

ESE 5310: Digital Signal Processing

Lecture 8: February 7, 2023

DT/CT Processing of CT/DT Signals

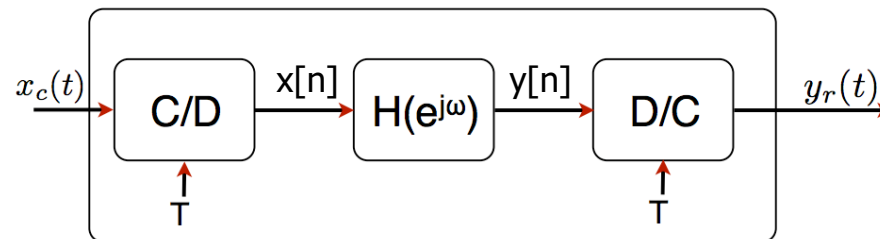


Lecture Outline

- ❑ Sampling/Recon Review
- ❑ DT processing of CT signals
 - Impulse Invariance
- ❑ CT processing of DT signals (why??)

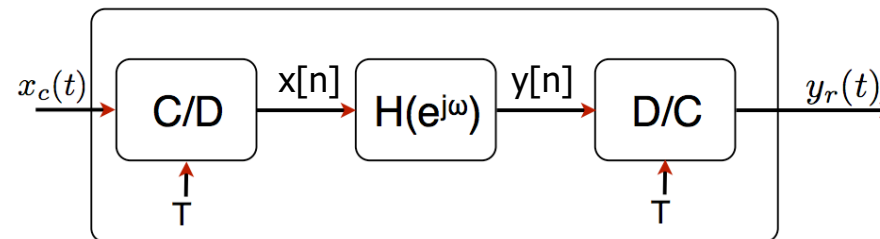
Signal Processing

- ❑ Use theory of sampling (C/D) and reconstruction (D/C) to implement signal processing systems
- ❑ Two cases:
 - Discrete-time processing of continuous-time signals

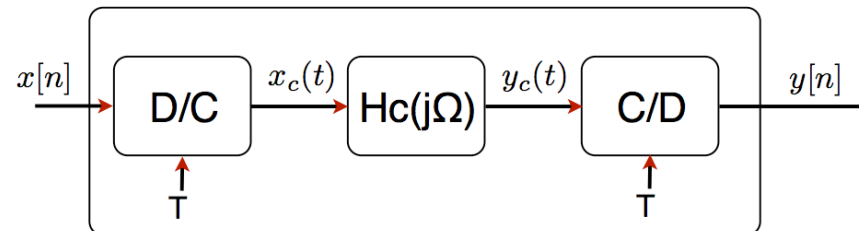


Signal Processing

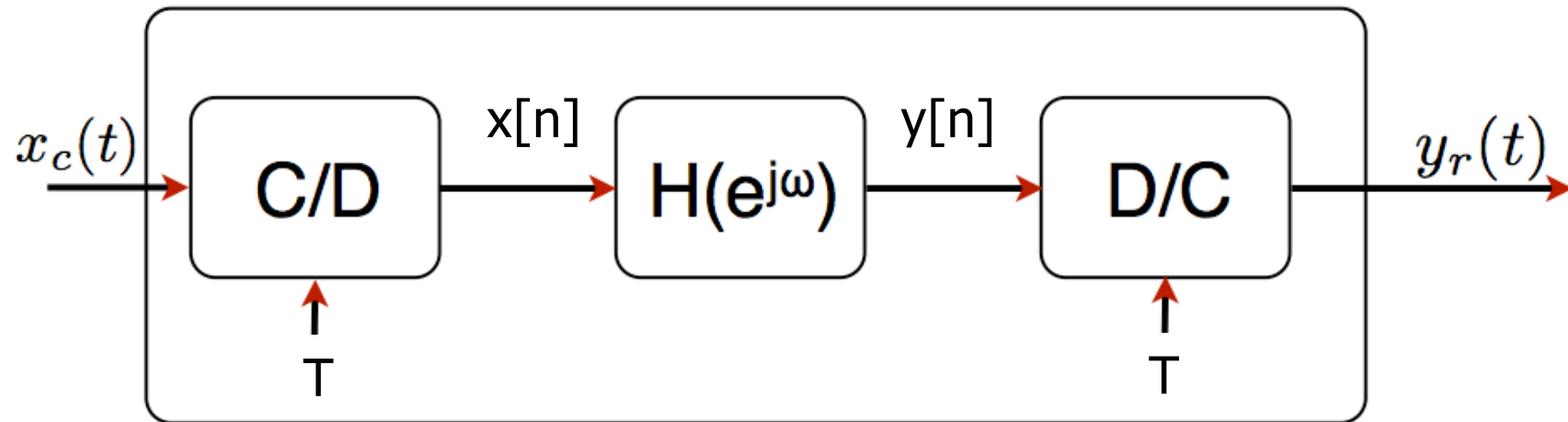
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- Two cases:
 - Discrete-time processing of continuous-time signals



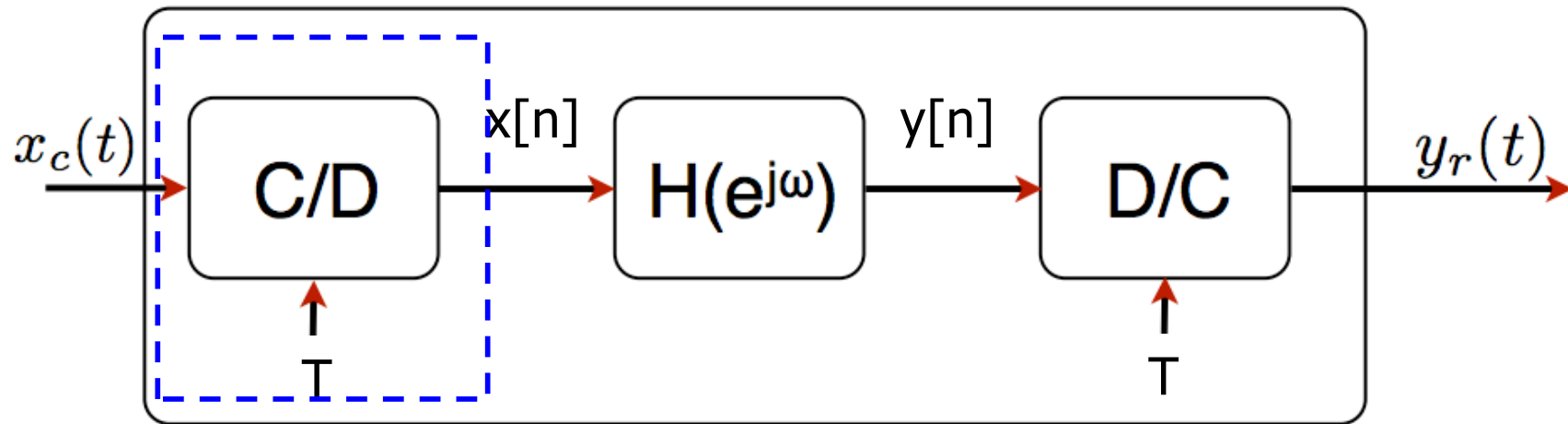
- Continuous-time processing of discrete-time signals



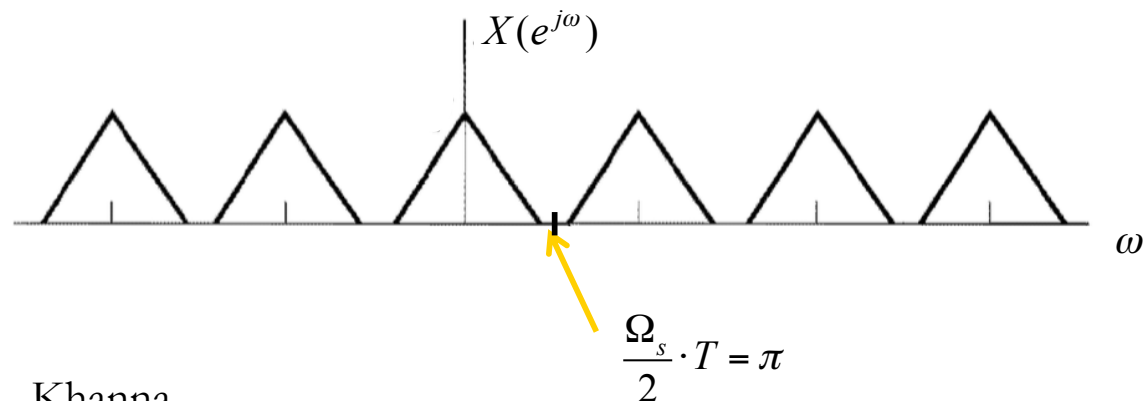
Discrete-Time Processing of Continuous Time



