**University of Pennsylvania**
**Department of Electrical and System Engineering**
**Digital Signal Processing**

# HW6: Frequency Response, All Pass/Min Phase Systems

Sunday, Mar. 10

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**Due:** Sunday, Mar. 17th, 11:59PM

- **Homework Problems:** All problems must be turned in and are not optional for full credit

  1. Homework problems from the book: 5.7, 5.24, 5.34
  2. Matlab problem: Group Delay
     The group delay is defined as the negative derivative of the phase of the frequency response. However, computation of the group delay is best done without explicitly evaluating the derivative with respect to $\omega$. The M-file below exploits the fact that multiplying by $n$ in the time domain will generate a derivative in the frequency domain. Furthermore this function is configured for the case where the signal $x[n]$ starts at an index other than $n = 0$.

```
 function [gd, w] = gdel(x, n, Lfft)
%GDEL compute the group delay of x[n]
%
% x:  Signal x[n] at the times (n)
% n:  Vector of time indices
% Lfft:  Length of the FFT used
% gd:  Group delay values on [-pi, pi)
% w:  List of frequencies over [-pi, pi)
%
% NOTE: Group delay of B(z)/A(z) = gdel(B) - gdel(A)
%
X = fft(x, Lfft);
dXdw = fft(n.*x, Lfft); % --- transform of nx[n]
gd = fftshift(real(dXdw./X )); % --- when X==0, gd=infinity
w = (2*pi/Lfft)*[0:(Lfft-1)] - pi;
```

     Test the group delay function with a shifted unit impulse signal. Define a unit impulse sequence $\delta[n - n_o]$ of lenght 128, over the range $-64 \leq n \leq 63$. Pick $n_o = 5$ or $n_o = -5$, and then make a plot of the signal with the time axis correctly labeled, to show that the impulse is located at $n = n_o$. In addition, compute and plot the group delay to verify the proper value is obtained. Submit your signal and group delay plots.

3. Matlab problem: Causal First-Order System
   Using the Matlab function `filter`, generate the impulse response of the causal system:

$$H_C(z) = \frac{1}{1 - 0.77z^{-1}} \quad ROC = \{z : |z| > 0.77\} \tag{1}$$

   (a) Plot the impulse response of the signal over the range $-64 \leq n \leq 63$.
   (b) Plot the frequency response magnitude and group delay. To generate the frequency response, compute the FFT form a finite section of the impulse response.
   (c) Repeat a and b for a pole closer to the unit circle; try 0.95 instead of 0.77. Describe the differences between the two cases.

4. Matlab problem: Frequency response for difference equations.
   For the following difference equation

$$y[n] - 1.8cos\left(\frac{\pi}{16}\right) y[n-1] + 0.81y[n-2] = x[n] + 0.5x[n-1] \tag{2}$$

   do the following frequency-domain computations:

   (a) Make plots for the magnitude and phase responses with 512 frequency samples around the entire unit circle (i.e from 0 to $2\pi$). Use `freqz` to generate your plots.
   **HINT:** The command `[H, W] = freqz(b, a, N, 'whole')` will evaluate the frequency response of a filter at `N`, equally spaced in radian frequency around the unit circle. If you do not use the `'whole'` option, `freqz` will use only the upper half of the unit circle (from 0 to $\pi$), which is sufficient for filter with real coefficients. The output vectors `H` and `W` will return `N` frequency response samples (`H`) and `N` equally spaced values of $\omega$ (`W`).
   (b) Now redo the frequency response using only the upper half of the unit circle (for $0 \leq \omega \leq \pi$). Submit your plot and explain why this is sufficient?
   (c) Specify the type of filter defined by this difference equation: high-pass, low-pass, all-pass, band-pass, or bandstop.
   (d) Redo a-c for the difference equation:

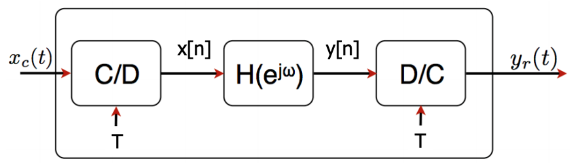
$$y[n]+0.13y[n-1]+0.52y[n-2]+0.3y[n-3] = 0.16x[n]-0.48x[n-1]+0.48x[n-2]-0.16x[n-3] \tag{3}$$

   (e) Redo a-c for the difference equation:

$$y[n] - 0.268y[n-2] = 0.634x[n] - 0.634x[n-2] \tag{4}$$

   (f) Experiment with your own difference equations! (nothing to turn in)

5. Matlab problem: Frequency response of a notch filter.
A notch filter attempts to remove one particular frequency. Suppose that a bandlimited continuous-time signal is known to contain a 60-Hz interference component, which we want to remove by processing with the standard system below for filtering a continuous-time signal with a discrete-time filter.




(a) Assume that the value of the sampling period is $T_S = 1$ms. What is the highest frequency that the analog signal can contain if aliasing is to be avoided?

(b) The discrete-time system to be used has a frequency response

$$H(e^{j\omega}) = \frac{[1 - e^{-j(\omega-\omega_0)}][1 - e^{-j(\omega+\omega_0)}]}{[1 - 0.9e^{-j(\omega-\omega_0)}][1 - 0.9e^{-j(\omega+\omega_0)}]} \tag{5}$$

Plot (in Matlab) the magnitude and phase of $H(e^{j\omega})$. Pick a trial value of $\omega_0 = 2\pi/5$. Submit your plots.

(c) What value should be chosen for $\omega_0$ to eliminate the 60-Hz component? Will the gain at the other frequencies be equal to 1?

(d) Make a Matlab plot of the frequency response magnitude only using the value from part (c). Submit your plot.

(e) Generate 150 samples of a 60-Hz sine wave sampled at $f_S = 1/T_S = 1000Hz$. Use the function `filter` to process this input signals with the system from (b) and the value of $\omega_0$ from (c). Display the output signal to illustrate that the filter actually removes the 60-Hz sinusoid. Submit the input and output signals.

(f) Since the DTFT is a frequency response, it describes the steady-state behaviour of the filter. Thus you should observe a "transient" response before the zero of the filter at 60Hz rejects the input completely. Measure the duration of this transient (in milliseconds) from the beginning of the signal until a point where the output is less than 1% of the input signal amplitude.

(g) EXTRA (not to turn in): Generate 150 samples of a 60-Hz + 20-Hz sine waves of different amplitudes. Use the function `filter` to process this input signals with the system from (b) and the value of $\omega_0$ from (c). Display the output signal to illustrate that the filter actually removes the 60-Hz sinusoid.

- **Recommended Problems for Practice:** From the book: 5.10, 5.13, 5.36, 5.43, 5.51