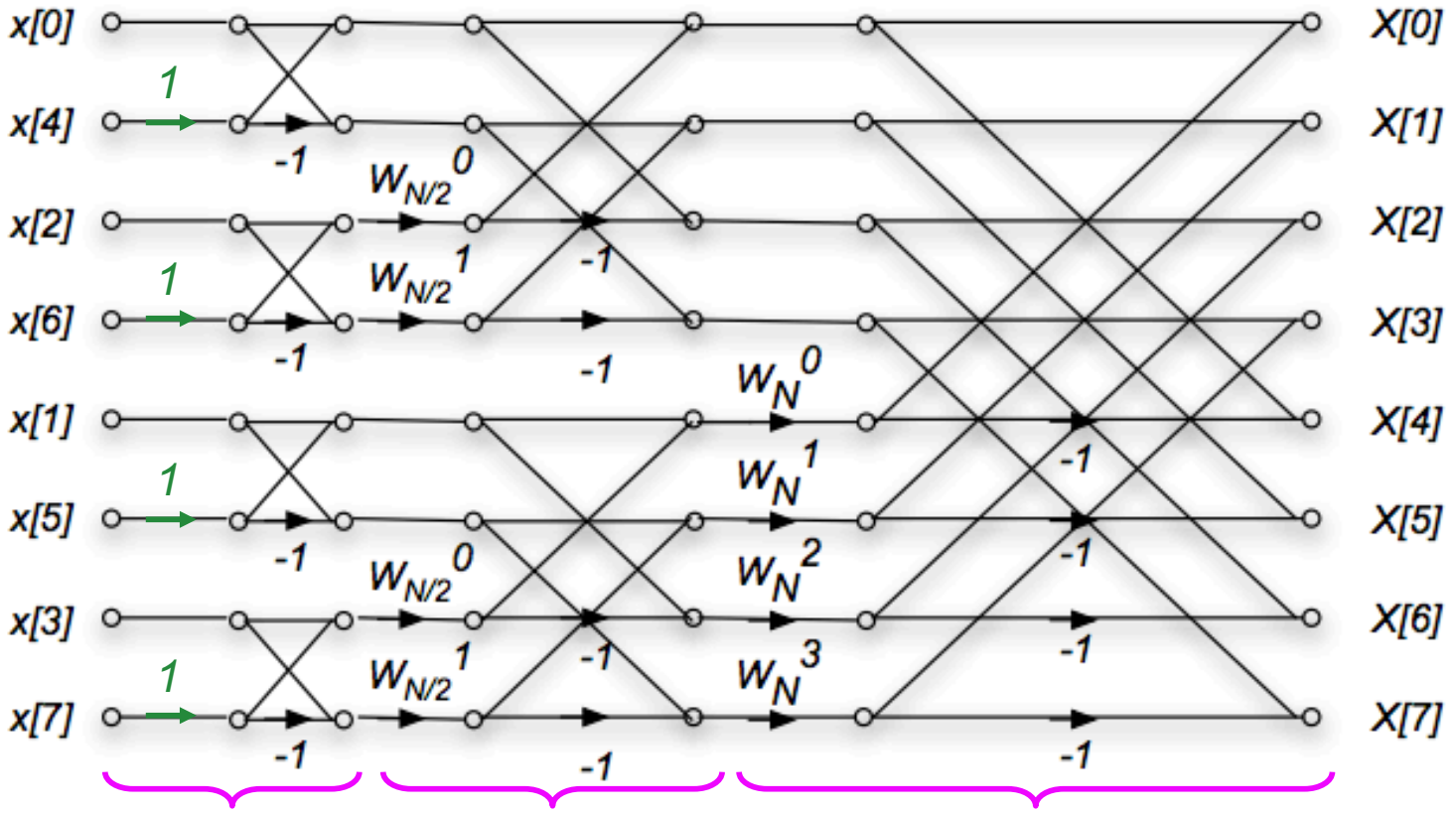
Decimation-in-Time FFT

- Replace N/2-point DFT with 2-point DFT and butterfly operations



Decimation-in-Time FFT

Combining all these stages, the diagram for the 8 sample DFT is:



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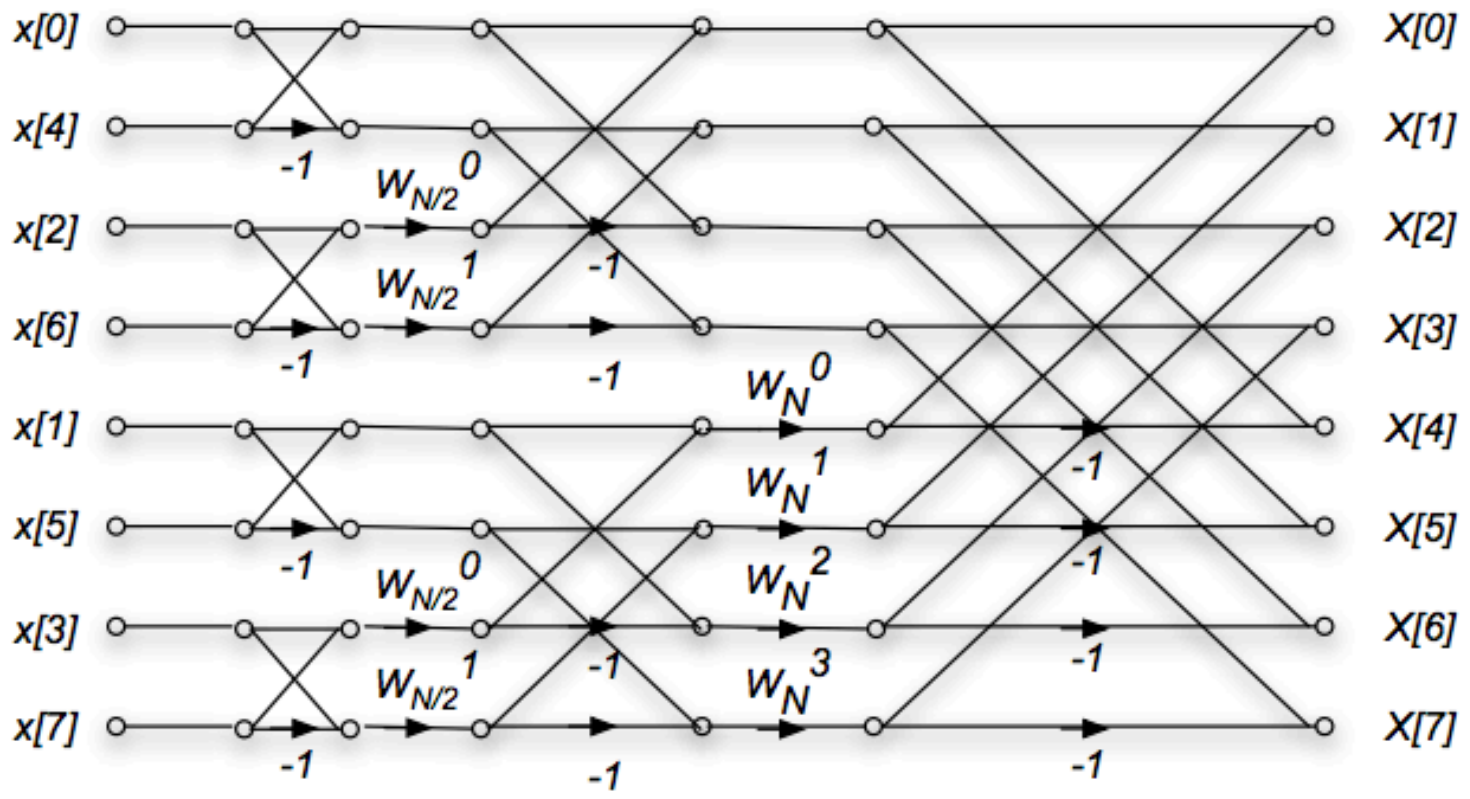

- $3=\log_2(N)=\log_2(8)$ stages
- $4=N/2=8/2$ multiplications in each stage
  - 1st stage has trivial multiplication

Decimation-in-Time FFT

- In general, there are $\log_2 N$ stages of decimation-in-time.
- Each stage requires N/2 complex multiplications, some of which are trivial.
- The total number of complex multiplications is $(N/2)\log_2 N$, or is $O(N \log_2 N)$

Decimation-in-Time FFT

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- Each stage requires N/2 complex multiplications, some of which are trivial.
- The total number of complex multiplications is $(N/2) \log_2 N$, or is $O(N \log_2 N)$

- The order of the input to the decimation-in-time FFT algorithm must be permuted.
  - Net effect is reversing the bit order of indexes