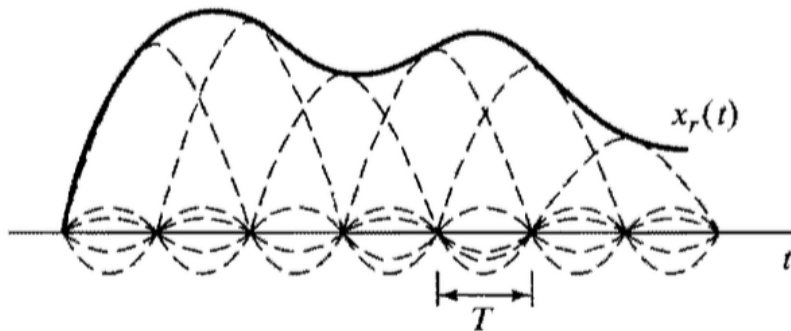
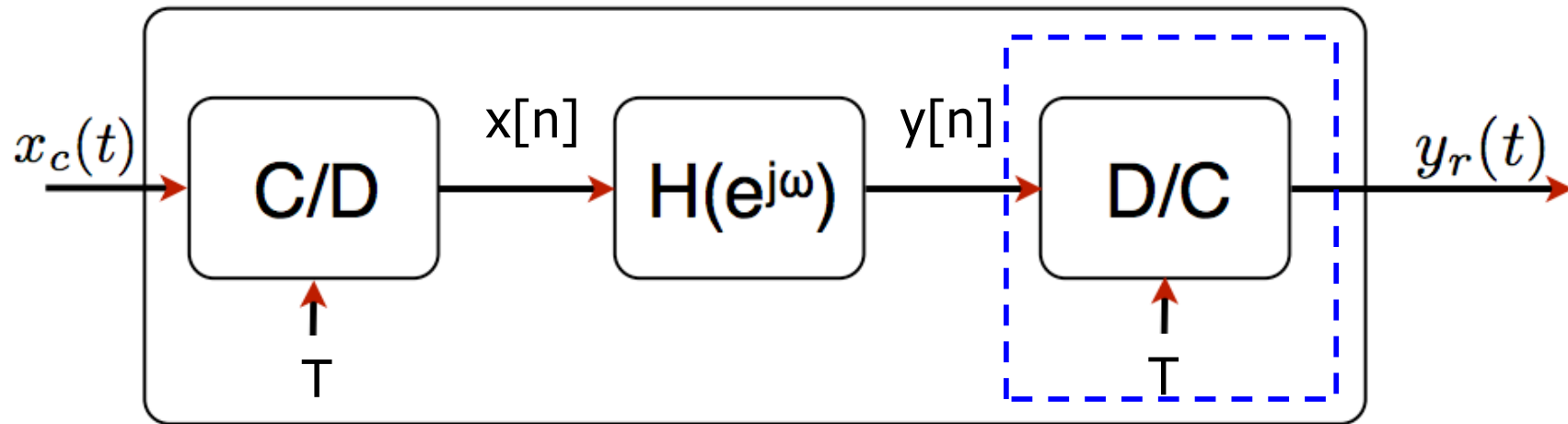
Discrete-Time Processing of Continuous Time



$$X(e^{j\omega}) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c \left[j \left(\frac{\omega}{T} - \frac{2\pi k}{T} \right) \right]$$



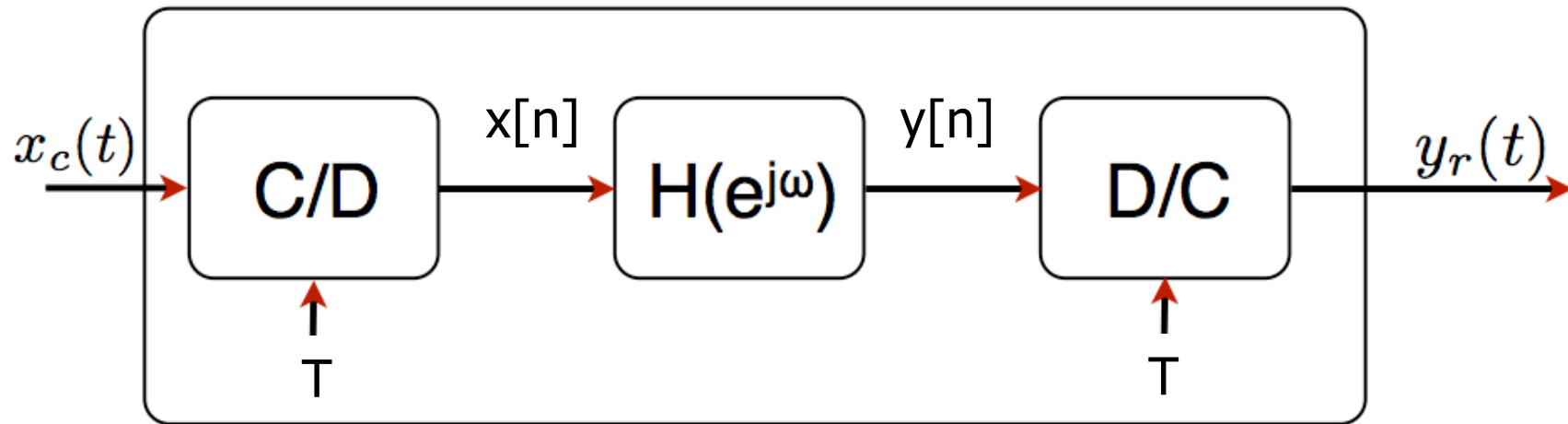
Discrete-Time Processing of Continuous Time



Sum of scaled
shifted sincs

$$y_r(t) = \sum_{n=-\infty}^{\infty} y[n] \frac{\sin[\pi(t - nT) / T]}{\pi(t - nT) / T}$$

Discrete-Time Processing of Continuous Time

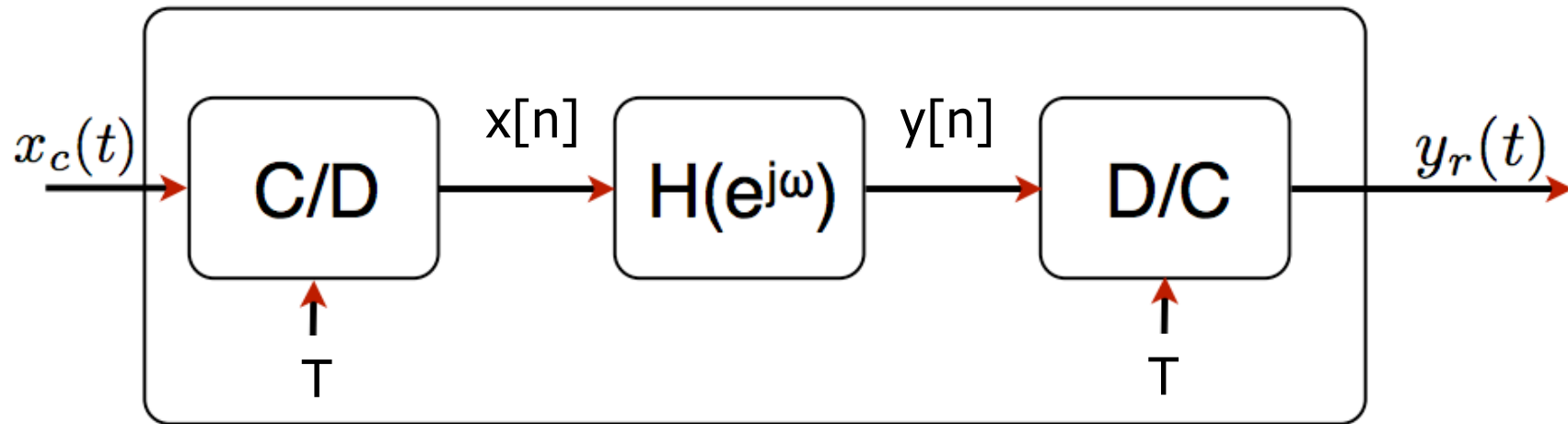


$$X(e^{j\omega}) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c \left[j \left(\frac{\omega}{T} - \frac{2\pi k}{T} \right) \right] \quad y_r(t) = \sum_{n=-\infty}^{\infty} y[n] \frac{\sin[\pi(t - nT) / T]}{\pi(t - nT) / T}$$

□ If $h[n]/H(e^{j\omega})$ is LTI

■ Is the whole system, H_{eff} , from $x_c(t) \rightarrow y_r(t)$ LTI?

