University of Pennsylvania
Department of Electrical and System Engineering
Digital Signal Processing

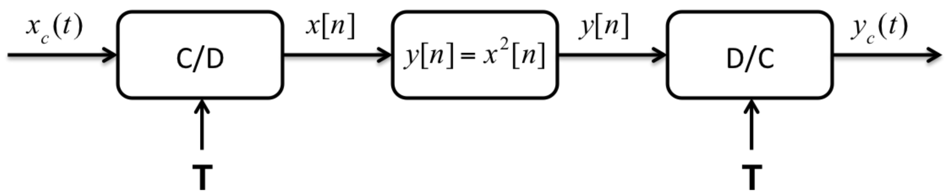
HW3: Reconstruction, DT/CT Systems, Re-Sampling Sunday, February 11th

---

Due: Sunday, February 18, 11:59PM

- **Homework Problems:** All problems must be turned in and are not optional for full credit

  1. Homework problems from the book: 4.28, 4.30, 4.32
  2. Consider a standard system (shown below) consisting of a C/D converter, a discrete-time system, and a D/C converter. Suppose that the Fourier transform $X_c(j\Omega)$ of $x_c(t)$ obeys $X_c(j\Omega) = 0$ for $|\Omega| \geq 2\pi \times 1000$ (or $|f| \geq 1000Hz$), and that the discrete-time system is a squarer, i.e. $y[n] = x^2[n]$. What is the largest value of T such that $y_c(t) = x_c^2(t)$? Justify your answer.



  3. Matlab problem: Frequency-Domain View of Sampling

     When a continuous-time signal is sampled, its spectrum shows the aliasing effect as we saw in class. To show this effect in reality, an oscilloscope is needed. In MATLAB the effect can only be simulated. To simulate the analog signals, a very high sampling rate will have to be used–at least five times the highest frequency that any analog signal will be allowed to have. Thus we need two "sampling rates"–one for the actual sampling under investigation and the other for simulating the continuous-time signals. A second issue is how to display the Fourier transform of the continuous-time signals. The following Matlab function should be used to plot the analog spectra. Notice that one of its inputs is the `dt` for simulation (I.e the sampling period under investigation). Make sure you understand what the code is doing.

```
 function fmagplot( xa, dt )
%FMAGPLOT
%      fmagplot( xa, dt )
%
%           xa:  the "ANALOG" CT signal
```

```
%           dt:  the sampling interval for the
%                  simulation of the CT signal, xa(t)
%
L = length(xa);
Nfft = round( 2 .^ round(log2(5*L)) ); % <-- next power of 2
Xa = fft(xa, Nfft);
range = 0:(Nfft/4);
ff = range/Nfft/dt;
plot( ff/1000, abs( Xa(1:length(range)) ) )
title('CONT-TIME FOURIER TRANSFORM (MAG)')
xlabel('FREQUENCY (kHz)'), grid
pause
```

(a) Generate a simulated sinusoid analog signal that is a cosine wave with analog frequency f_o.

$$x(t) = cos(2\pi f_0 t + \phi) \qquad 0 \leq t \leq T$$

Choose some signal frequency, f_0, much less than the simulation frequency, f_{sim} and take the phase to be random. Generate samples at the rate $f_{sim} = 80khz$ over a time interval of length T. Choose T so that you get about 900 to 1000 samples of the simulated analog signal. Plot the time signal with `plot` so that the samples are connected. Make sure you label the x-axis correctly. Submit your plot.

(b) Plot the Fourier transform of your signal in part(a) with `fmagplot`.

(c) We want to now sample our simulated CT signal with a sampling period of T_s to generate a DT signal. To avoid unnecessary complication, the ratio of f_{sim} to f_s should be an integer L. Then every Lth sample o fthe x(t) vector can be selected to simulate the sampling. (NOTE: This sampling will actually perform an A/D operation, since the sample is also being quantized to a digital value). Plot the resulting DT signal when $f_s = 8kHz$. Submit your stem plot with axis labeled.

(d) Compute the DTFT of the DT signal and explain how it is related to the Fourier transform from part (b).

(e) EXTRA PRACTICE (not to turn in): Repeat this exercise with different values of signal frequency f_o to see how the frequency domain representations change.

(f) EXTRA PRACTICE (not to turn in): Try repeating this exercise with CT signals that aren't pure sinusoids.

4. Matlab problem: Reconstruction Filter Design and D/A Conversion
This is a follow up from Matlab problem 1 where you created a sampled signal. The D/A conversion phase consists of creating an analog signal, $\hat{x}(t)$ from the discrete-time signal $x[n]$ by inserting a number of zeros (The number depends on the ratio $\frac{f_{sim}}{f_s}$.) between each sample and then filtering with the reconstruction filter.

(a) The reconstruction filter will, of course, have to be a digital filter to simulate the true analog filter. Use the MATLAB filter design function `cheby2` to design this filter: `[b,a] = cheby2(9, 60, fcut)`. This will design a ninth-order filter with 60db of stopband attenuation. The analog cutoff frequency has to be at $\frac{1}{2}f_s$. For MATLAB this has to be scaled to `fcut = 2*(fsamp/2)/fsim`. Use `freqz` to plot the frequency response of the simulated reconstruction filter. To get its true analog cutoff frequency on the plot, you must remember that this is a digital filter, where the frequency $\omega = \pi$ is mapped to $\frac{1}{2}f_{sim}$. Submit the magnitude and phase with your homework. Make sure to label your axes appropriately.

(b) Carry-out the zero-insert operation with the appropriate number of zeros to create $\hat{x}(t)$ and apply the Chebyshev reconstruction filter to get the smoothed output, $x_r(t)$. Submit a plot of your reconstructed signal, $x_r(t)$ and its Fourier transform.

- **Recommended Problems for Practice:** From the book: 4.11, 4.12, 4.19, 4.21, 4.24, 4.27, 4.29, 4.31