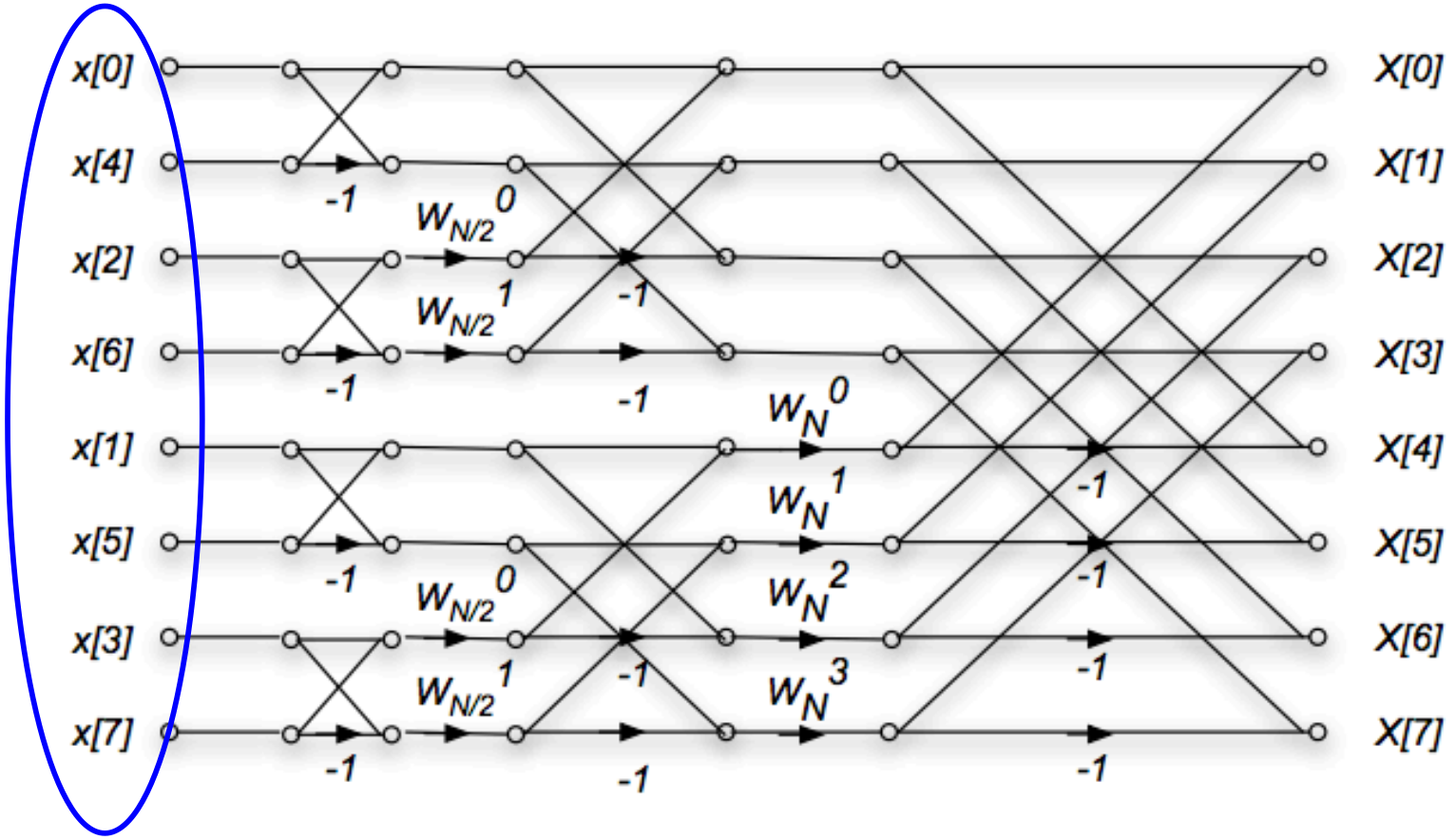
Decimation-in-Time FFT

This is illustrated in the following table for $N = 8$.

Decimal	Binary	Bit-Reversed Binary	Bit-Reversed Decimal
0	000	000	0
1	001	100	4
2	010	010	2
3	011	110	6
4	100	001	1
5	101	101	5
6	110	011	3
7	111	111	7

Decimation-in-Time FFT

Combining all these stages, the diagram for the 8 sample DFT is:



Decimation-in-Frequency FFT

The DFT is

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk}$$

Decimation-in-Frequency FFT

The DFT is

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk}$$

If we only look at the even samples of $X[k]$, we can write $k = 2r$,

$$X[2r] = \sum_{n=0}^{N-1} x[n] W_N^{n(2r)}$$

Decimation-in-Frequency FFT

The DFT is

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk}$$

If we only look at the even samples of $X[k]$, we can write $k = 2r$,

$$X[2r] = \sum_{n=0}^{N-1} x[n] W_N^{n(2r)}$$

We split this into two sums, one over the first $N/2$ samples, and the second of the last $N/2$ samples.

$$X[2r] = \sum_{n=0}^{(N/2)-1} x[n] W_N^{2rn} + \sum_{n=0}^{(N/2)-1} x[n + N/2] W_N^{2r(n+N/2)}$$

Decimation-in-Frequency FFT

But $W_N^{2r(n+N/2)} = W_N^{2rn} W_N^{rN} = W_N^{2rn}$

We can then write

$$X[2r] = \sum_{n=0}^{(N/2)-1} x[n] W_N^{2rn} + \sum_{n=0}^{(N/2)-1} x[n+N/2] W_N^{2r(n+N/2)}$$

$$= \sum_{n=0}^{(N/2)-1} x[n] W_N^{2rn} + \sum_{n=0}^{(N/2)-1} x[n+N/2] W_N^{2rn}$$

Decimation-in-Frequency FFT

But $W_N^{2r(n+N/2)} = W_N^{2rn} W_N^{rN} = W_N^{2rn} = W_{N/2}^{rn}$.
We can then write

$$
\begin{aligned}
X[2r] &= \sum_{n=0}^{(N/2)-1} x[n] W_N^{2rn} + \sum_{n=0}^{(N/2)-1} x[n+N/2] W_N^{2r(n+N/2)} \\
&= \sum_{n=0}^{(N/2)-1} x[n] W_N^{2rn} + \sum_{n=0}^{(N/2)-1} x[n+N/2] W_N^{2rn} \\
&= \sum_{n=0}^{(N/2)-1} (x[n] + x[n+N/2]) W_{N/2}^{rn}
\end{aligned}
$$

This is the $N/2$-length DFT of first and second half of $x[n]$ summed.

Decimation-in-Frequency FFT

$$X[2r] = \text{DFT}_{\frac{N}{2}}\{(x[n] + x[n + N/2])\}$$
$$X[2r+1] = \text{DFT}_{\frac{N}{2}}\{(x[n] - x[n + N/2])\, W_N^n\}$$

(By a similar argument that gives the odd samples)

Decimation-in-Frequency FFT

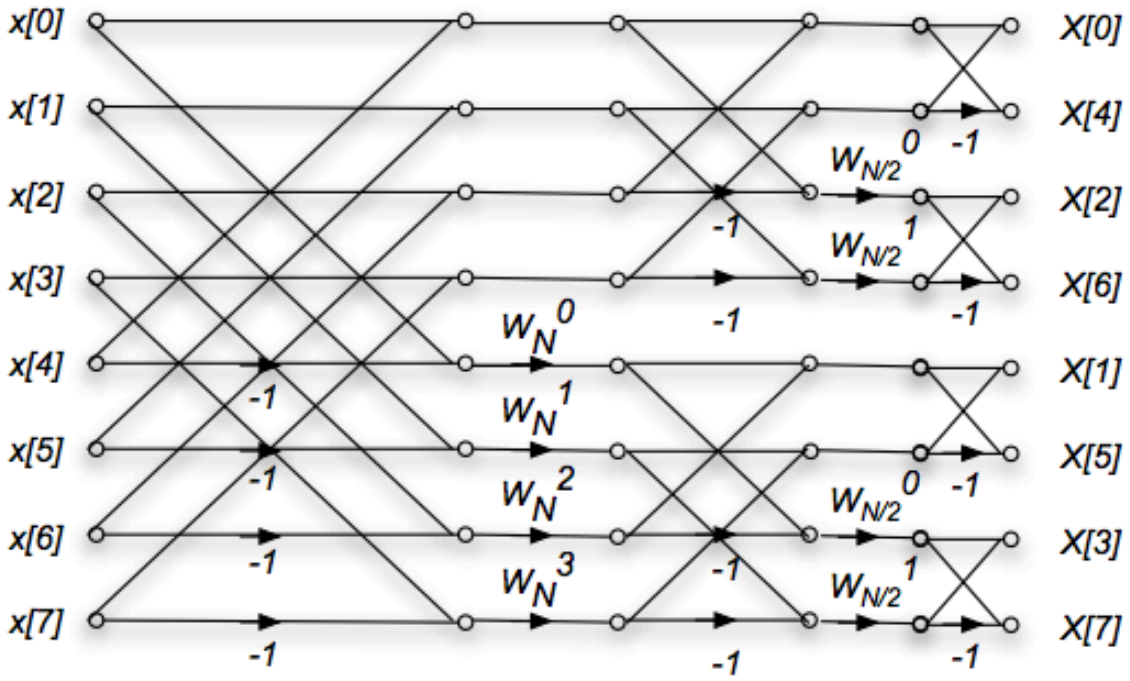
$$X[2r] = \text{DFT}_{\frac{N}{2}}\{(x[n] + x[n + N/2])\}$$
$$X[2r+1] = \text{DFT}_{\frac{N}{2}}\{(x[n] - x[n + N/2])\, W_N^n\}$$

(By a similar argument that gives the odd samples)

- Continue the same approach on the N/2 DFTs, and N/4 DFTs until we reach the 2-point DFT, which is a simple butterfly operation