Discrete-Time Processing of Continuous Time



$$X(e^{j\omega}) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c \left[j \left(\frac{\omega}{T} - \frac{2\pi k}{T} \right) \right] \quad y_r(t) = \sum_{n=-\infty}^{\infty} y[n] \frac{\sin[\pi(t - nT)/T]}{\pi(t - nT)/T}$$

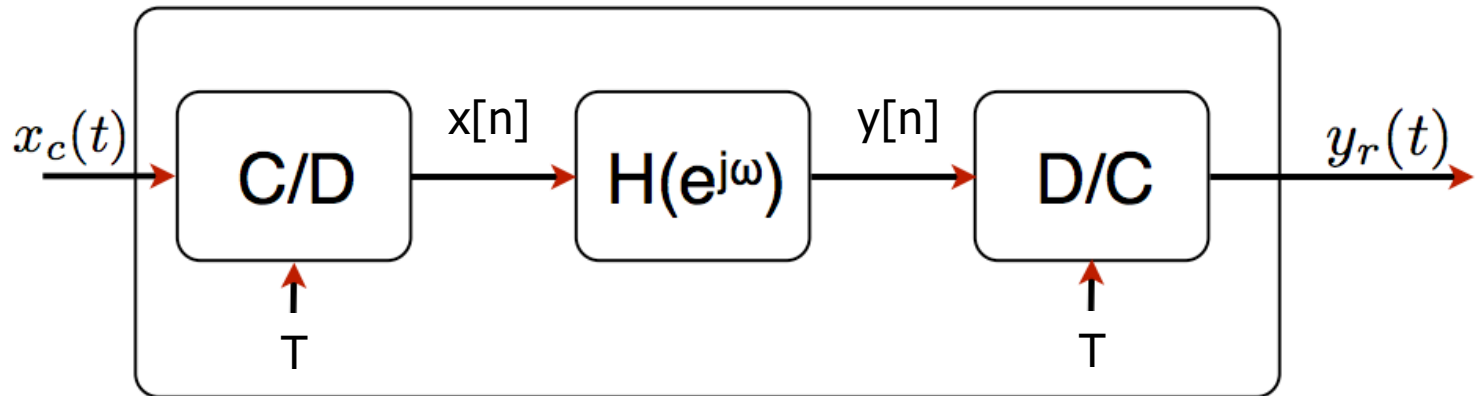
□ If $x_c(t)$ is bandlimited by $\Omega_s/2 = \pi/T$, then,

$$\frac{Y_r(j\Omega)}{X_c(j\Omega)} = H_{eff}(j\Omega) = \begin{cases} H(e^{j\omega}) \big|_{\omega=\Omega T} & |\Omega| < \pi/T \\ 0 & else \end{cases}$$



Example 1

- Consider the following system



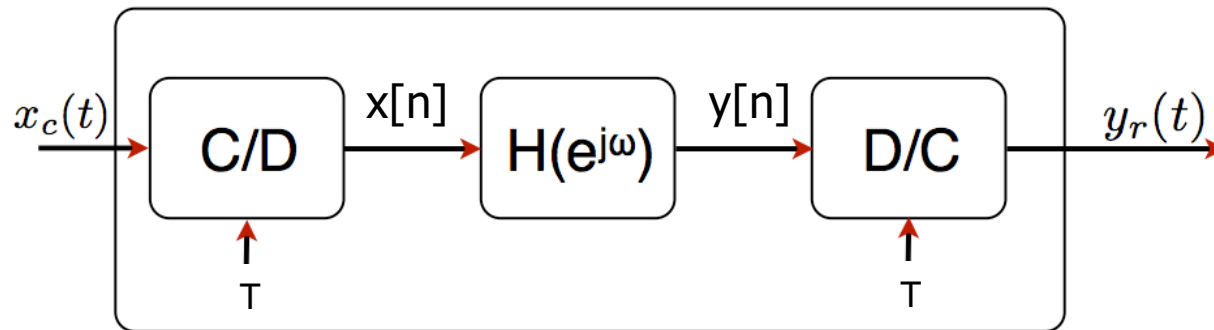
- Where
$$H(e^{j\omega}) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & \omega_c < |\omega| \leq \pi \end{cases}$$

- What is the effective frequency response of the system? What happens to a signal bandlimited by Ω_N ?



Example 2

- DT implementation of an ideal CT bandlimited differentiator



- The ideal CT differentiator is defined by

$$y_c(t) = \frac{d}{dt}[x_c(t)]$$

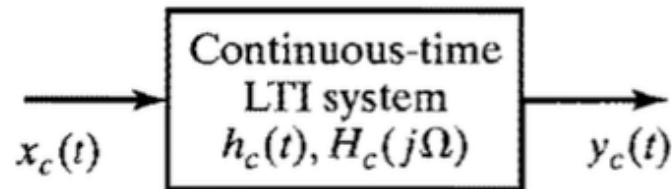
- With corresponding

$$H_c(j\Omega) = j\Omega$$



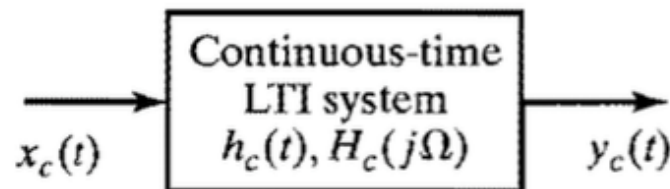
Impulse Invariance

- Want to implement continuous-time system...

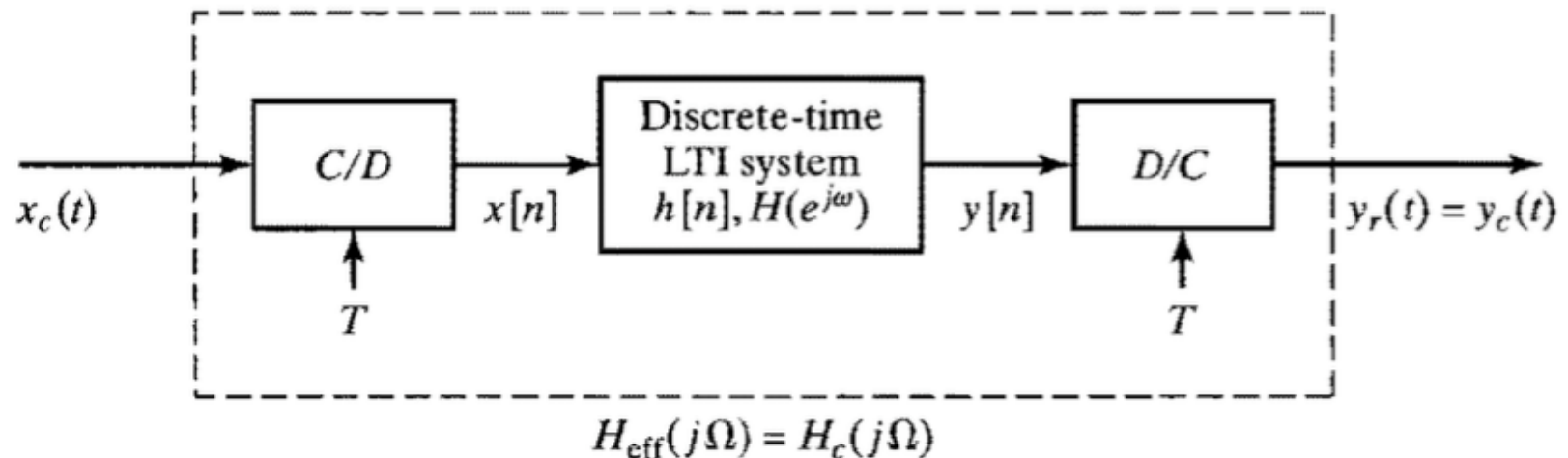


Impulse Invariance

- Want to implement continuous-time system...



