ESE 5310: Digital Signal Processing

Lecture 4: February 1, 2024

Discrete Time Fourier Transform,

Z-Transform

Lecture Outline

- Eigenfunctions
- Discrete Time Fourier Transform
  - Definition
  - Properties
- Frequency Response of LTI Systems
- z-Transform

Eigenfunctions

Eigenfunction

- $x[n]=e^{j\omega n}$

- Eigenfunction: function when acted upon by a linear operator is scaled by an eigenvalue
  - $\boldsymbol{D}f = \lambda f$

Eigenvalue (frequency response)

- $x[n]=e^{j\omega n}$

$$y[n] = \sum_{k=-\infty}^{\infty} x[n-k]h[k]$$

$$= \sum_{k=-\infty}^{\infty} e^{j\omega(n-k)}h[k]$$

$$= e^{j\omega n} \sum_{k=-\infty}^{\infty} h[k]e^{-j\omega k}$$

$$= H(e^{j\omega})e^{j\omega n}$$

$$\boxed{H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} h[k]e^{-j\omega k}}$$

- Describes the change in amplitude and phase of signal at frequency ω
- Frequency response
- Complex value
  - Re and Im
  - Mag and Phase

DT Frequency Response

- $H(e^{j(\omega+2\pi)})$?

$$\boxed{H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} h[k]e^{-j\omega k}}$$

DT Frequency Response

- $H(e^{j(\omega+2\pi)})$?

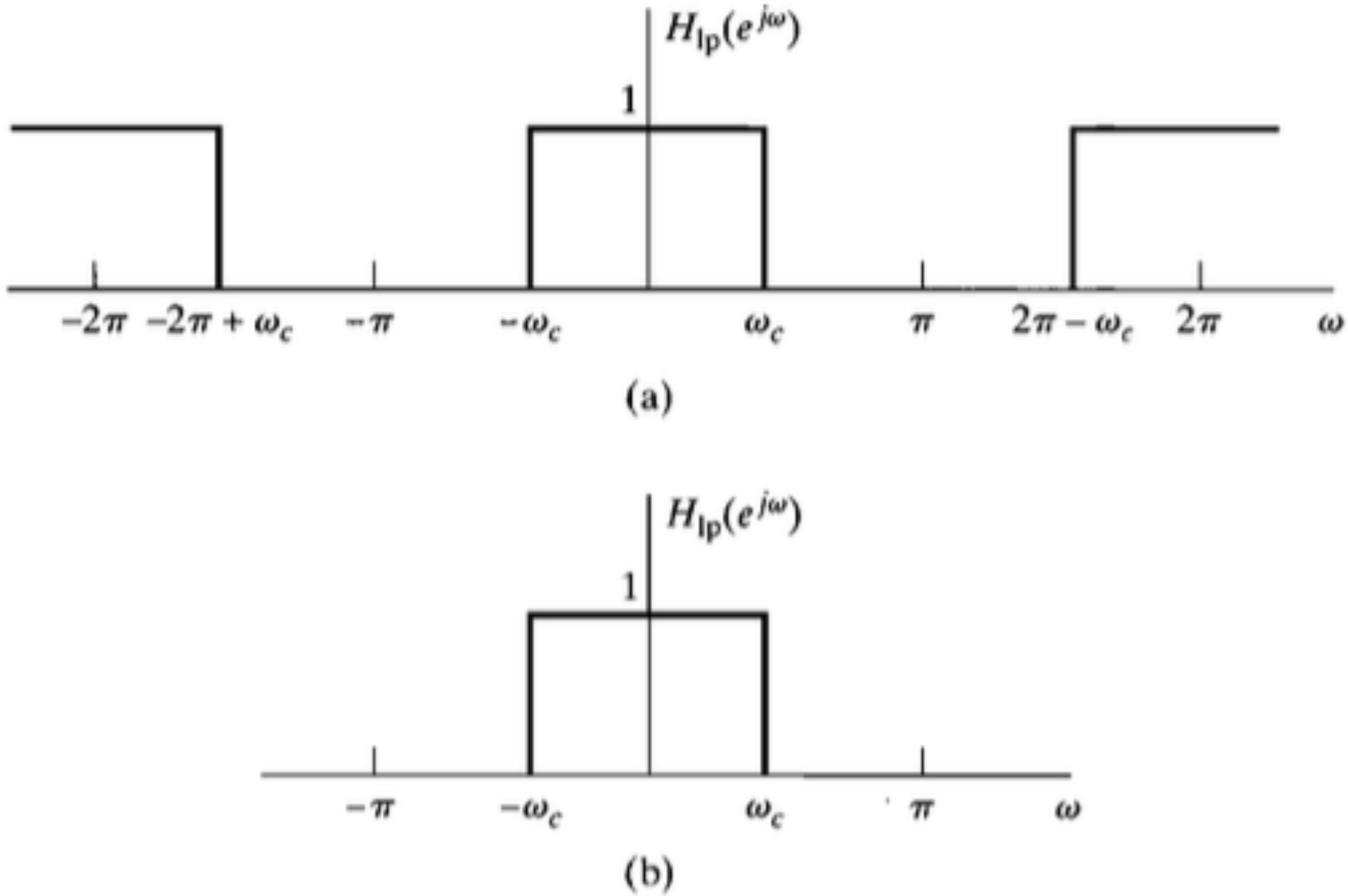
$$H\left(e^{j(\omega+2\pi)}\right) = \sum_{k=-\infty}^{\infty} h[k]e^{-j(\omega+2\pi)k}$$