Decimation-in-Frequency FFT

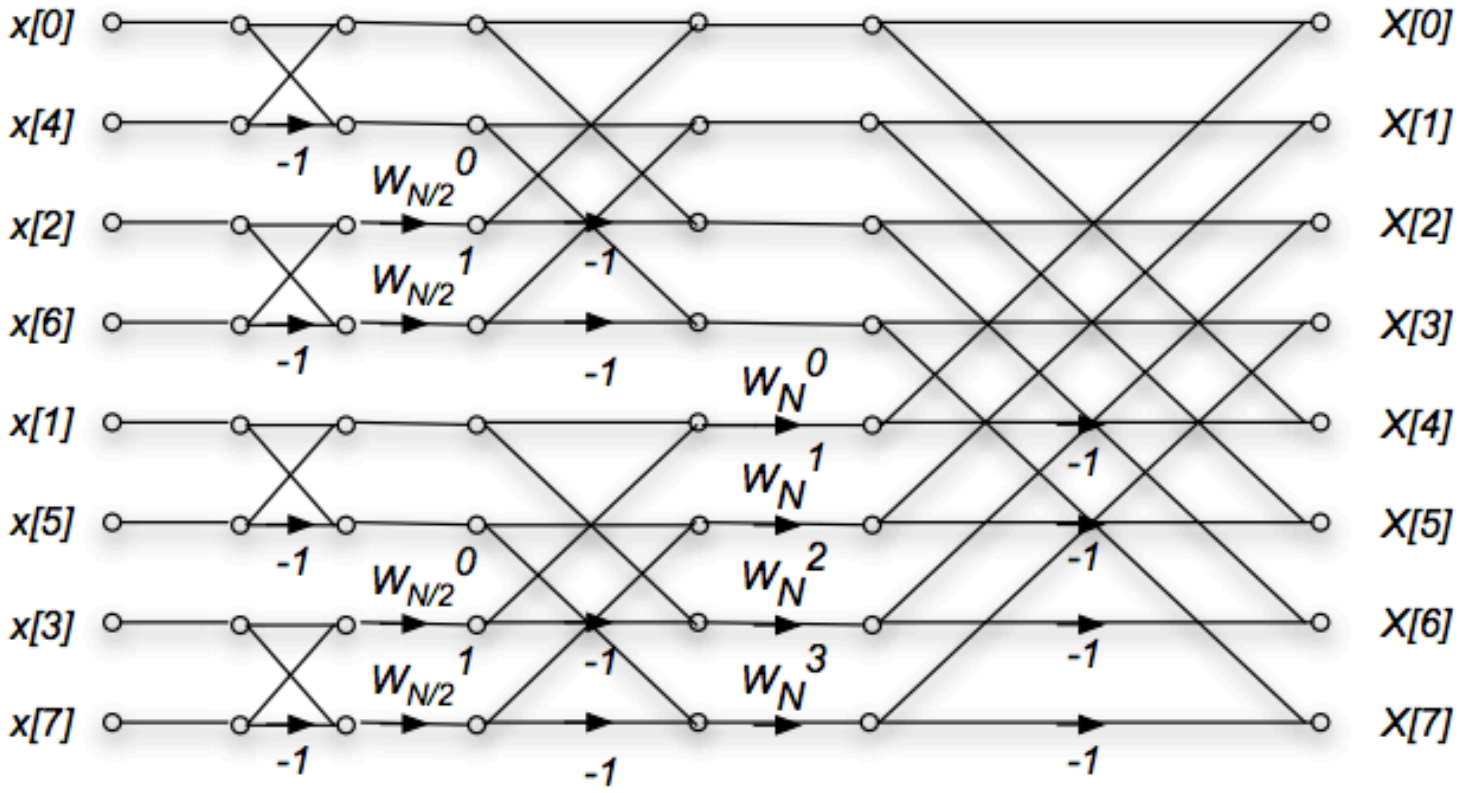
The diagram for and 8-point decimation-in-frequency DFT is as follows



This is just the decimation-in-time algorithm reversed!
The inputs are in normal order, and the outputs are bit reversed.

Decimation-in-Time FFT

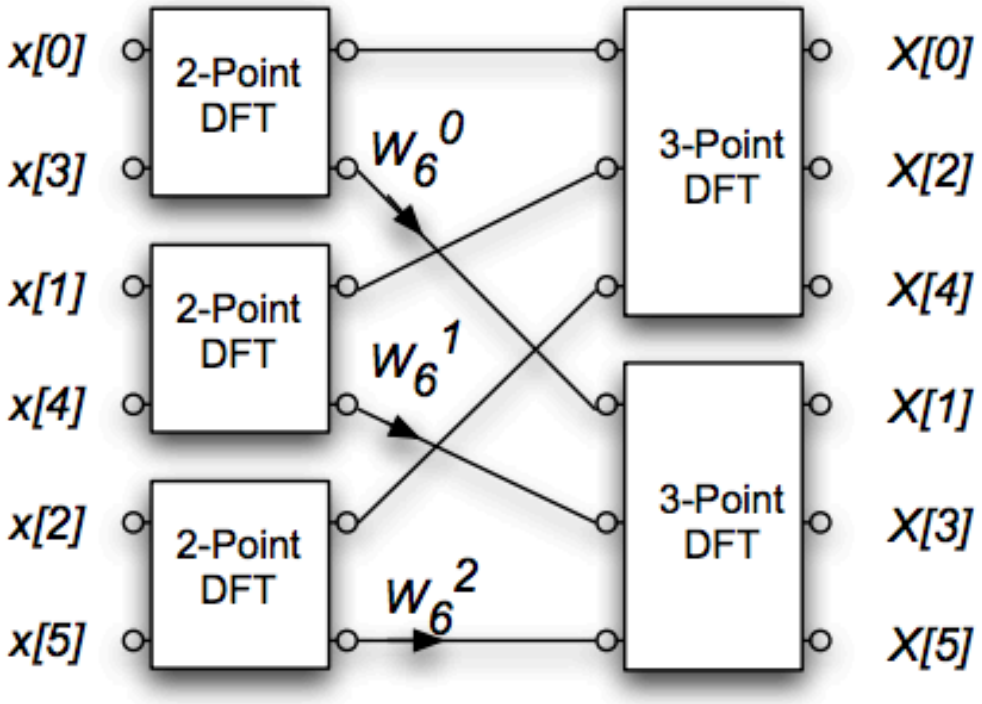
Combining all these stages, the diagram for the 8 sample DFT is:



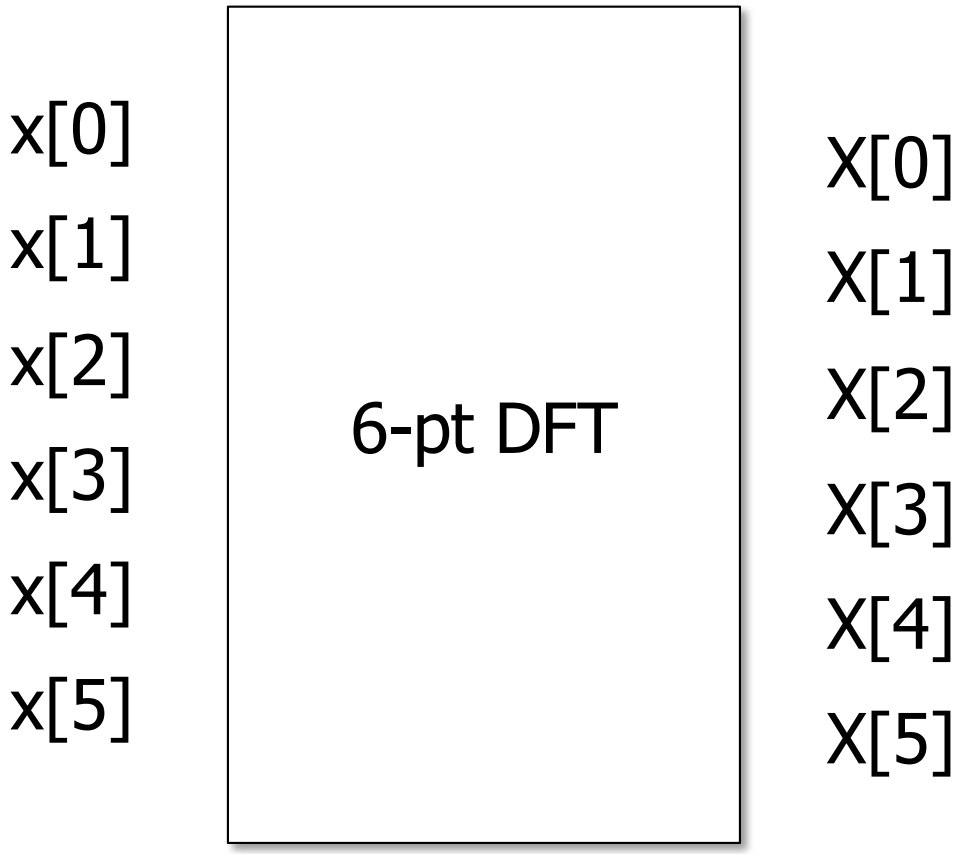
- $3=\log_2(N)=\log_2(8)$ stages
- $4=N/2=8/2$ multiplications in each stage
  - 1st stage has trivial multiplication