- ...in discrete-time





Impulse Invariance

- With $H_c(j\Omega)$ bandlimited, choose

$$H(e^{j\omega}) = H_c(j\Omega) \Big|_{\Omega = \frac{\omega}{T}}, \quad |\omega| < \pi$$

- With the further requirement that T be chosen such that

$$H_c(j\Omega) = 0, \quad |\Omega| \geq \pi / T$$

Impulse Invariance

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$$h[n] = T h_c(nT)$$



Impulse Invariance

□ Let,

$$h[n] = h_c(nT)$$

Impulse Invariance

□ Let,

$$h[n] = h_c(nT)$$

$$H_c(j\Omega) = 0, \quad |\Omega| \geq \pi / T$$

□ If sampling at Nyquist Rate then

$$H(e^{j\omega}) = \frac{1}{T} \sum_{k=-\infty}^{\infty} H_c \left[j \left(\frac{\omega}{T} - \frac{2\pi k}{T} \right) \right]$$

$$H(e^{j\omega}) = \frac{1}{T} H_c \left(j \frac{\omega}{T} \right), \quad |\omega| < \pi$$

Impulse Invariance

□ Let,

$$h[n] = T h_c(nT)$$

$$H_c(j\Omega) = 0, \quad |\Omega| \geq \pi / T$$

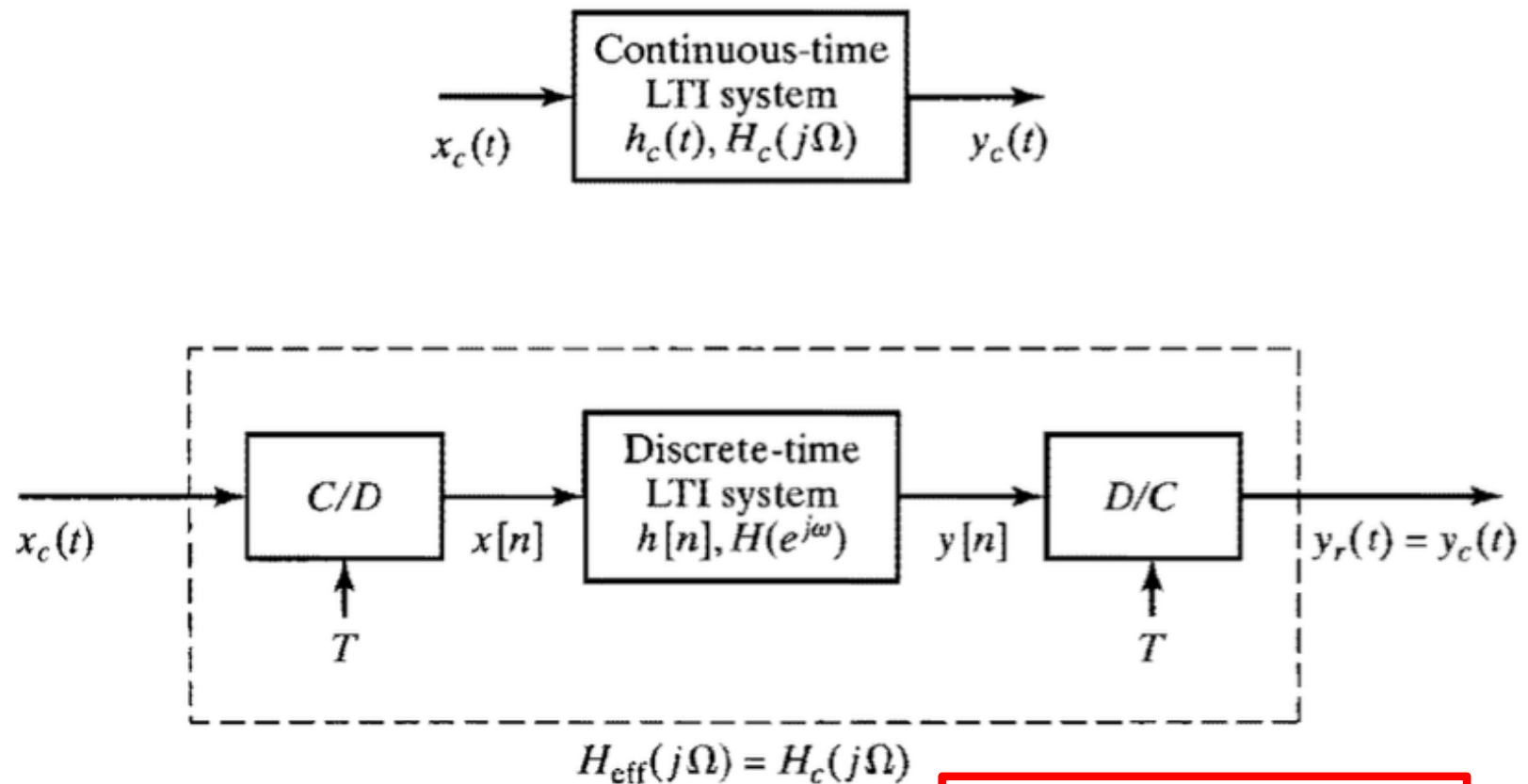
□ If sampling at Nyquist Rate then

$$H(e^{j\omega}) = \frac{T}{T} \sum_{k=-\infty}^{\infty} H_c \left[j \left(\frac{\omega}{T} - \frac{2\pi k}{T} \right) \right]$$

$$H(e^{j\omega}) = \frac{T}{T} H_c \left(j \frac{\omega}{T} \right), \quad |\omega| < \pi$$

Impulse Invariance

- Want to implement continuous-time system in discrete-time

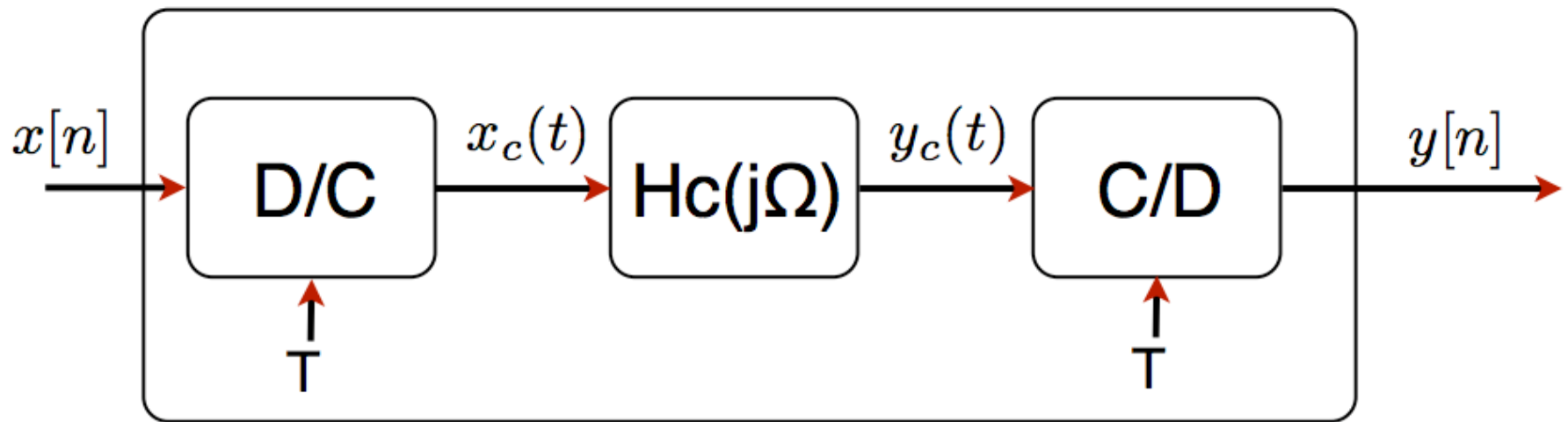


$$h[n] = Th_c(nT)$$



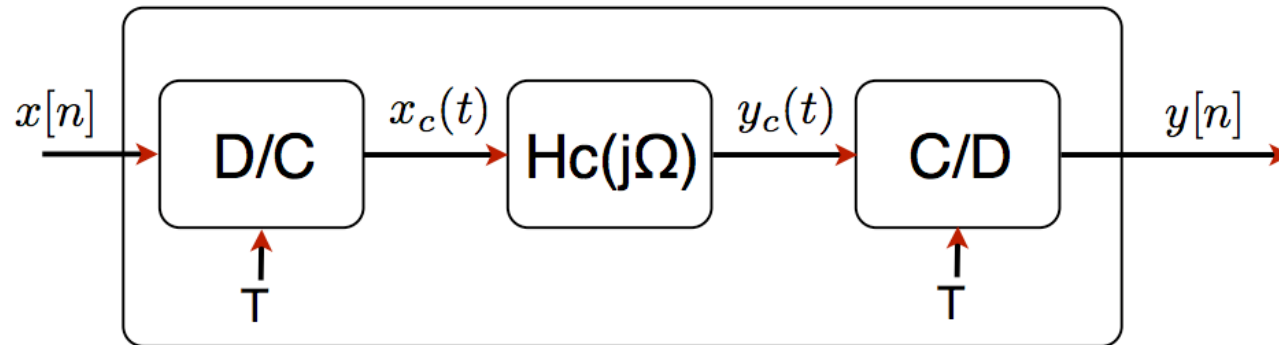
Continuous-Time Processing of Discrete-Time

- Useful to interpret DT systems with no simple interpretation in discrete time



Is the $H_{\text{eff}}(e^{j\omega})$ LTI?