$$= \sum_{k=-\infty}^{\infty} h[k]e^{-j\omega k}e^{-j2\pi k}$$

$$= \sum_{k=-\infty}^{\infty} h[k]e^{-j\omega k}$$

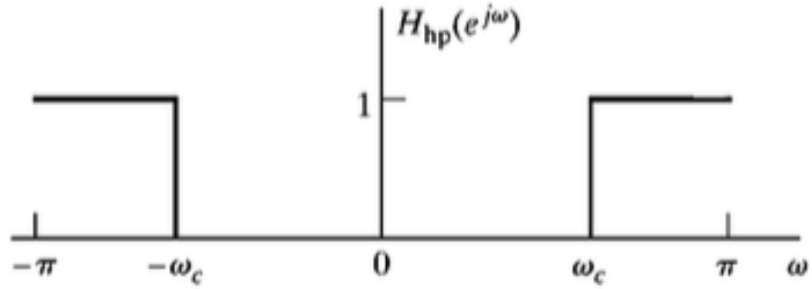
$$= H(e^{j\omega})$$

Periodicity of Low Pass Freq Response



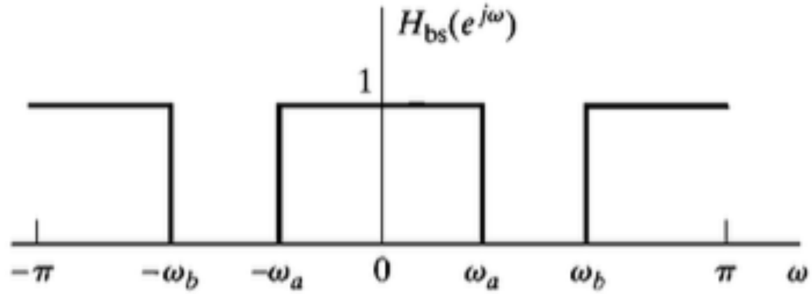
Other Filters

High-pass



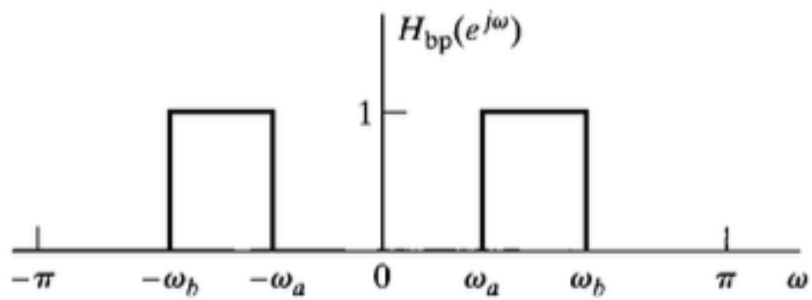
(a)

Band-stop



(b)

Band-pass



(c)