Non-Power-of-2 FFTs

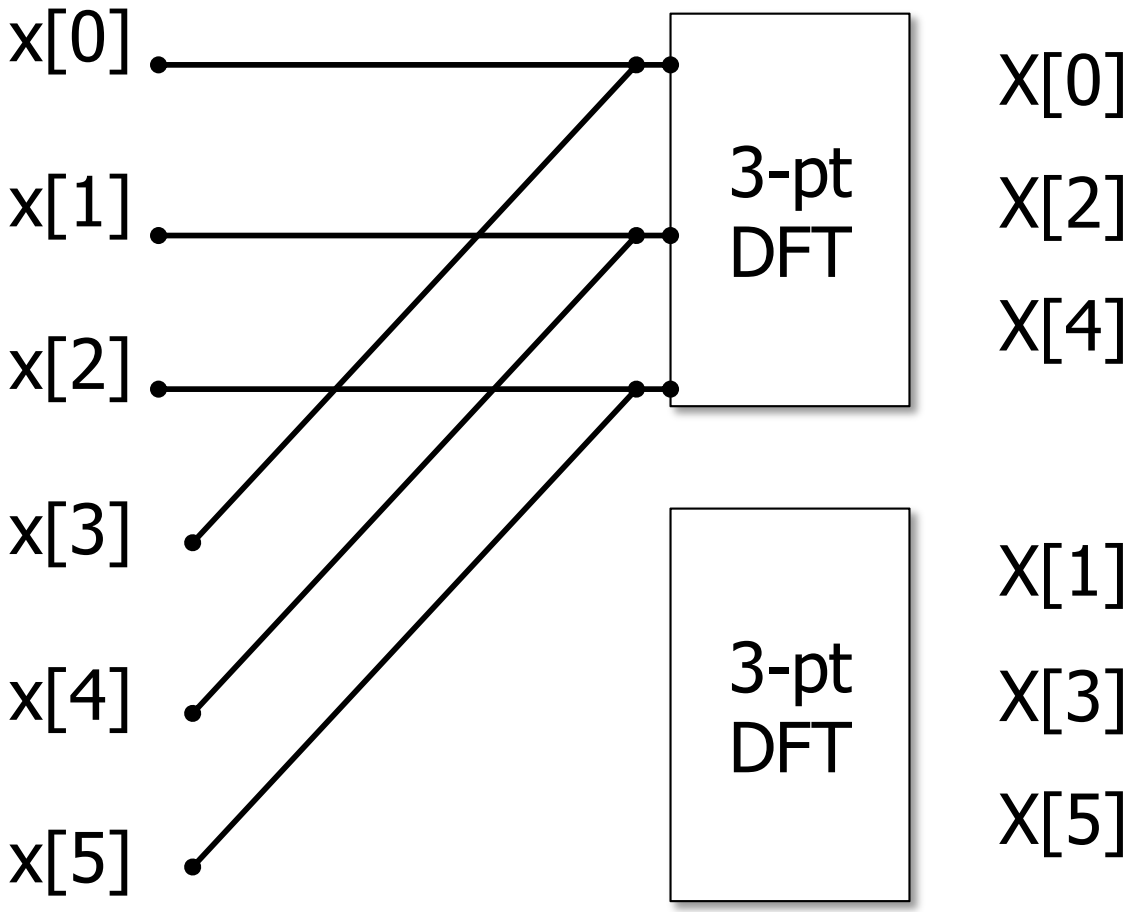
- A similar argument applies for any length DFT, where the length N is a composite number
- For example, if N=6, with decimation-in-frequency you could compute three 2-point DFTs followed by two 3-point DFTs



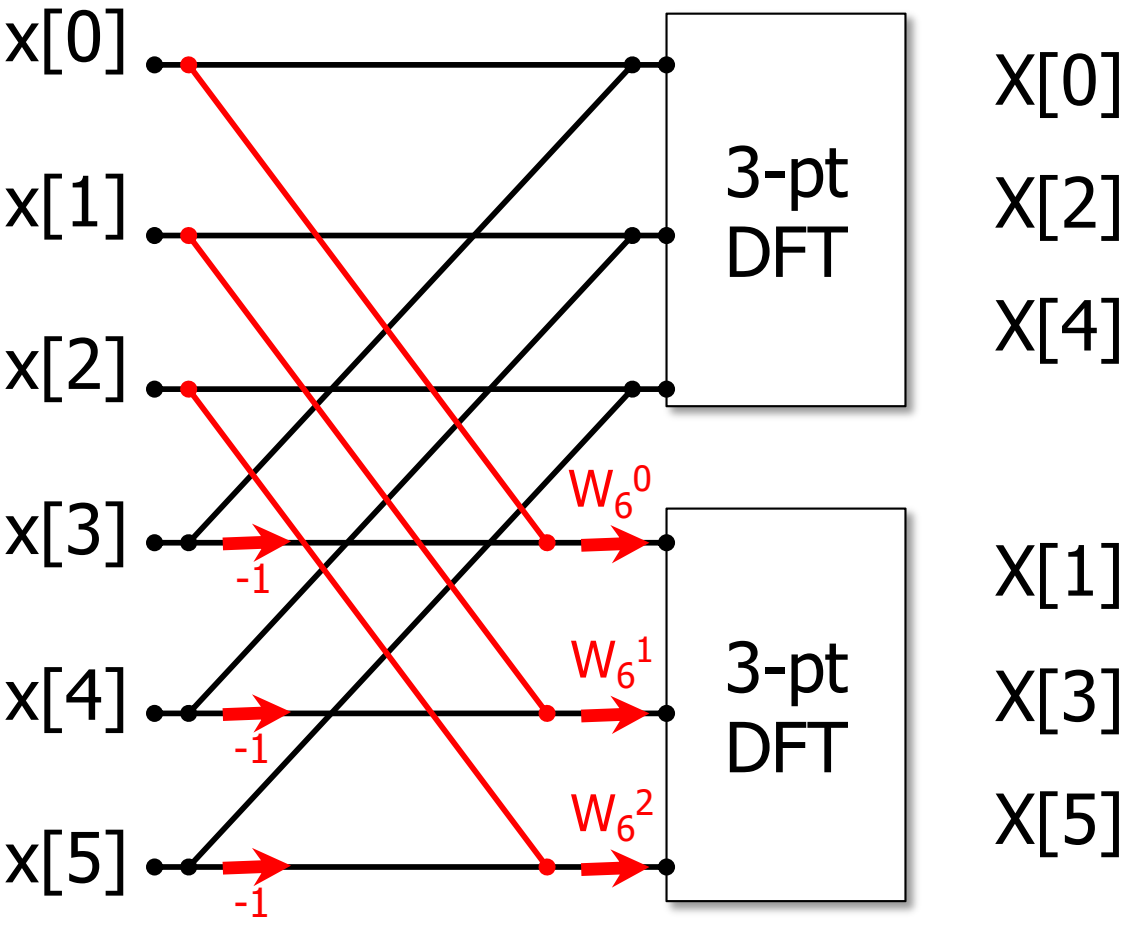
Example: Non-Power-of-2 FFTs



Example: Non-Power-of-2 FFTs



Example: Non-Power-of-2 FFTs



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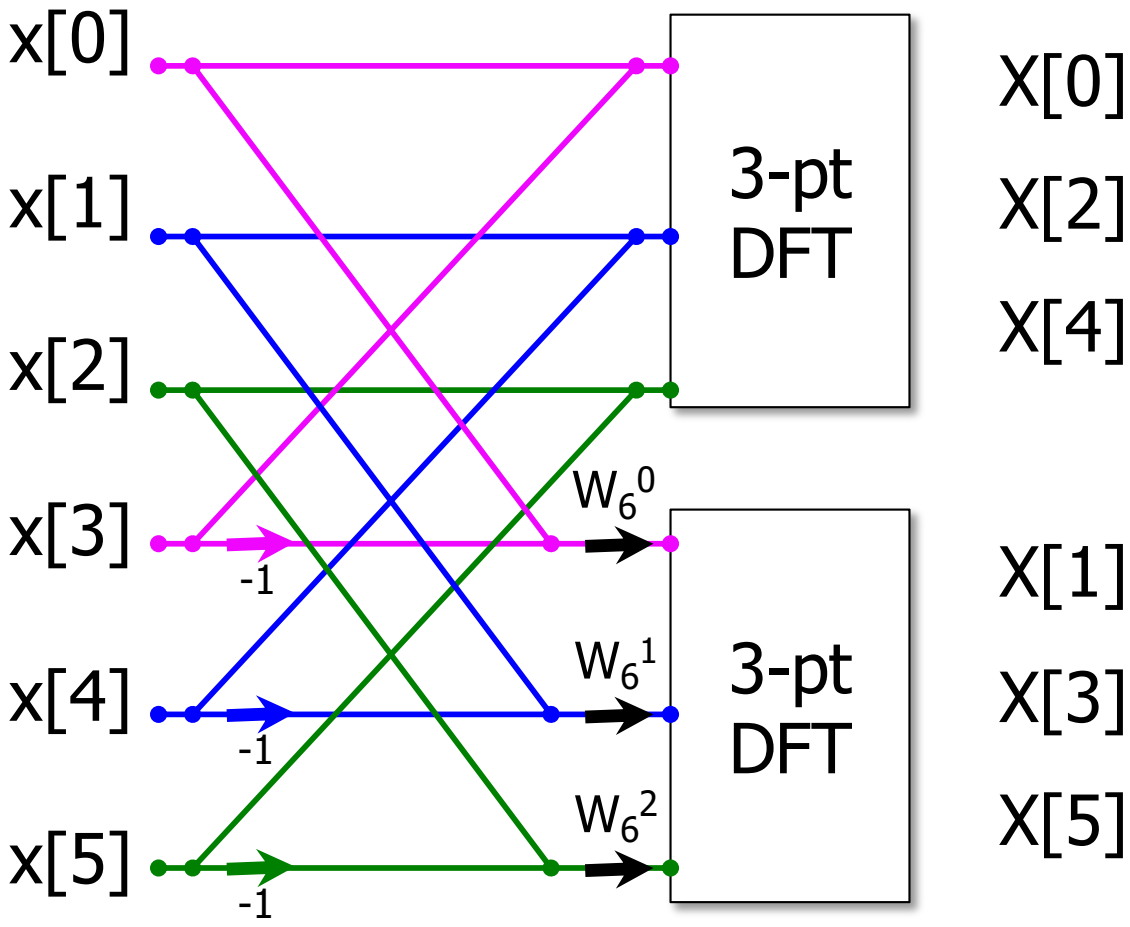