Continuous-Time Processing of Discrete-Time

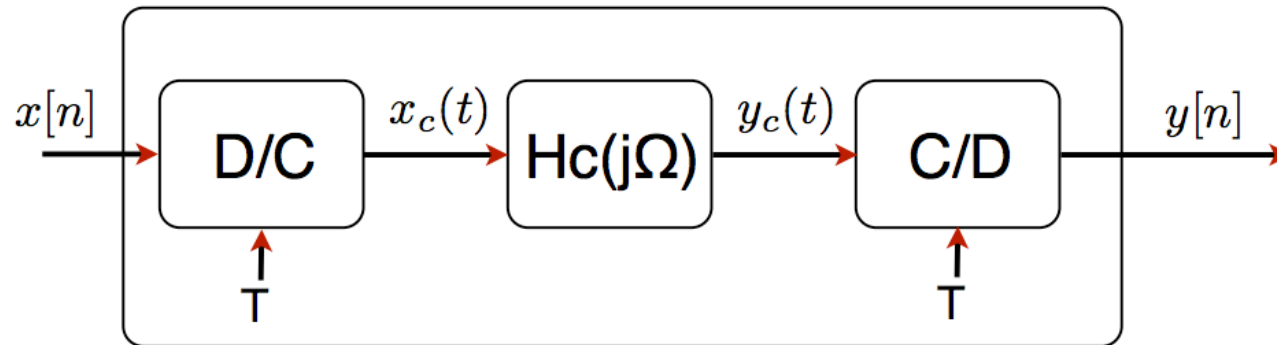


$$X_c(j\Omega) = \begin{cases} TX(e^{j\Omega T}) & |\Omega| < \pi / T \\ 0 & \text{else} \end{cases}$$

Also bandlimited

→ $Y_c(j\Omega) = H_c(j\Omega)X_c(j\Omega)$

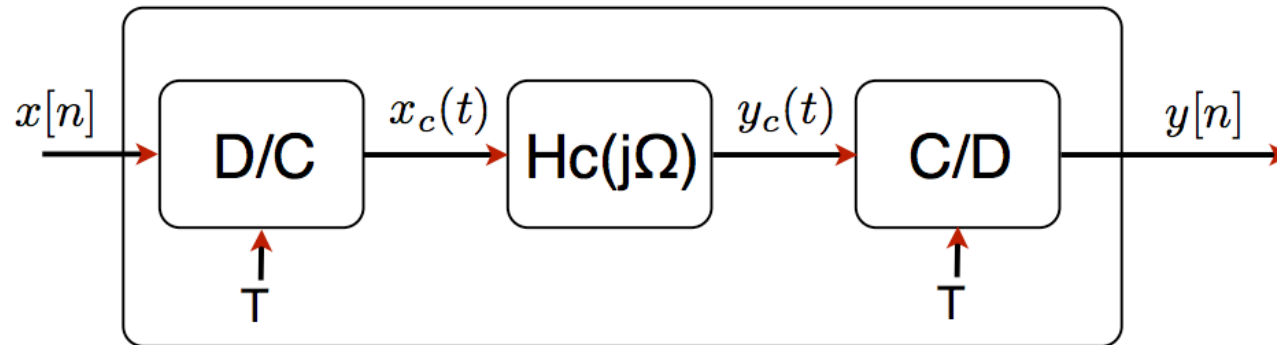
Continuous-Time Processing of Discrete-Time



$$Y_c(j\Omega) = H_c(j\Omega)X_c(j\Omega)$$

$$Y(e^{j\omega}) = \frac{1}{T} \sum_{k=-\infty}^{\infty} Y_c \left[j \left(\Omega - k\Omega_s \right) \right] \Big|_{\Omega=\omega/T} = \frac{1}{T} Y_c(j\Omega) \Big|_{\Omega=\omega/T}$$

Continuous-Time Processing of Discrete-Time



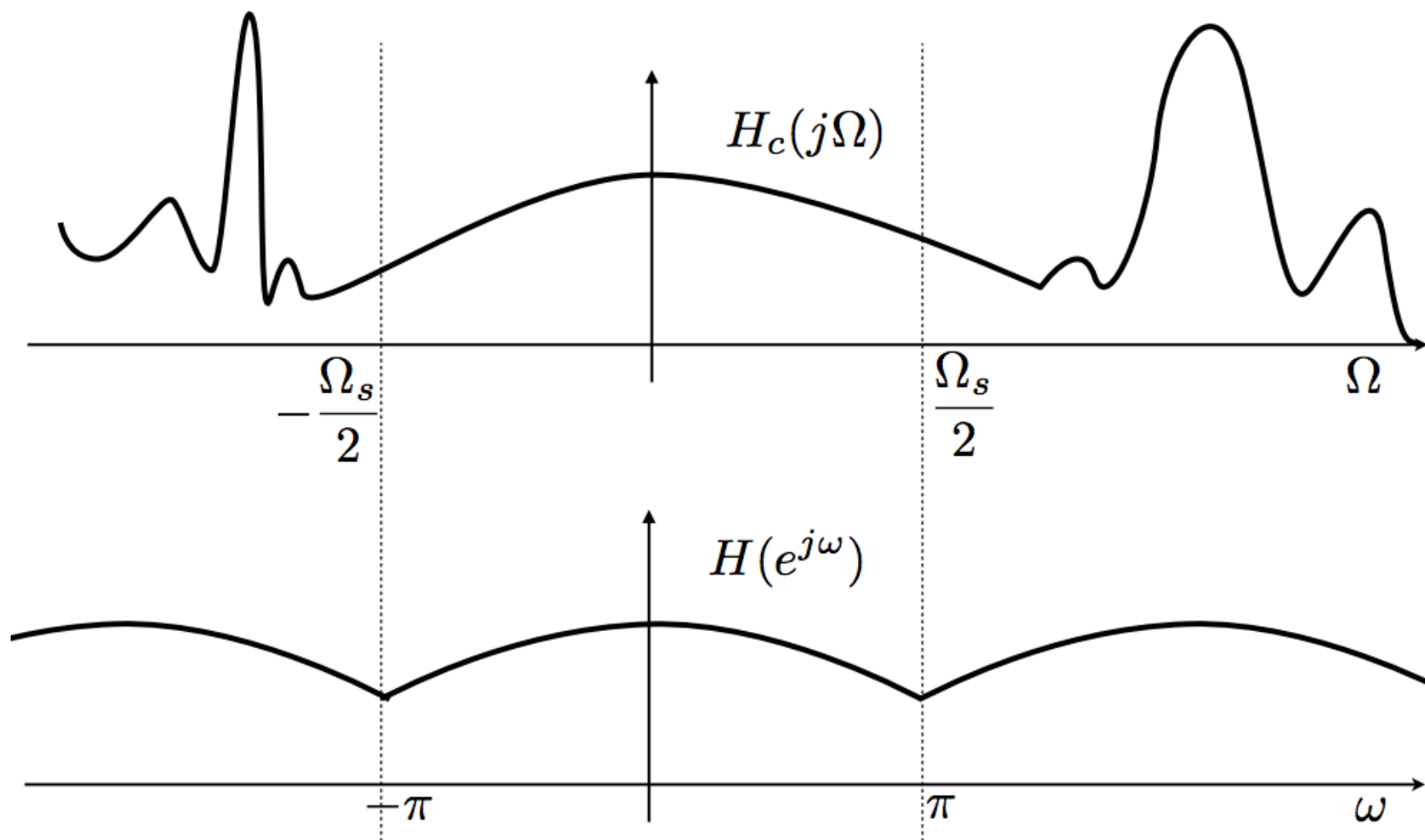
$$Y_c(j\Omega) = H_c(j\Omega)X_c(j\Omega) \qquad Y(e^{j\omega}) = \frac{1}{T} Y_c(j\Omega) \Big|_{\Omega=\omega/T}$$

$$\begin{aligned} Y(e^{j\omega}) &= \frac{1}{T} H_c(j\Omega) \Big|_{\Omega=\omega/T} X_c(j\Omega) \Big|_{\Omega=\omega/T} \\ &= \frac{1}{T} H_c(j\Omega) \Big|_{\Omega=\omega/T} (TX(e^{j\omega})) \qquad |\omega| < \pi \end{aligned}$$