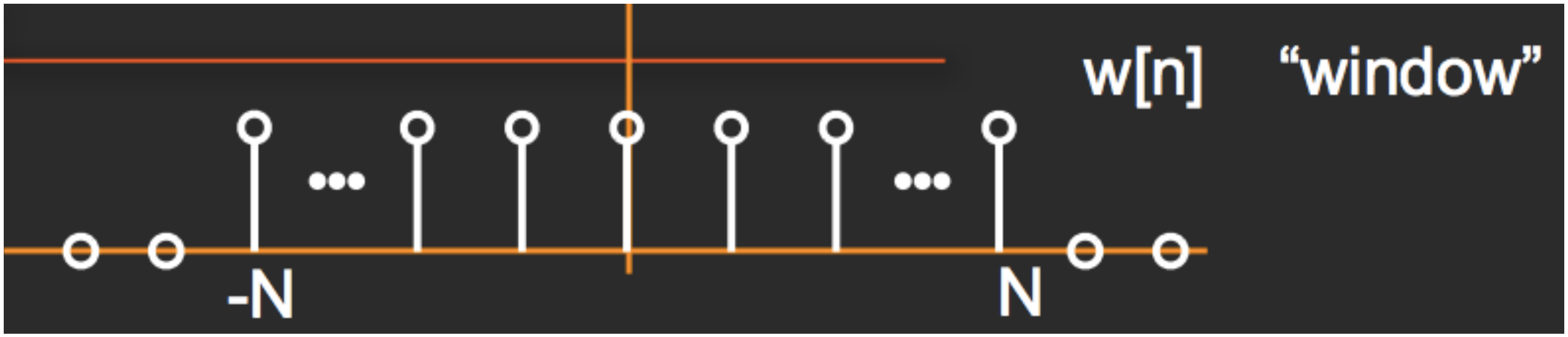
Discrete-Time Fourier Transform (DTFT)

DTFT Definition

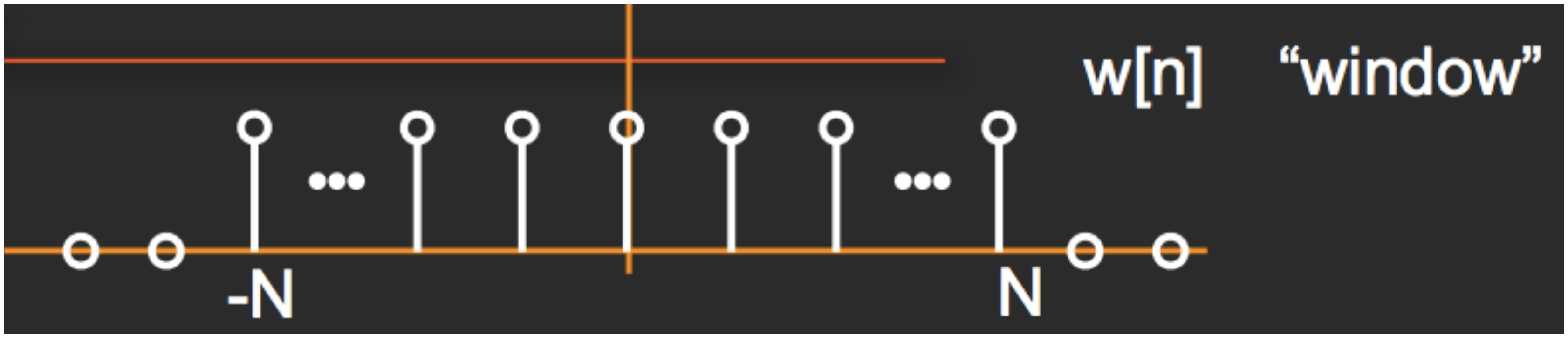
$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} x[k]e^{-j\omega k}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

Example: Window DTFT



Example: Window DTFT



$$W(e^{j\omega}) = \sum_{k=-\infty}^{\infty} w[k] e^{-j\omega k}$$

$$= \sum_{k=-N}^{N} e^{-j\omega k}$$

Example: Window DTFT

$$W(e^{j\omega}) = \sum_{k=-N}^{N} e^{-j\omega k}$$

$$= e^{j\omega N} + e^{j\omega(N-1)} + \ldots + e^{j\omega 0} + \ldots e^{-j\omega(N-1)} + e^{-j\omega N}$$

$$= e^{-j\omega N}(1 + e^{j\omega} + \ldots + e^{j\omega N} + \ldots + e^{j\omega(2N-1)} + e^{j\omega 2N})$$

Useful sum: $1 + p + p^2 + \ldots + p^M = \dfrac{1 - p^{M+1}}{1 - p}$

Example: Window DTFT

$$W(e^{j\omega}) = \sum_{k=-N}^{N} e^{-j\omega k}$$

$$= e^{j\omega N} + e^{j\omega(N-1)} + \ldots + e^{j\omega 0} + \ldots e^{-j\omega(N-1)} + e^{-j\omega N}$$

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Useful sum: $1 + p + p^2 + \ldots + p^M = \dfrac{1 - p^{M+1}}{1 - p}$

$$p = e^{j\omega} \qquad M = 2N$$

$$W(e^{j\omega}) = e^{-j\omega N} \frac{1 - e^{j\omega(2N+1)}}{1 - e^{j\omega}}$$