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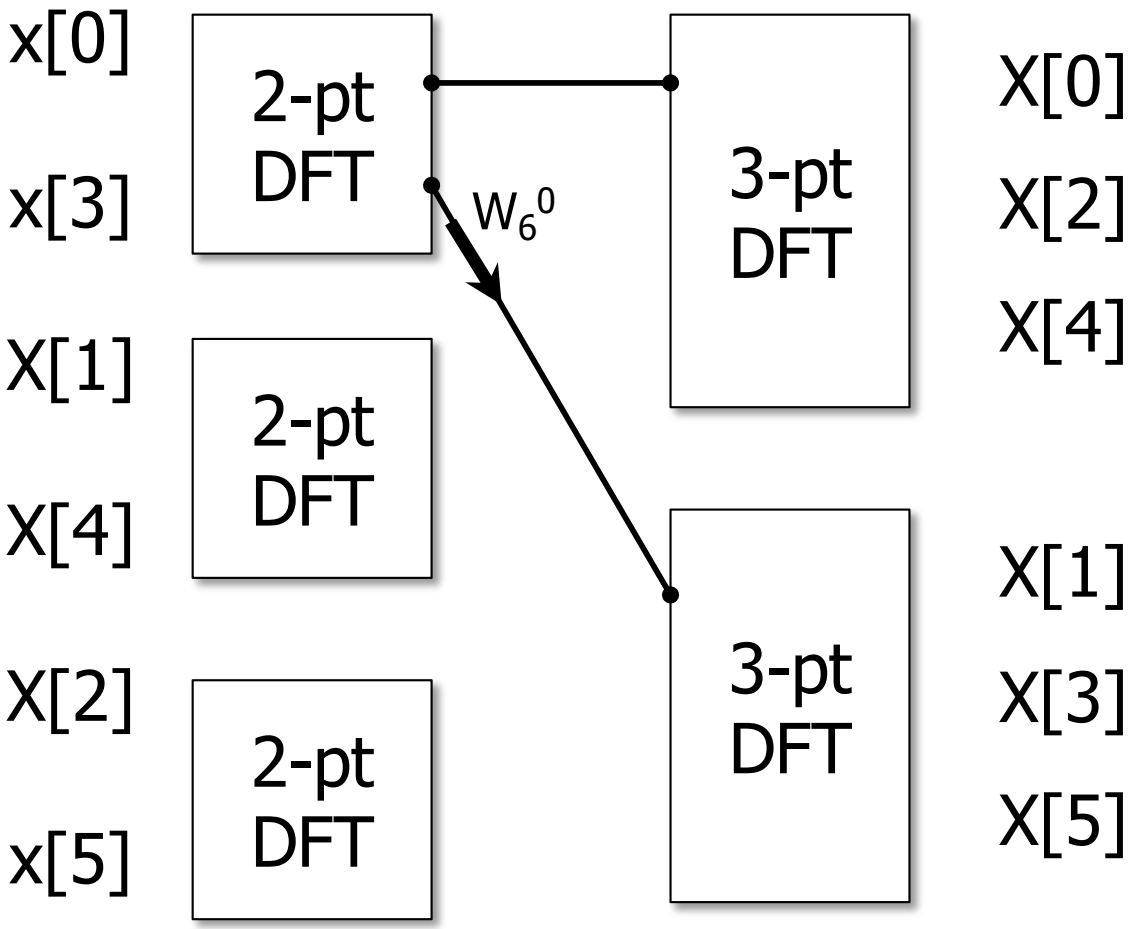

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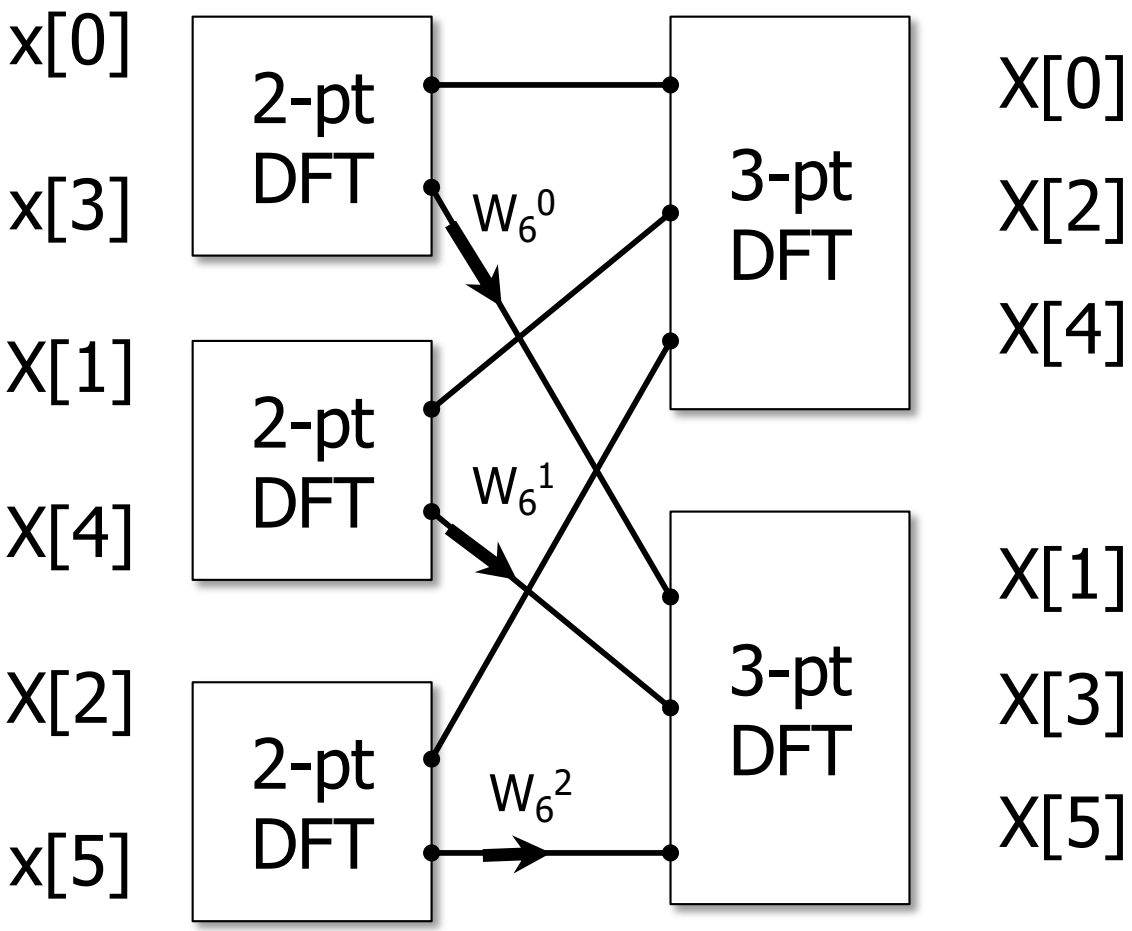


Non-Power-of-2 FFTs

- Good component DFTs are available for lengths up to 20(ish). Many of these exploit the structure for that specific length
  - For example, a factor of

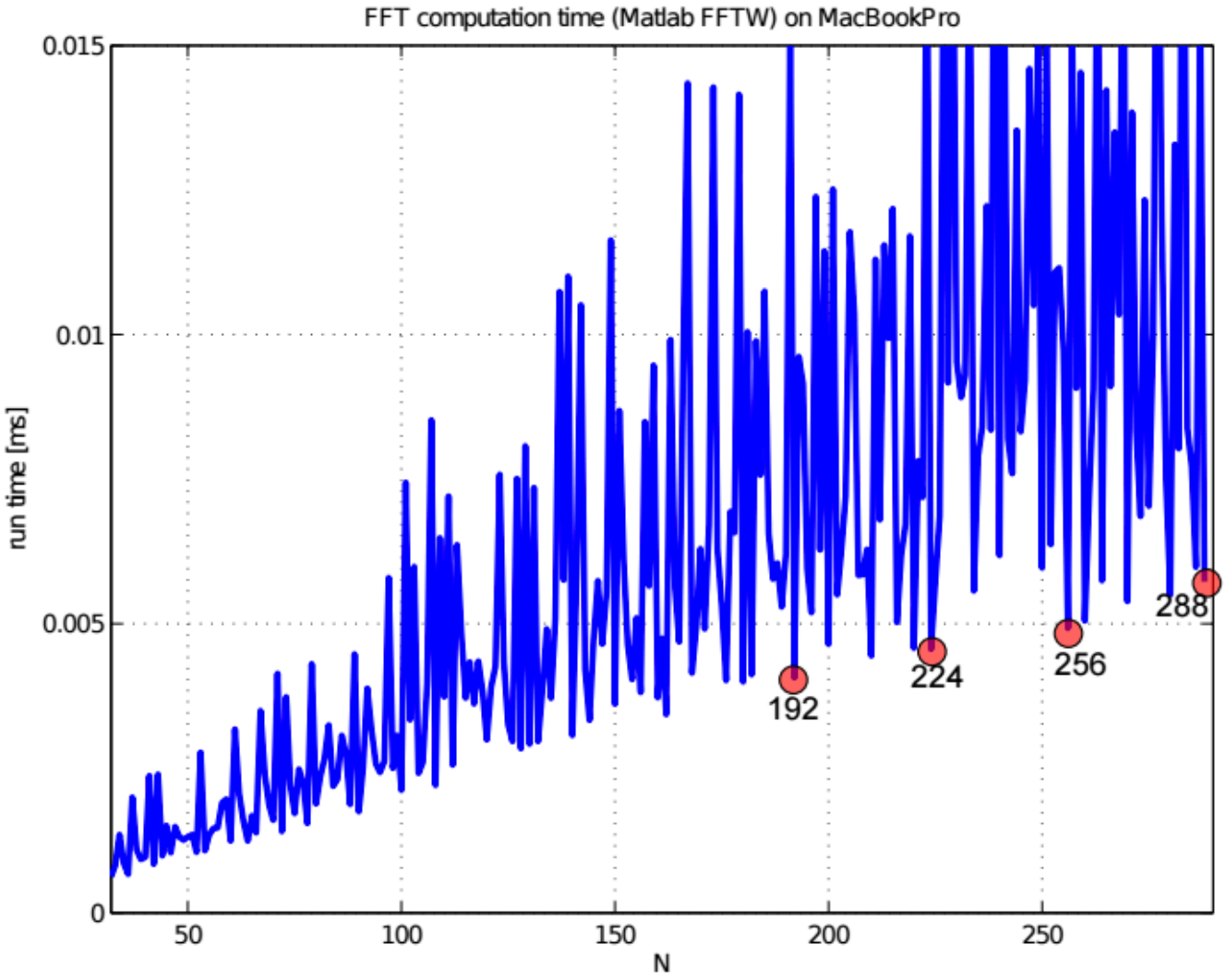
$$W_N^{N/4} = e^{-j\frac{2\pi}{N}(N/4)} = e^{-j\frac{\pi}{2}} = -j$$