$$H(e^{j\omega})$$



Example



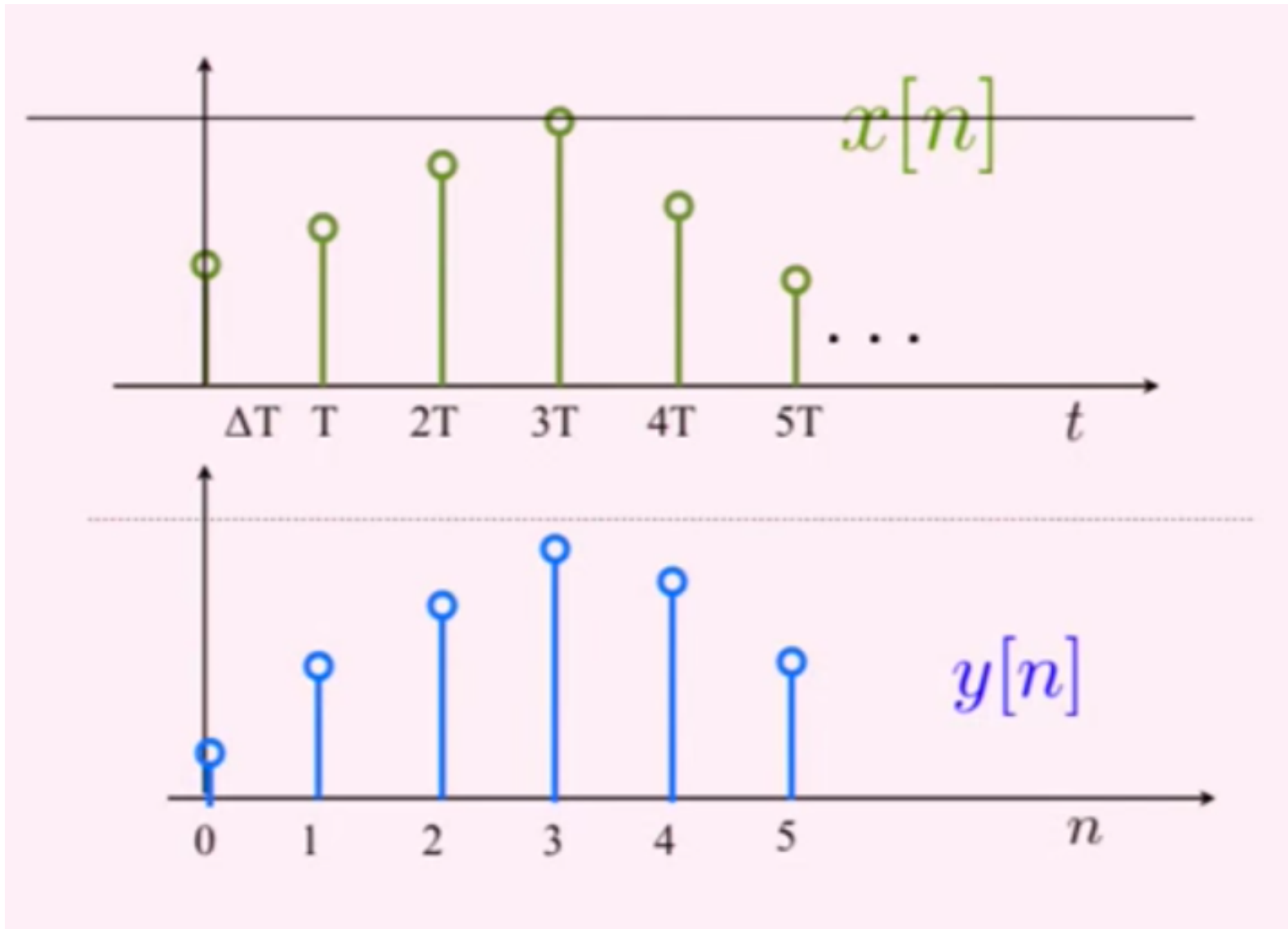
Example: Non-integer Delay

- What is the time domain operation when Δ is non-integer? I.e $\Delta=1/2$

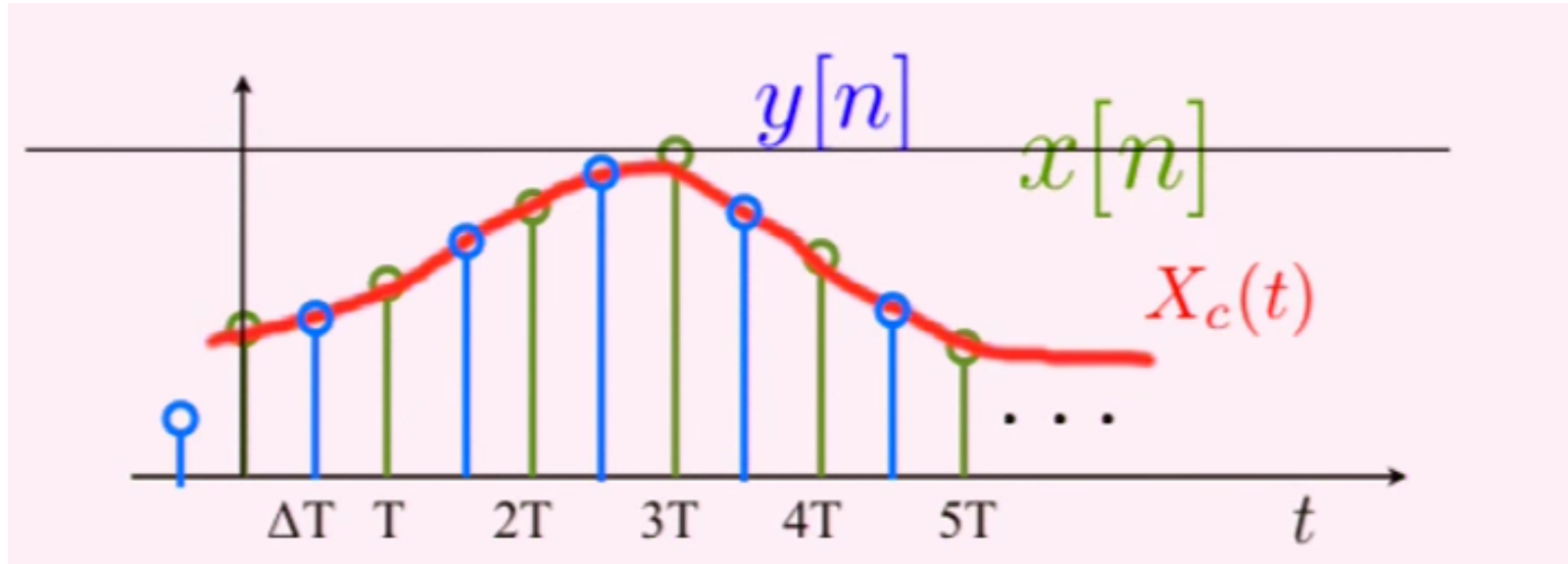
$$H(e^{j\omega}) = e^{-j\omega\Delta}$$

$$\begin{aligned}\delta[n] &\leftrightarrow 1 \\ \delta[n - n_d] &\leftrightarrow e^{-j\omega n_d}\end{aligned}$$

