Example 1:

A long *periodic* sequence x of period $N = 2^r$ (r is an integer) is to be convolved with a finite-length sequence h of length K.

(a) *Show* that the output y of this convolution (filtering) is *periodic*. What is its period?
(b) Let $K = m\,N$ where m is an integer; N is large. How would you implement this convolution *efficiently*? Explain your analysis clearly. Compare the *total number of multiplications* required in your scheme to that in the direct implementation of FIR filtering. (Consider the case $r = 10$, $m = 10$).

Example 2:

A sequence $x = \{x[n],\ n = 0,1,\dots, N - 1\}$ is given; let $X(e^{j\omega})$ be its DTFT.

(a) Suppose $N = 10$. You want to evaluate both $X(e^{j2\pi\, 7/12})$ and $X(e^{j2\pi\, 3/8})$. The only computation you can perform is one DFT, on any one input sequence of your choice. Can you find the desired DTFT values? *(Show your analysis and explain clearly.)*

(b) Suppose N is large. You want to obtain $X(e^{j\omega})$ at the following $2M$ frequencies:

$$\omega = \frac{2\pi}{M} m\ ,\ m = 0,1,\dots,M\text{-}1 \quad \text{and} \quad \omega = \frac{2\pi}{M} m + \frac{2\pi}{N}\ \ m = 0,1,\dots,M\text{-}1$$

Here $M = 2^{\mu} << N = 2^{v}$

A standard radix-2 FFT algorithm is available. You may execute the FFT algorithm *once* or *more than once*, and *multiplications* and *additions* outside of the FFT are *allowed*, if necessary.

i. You want to get the $2M$ DTFT values with as few *total multiplications* as possible (*including those in the FFT*). Give explicitly the best method you can find for this, with an estimate of the *total number of multiplications* needed in terms of M and N.
ii. Does your result change if extra multiplications outside of FFTs are *not* allowed?