    Just swaps the real and imaginary components of a complex number. Hence a DFT of length 4 doesn't require any complex multiples.
  - Half of the multiples of an 8-point DFT also don't require multiplication
  - Composite length FFTs can be very efficient for any length that factors into terms of this order

Non-Power-of-2 FFTs

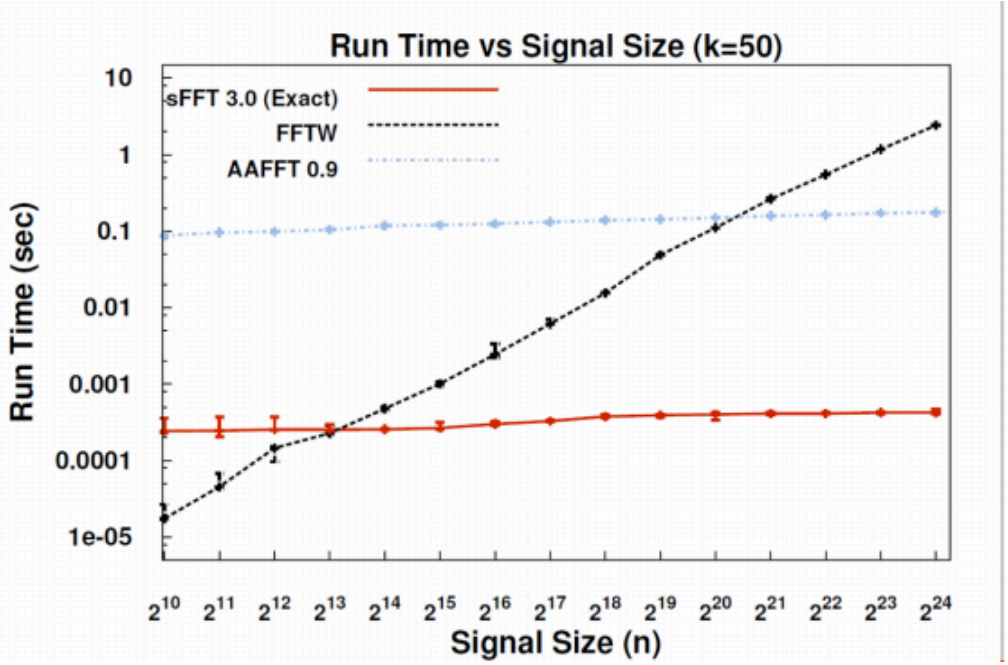
- Historically, the power-of-two FFTs were much faster (better written and implemented).
- For non-power-of-two length, it was faster to zero pad to power of two.
- Recently this has changed. The free FFTW package implements very efficient algorithms for almost any filter length. Matlab has used FFTW since version 6

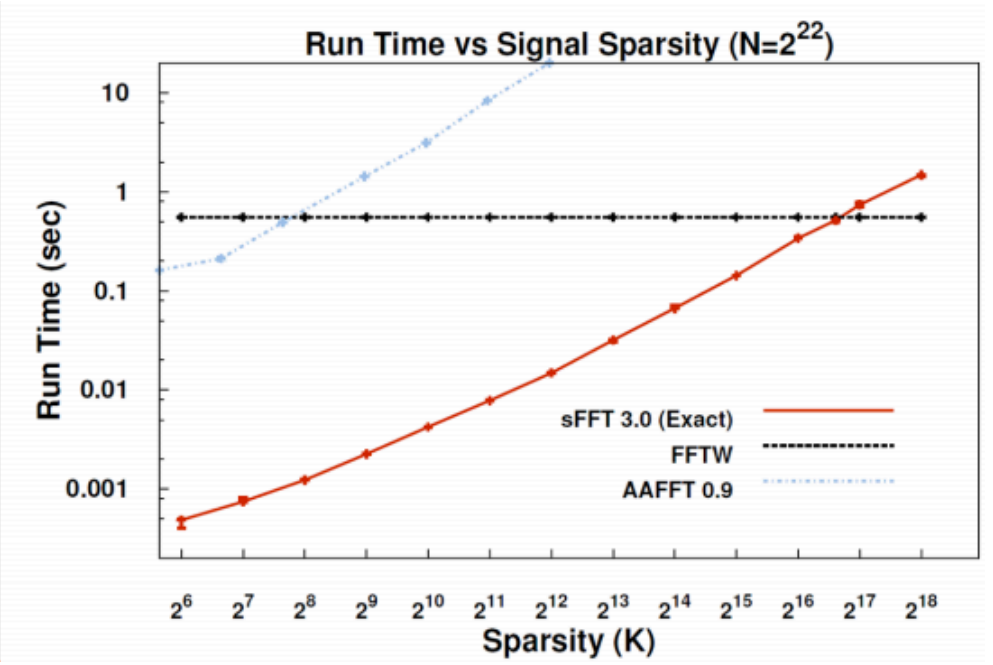
FFT Computation Time



Beyond NlogN

- What if the signal x[n] has a k sparse frequency
  - A. Gilbert et. al, "Near-optimal sparse Fourier representations via sampling
  - H. Hassanieh et. al, "Nearly Optimal Sparse Fourier Transform"
  - Others…
  - O(K Log N) instead of O(N Log N)





From: http://groups.csail.mit.edu/netmit/sFFT/paper.html

Example 1:

A long *periodic* sequence x of period $N = 2^r$ (r is an integer) is to be convolved with a finite-length sequence h of length K.

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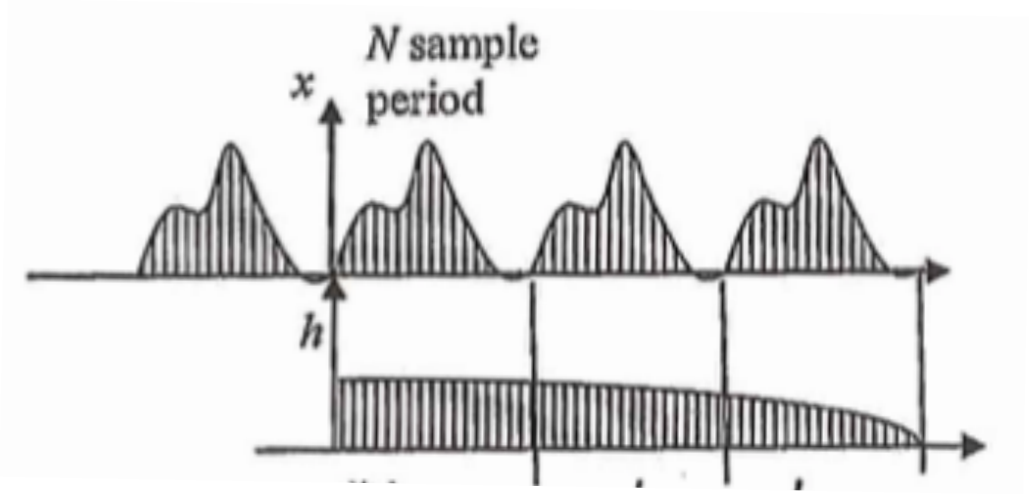
Compare the *total number of multiplications* required in your scheme to that in the direct implementation of FIR filtering. (Consider the case $r = 10$, $m = 10$).