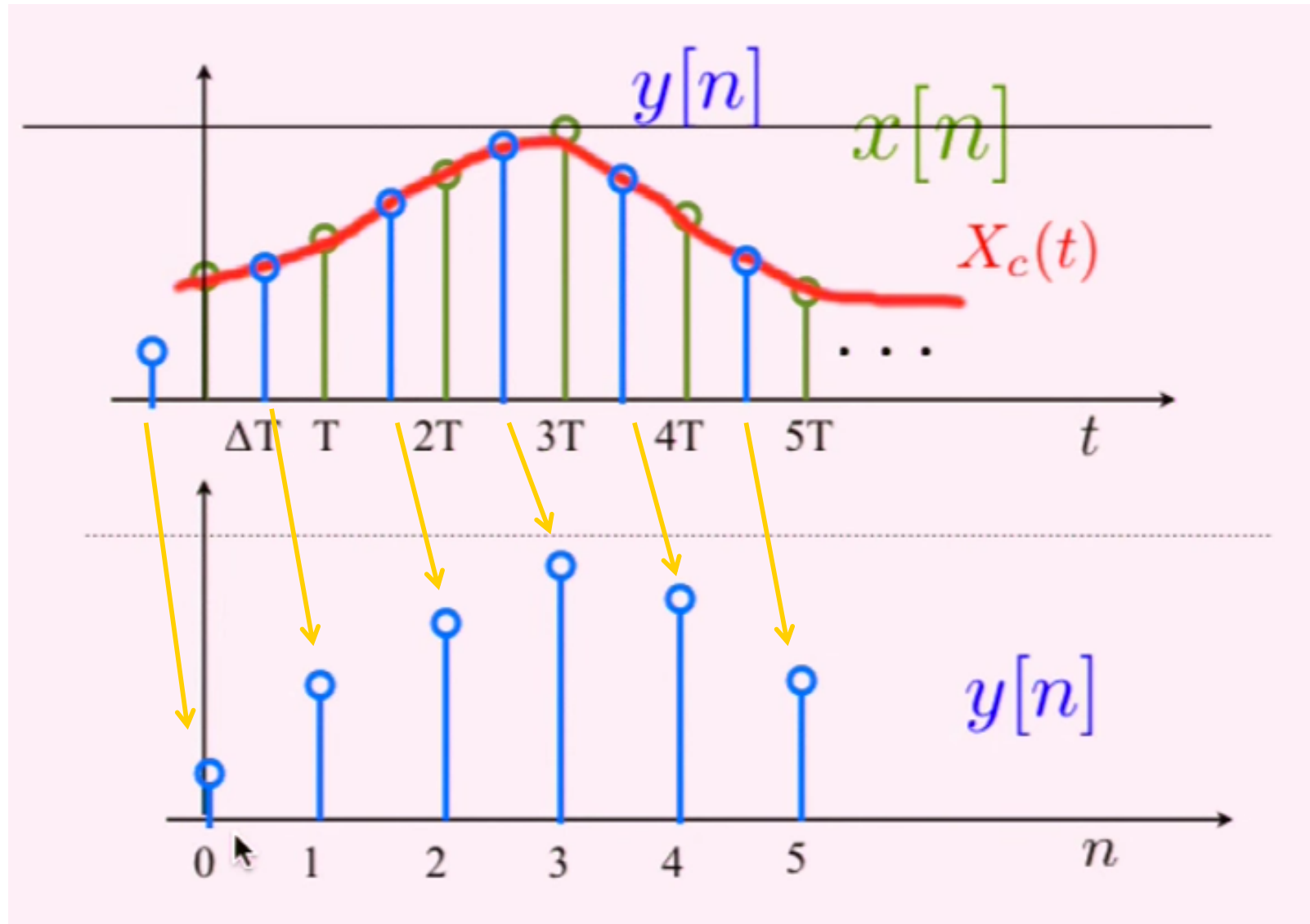
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Example: Non-integer Delay

- What is the time domain operation when Δ is non-integer? I.e $\Delta=1/2$

$$H(e^{j\omega}) = e^{-j\omega\Delta} \quad h[n]?$$

Let: $H_c(j\Omega) = e^{-j\Omega\Delta T}$ delay of ΔT in continuous time

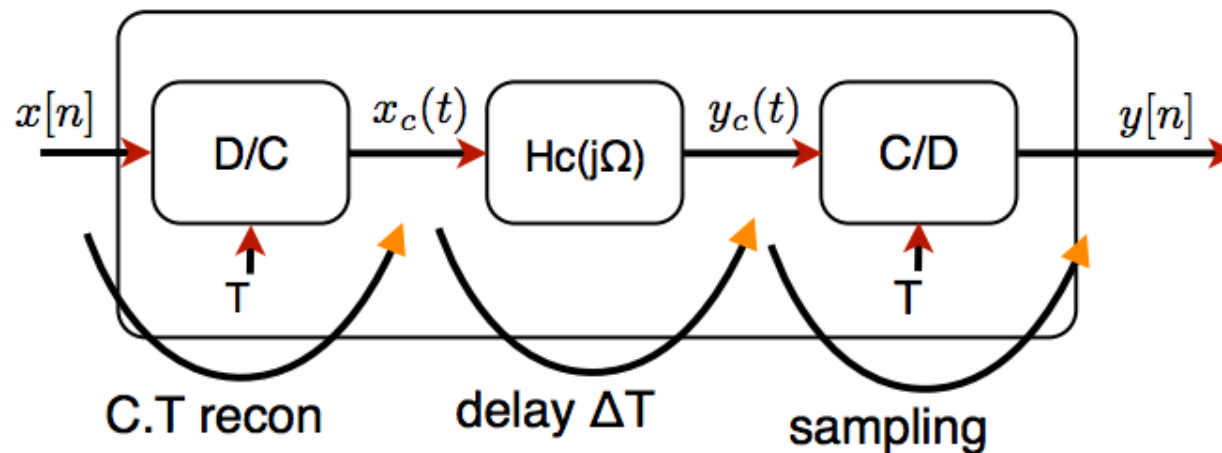
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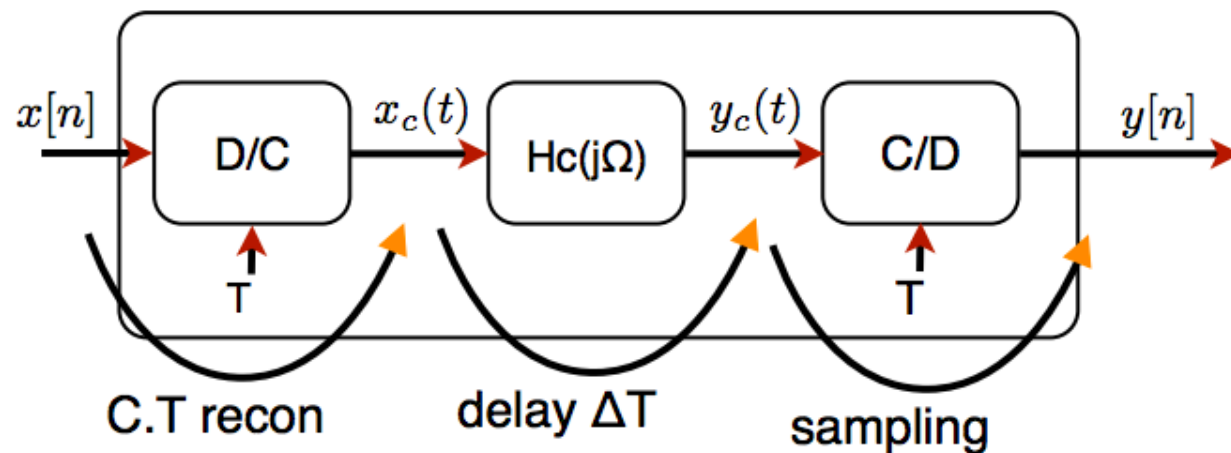
$h[n]?$

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Example: Non-integer Delay

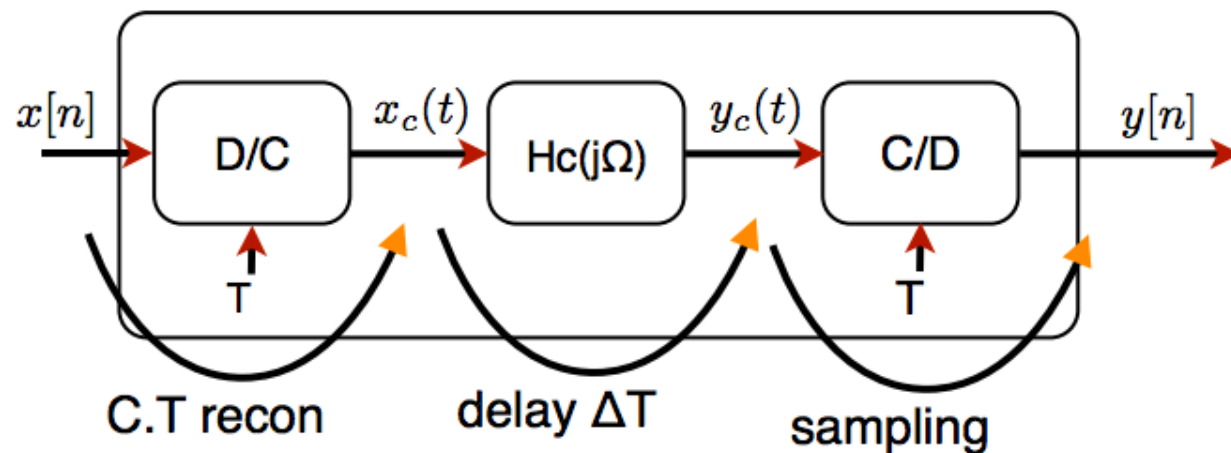
- ❑ The block diagram is for interpretation/analysis only



$$x_c(t) = \sum_k x[k] \text{sinc}\left(\frac{t - kT}{T}\right)$$

Example: Non-integer Delay

- ❑ The block diagram is for interpretation/analysis only

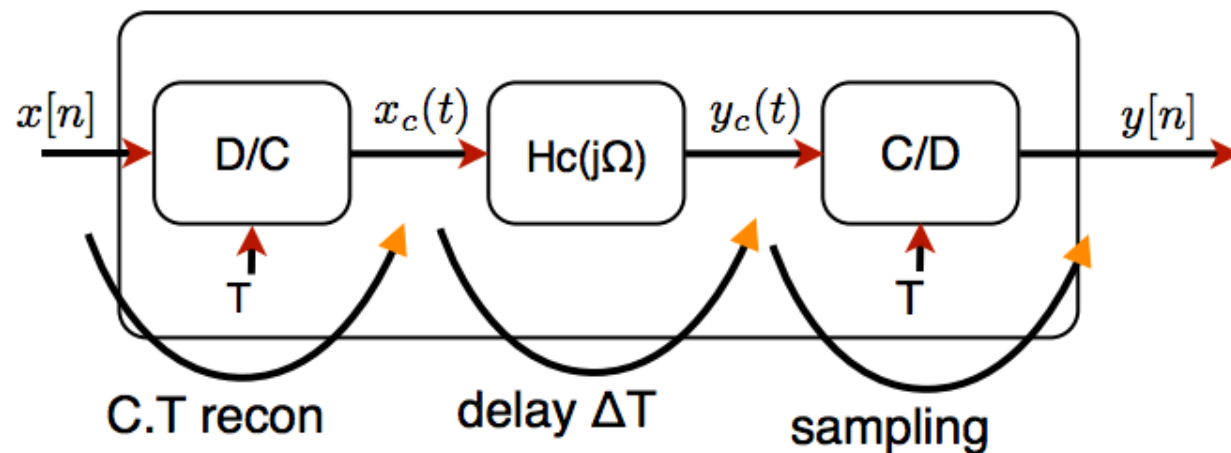


$$x_c(t) = \sum_k x[k] \text{sinc}\left(\frac{t - kT}{T}\right)$$

$$y_c(t) = x_c(t - T\Delta)$$

Example: Non-integer Delay

- The block diagram is for interpretation/analysis only



$$x_c(t) = \sum_k x[k] \text{sinc}\left(\frac{t - kT}{T}\right) \qquad y[n] = y_c(nT)$$

$$y_c(t) = x_c(t - T\Delta)$$

Example: Non-integer Delay

- The block diagram is for interpretation/analysis only

$$x_c(t) = \sum_k x[k] \text{sinc}\left(\frac{t - kT}{T}\right) \quad \longrightarrow \quad y_c(t) = x_c(t - T\Delta) \quad \longrightarrow \quad y[n] = y_c(nT)$$

Example: Non-integer Delay

- Delay system has an impulse response of a sinc with a continuous time delay

$$\begin{aligned} y[n] &= \sum_k x[k] \text{sinc}(n - \Delta - k) \\ &= x[n] * \text{sinc}(n - \Delta) \end{aligned}$$

$$\Rightarrow h[n] = \text{sinc}(n - \Delta)$$

Example: Non-integer Delay

- What is the time domain operation when Δ is non-integer? I.e $\Delta=1/2$

$$H(e^{j\omega}) = e^{-j\omega\Delta}$$

$$\begin{aligned}\delta[n] &\leftrightarrow 1 \\ \delta[n - n_d] &\leftrightarrow e^{-j\omega n_d}\end{aligned}$$

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Example: Non-integer Delay

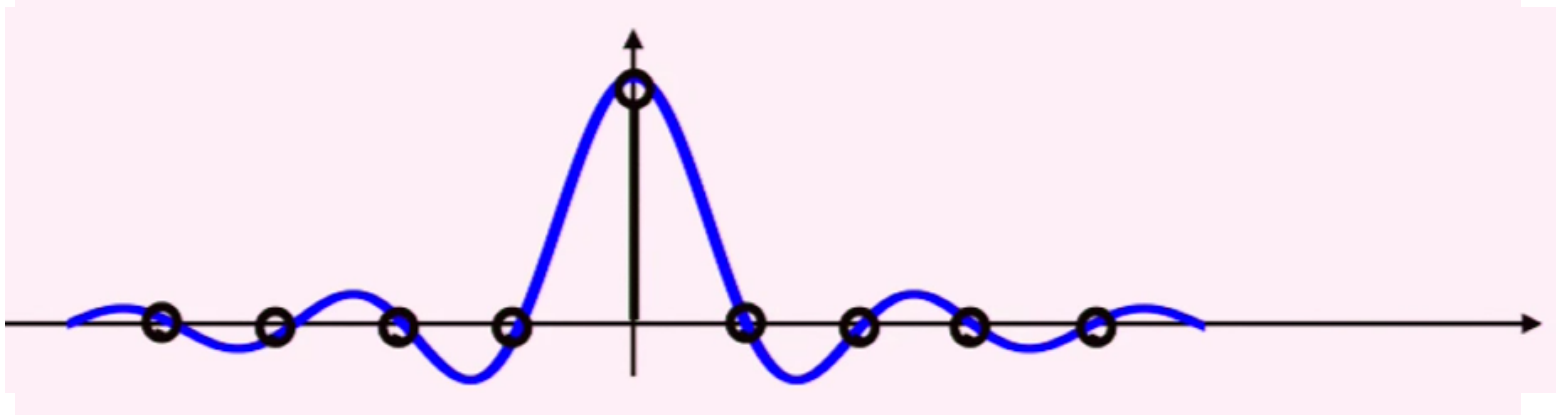
- My non-integer delay system has an impulse response of a sinc with a continuous time delay

$$H(e^{j\omega}) = e^{-j\omega\Delta} \longleftrightarrow h[n] = \text{sinc}(n - \Delta)$$

Example: Non-integer Delay

- My non-integer delay system has an impulse response of a sinc with a continuous time delay

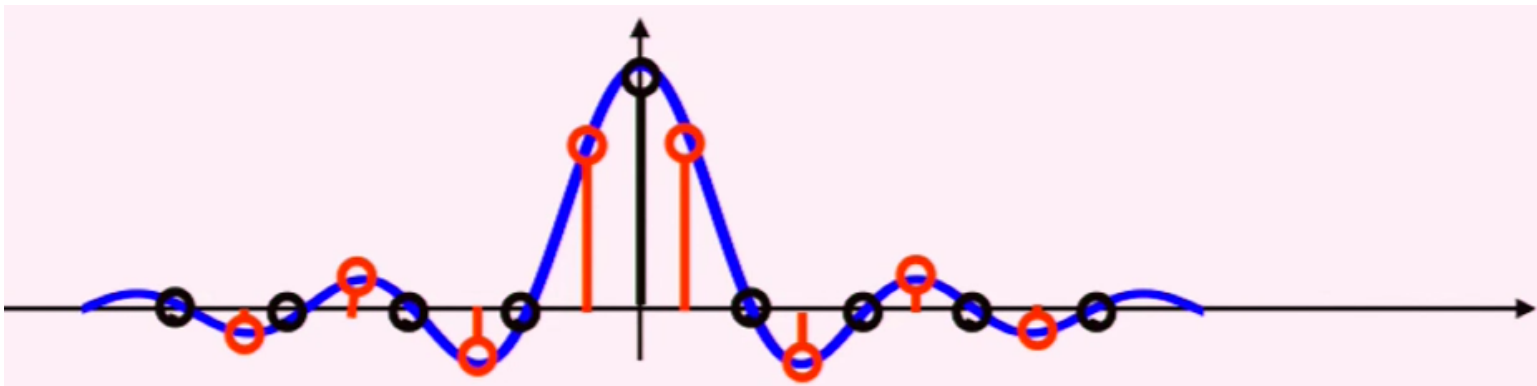
$$H(e^{j\omega}) = e^{-j\omega\Delta} \iff h[n] = \text{sinc}(n - \Delta)$$



Example: Non-integer Delay

- My non-integer delay system has an impulse response of a sinc with a continuous time delay

$$H(e^{j\omega}) = e^{-j\omega\Delta} \iff h[n] = \text{sinc}(n - \Delta)$$





Big Ideas

- ❑ DT processing of CT
 - Effectively LTI if no aliasing
- ❑ CT processing of DT
 - Always LTI
 - Useful for interpretation



Admin

- ❑ HW 3 out now
 - Due Sunday