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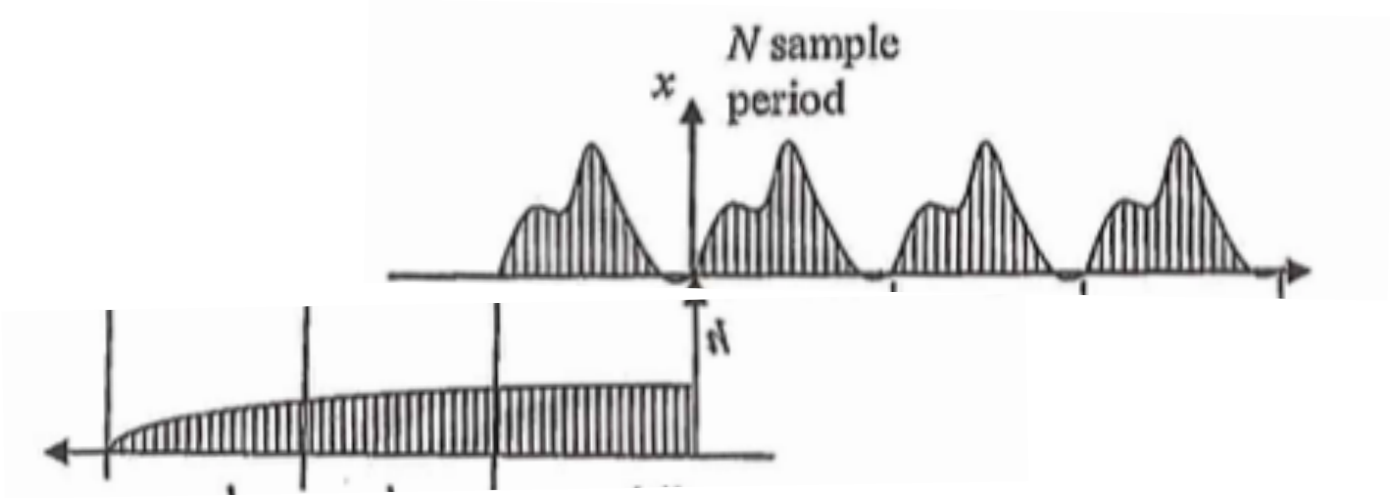

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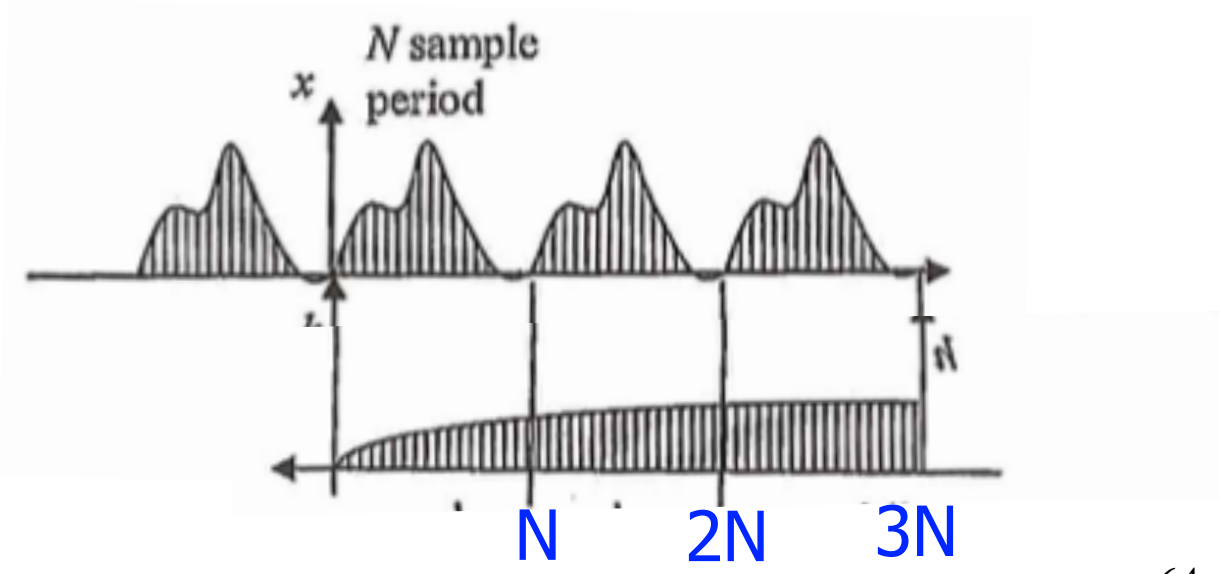

Example 2:

A sequence $x = \{x[n], n = 0,1,\ldots,N-1\}$ is given; let $X(e^{j\omega})$ be its DTFT.

(a) Suppose $N = 10$. You want to evaluate both $X(e^{j2\pi 7/12})$ and $X(e^{j2\pi 3/8})$. The only computation you can perform is one DFT, on any one input sequence of your choice. Can you find the desired DTFT values? *(Show your analysis and explain clearly.)*

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A sequence $x = \{x[n],\ n = 0,1,\ldots,N-1\}$ is given; let $X(e^{j\omega})$ be its DTFT.

(b) Suppose N is large. You want to obtain $X(e^{j\omega})$ at the following $2M$ frequencies:

$$\omega = \frac{2\pi}{M}m,\ \ m = 0,1,\ldots,M-1 \quad \text{and} \quad \omega = \frac{2\pi}{M}m + \frac{2\pi}{N},\ \ m = 0,1,\ldots,M-1.$$

Here $M = 2^{\mu} \ll N = 2^{\nu}$

A standard radix-2 FFT algorithm is available. You may execute the FFT algorithm *once* or *more than once*, and *multiplications* and *additions* outside of the FFT are *allowed*, if necessary.

(i) You want to get the $2M$ DTFT values with as few *total multiplications* as possible (*including those in the FFT*). Give explicitly the best method you can find for this, with an estimate of the *total number of multiplications* needed in terms of M and N.

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(ii) Does your result change if extra multiplications outside of FFTs are *not* allowed?

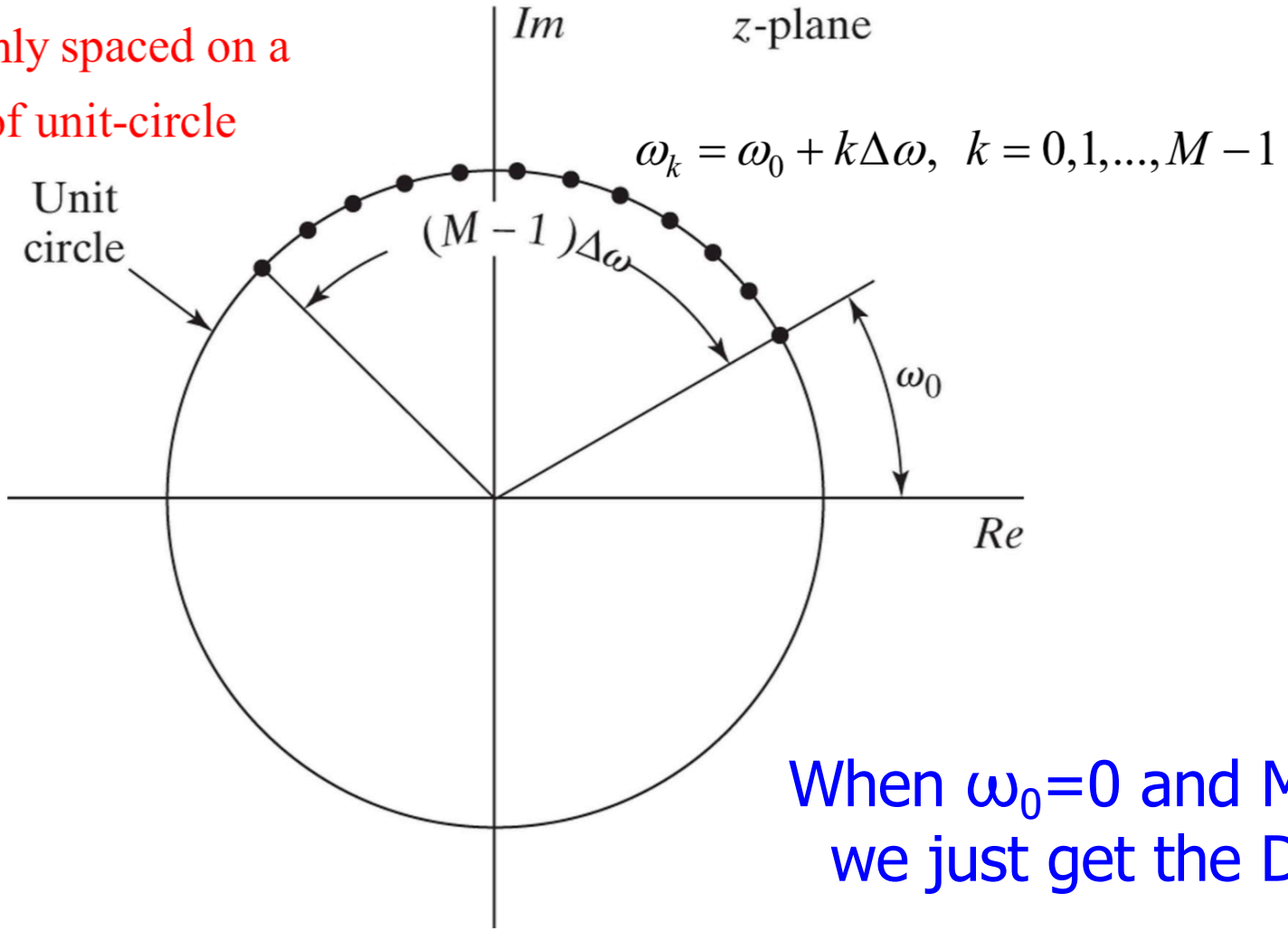
Chirp Transfer Algorithm

Chirp Transform Algorithm

- Uses convolution to evaluate the DFT
- This algorithm is not optimal in minimizing any measure of computational complexity, but it has been useful in a variety of applications, particularly when implemented in technologies that are well suited to doing convolution with a fixed, pre-specified impulse response.
- The CTA is also more flexible than the FFT, since it can be used to compute *any* set of equally spaced samples of the Fourier transform on the unit circle.

Chirp Transform Algorithm

For M points of DTFT uniformly spaced on a sector of unit-circle

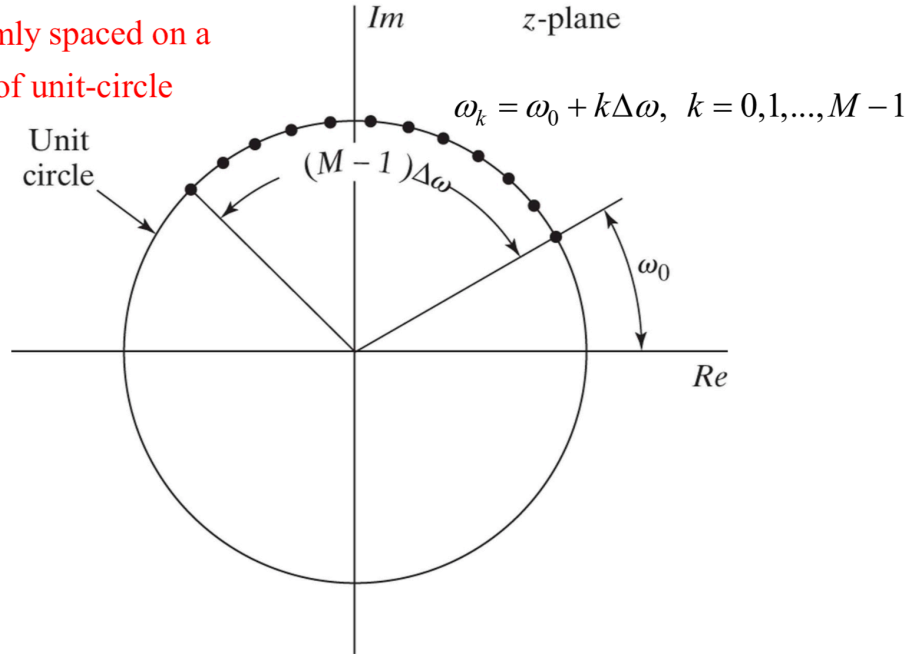


When ω_0=0 and M=N, we just get the DFT

Example: Chirp Transform Parameters

- We have a finite-length sequence x[n] that is nonzero only on the interval n = 0, ..., 25, (Length N=26) and we wish to compute 16 samples of the DTFT $X(e^{j\omega})$ at the frequencies $\omega_k = 2\pi/27 + 2\pi k/1024$ for k = 0, . . . , 15.

For M points of DTFT uniformly spaced on a sector of unit-circle



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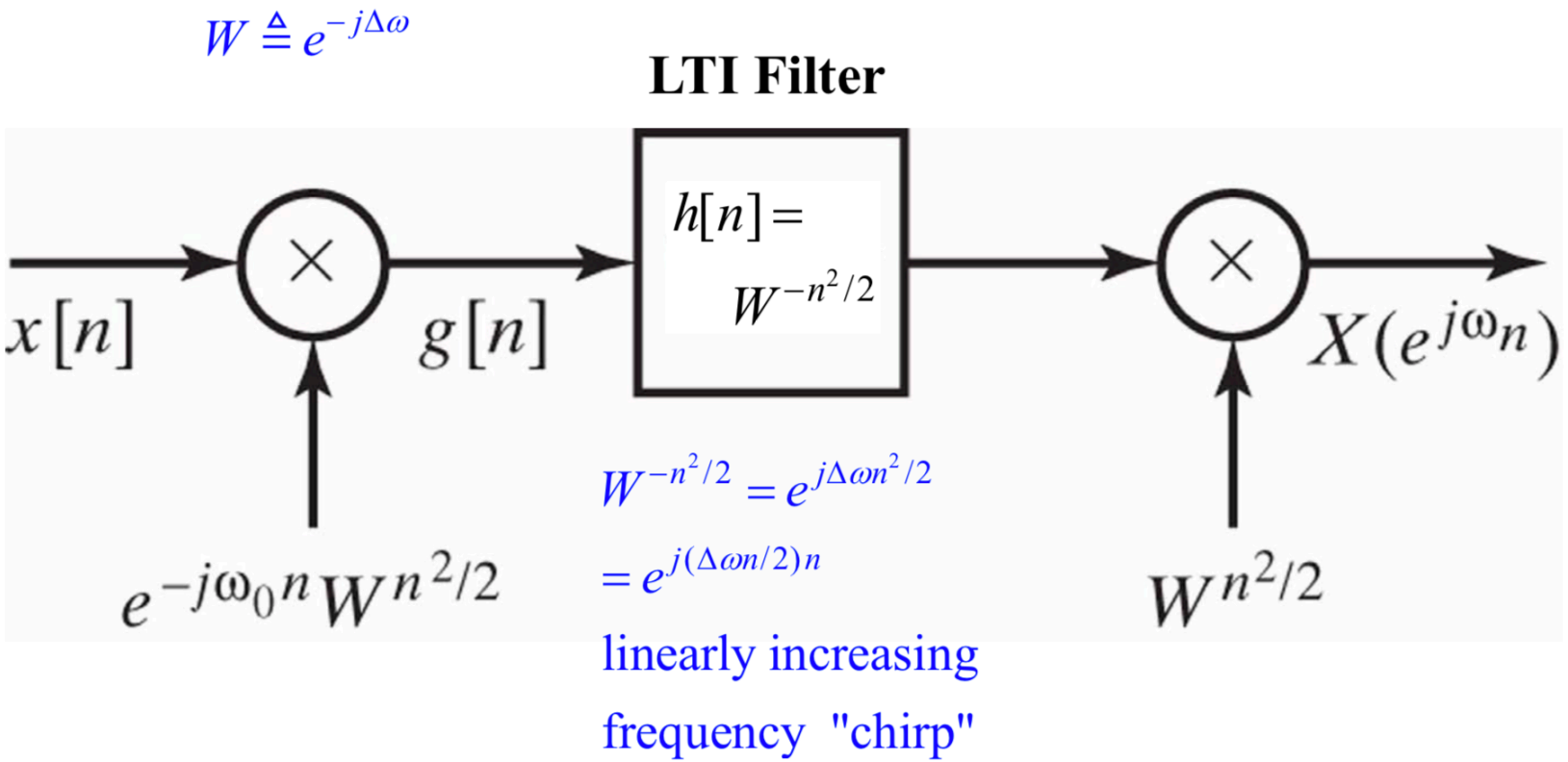
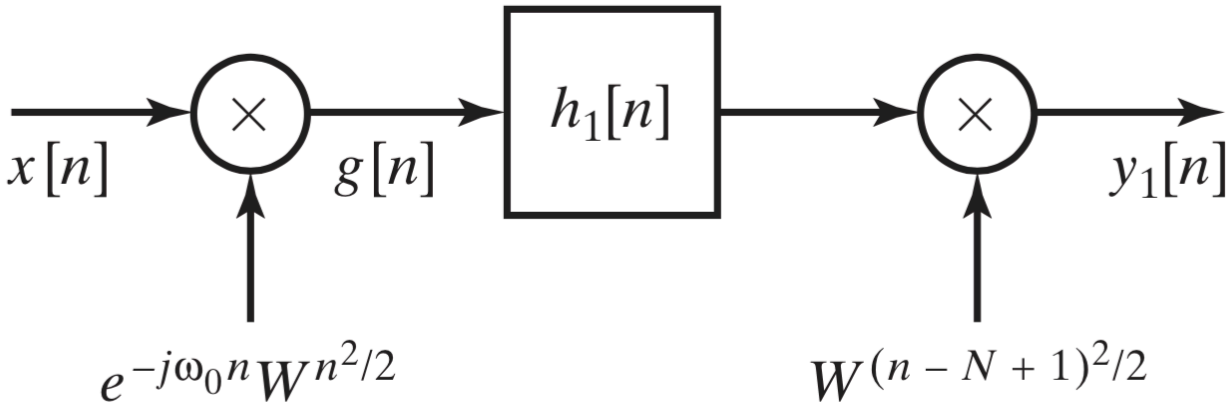


Causal FIR CTA

$$h_1[n] = \begin{cases} W^{-(n-N+1)^2/2}, & n = 0, 1, \ldots, M+N-2, \\ 0, & \text{otherwise.} \end{cases}$$



$$X(e^{j\omega_n}) = y_1[n+N-1], \qquad n = 0, 1, \ldots, M-1.$$

Big Ideas

- Fast Fourier Transform
  - Enable computation of an N-point DFT (or DFT^{-1}) with the order of just $N \cdot \log_2 N$ complex multiplications.
  - Most FFT algorithms decompose the computation of a DFT into successively smaller DFT computations.
    - Decimation-in-time algorithms
    - Decimation-in-frequency
  - Historically, power-of-2 DFTs had highest efficiency
  - Modern computing has led to non-power-of-2 FFTs with high efficiency
  - Sparsity leads to reduced computation on order $K \cdot \log N$
- Design DSP methods to minimize computations!

Admin

- HW 8 due 4/11
- Project 2 posted on 4/11
  - Can work in pairs