Example: Window DTFT

$$W(e^{j\omega}) = e^{-j\omega N}\frac{1-e^{j\omega(2N+1)}}{1-e^{j\omega}}$$

Example: Window DTFT

$$W(e^{j\omega}) = e^{-j\omega N}\frac{1-e^{j\omega(2N+1)}}{1-e^{j\omega}}$$

$$= \frac{e^{-j\omega N} - e^{j\omega(N+1)}}{1-e^{j\omega}}$$

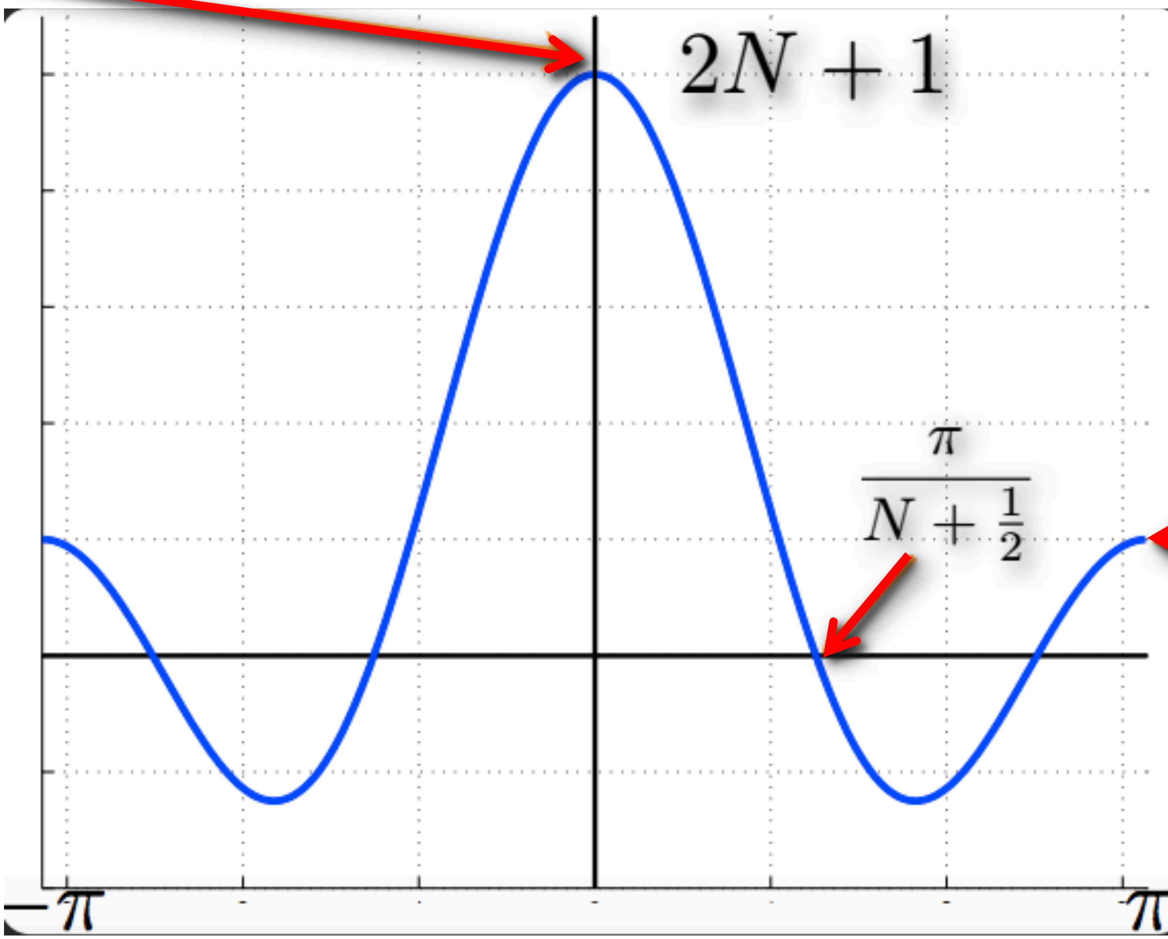
$$= \frac{e^{-j\omega N} - e^{j\omega(N+1)}}{1-e^{j\omega}} \times \frac{e^{-j\omega/2}}{e^{-j\omega/2}}$$

$$= \frac{e^{-j\omega(N+1/2)} - e^{j\omega(N+1/2)}}{e^{-j\omega/2} - e^{j\omega/2}} = \frac{\sin\left((N+1/2)\omega\right)}{\sin\left(\omega/2\right)}$$

Example: Window DTFT

$$W(e^{j\omega}) = \frac{\sin\left((N+1/2)\omega\right)}{\sin\left(\omega/2\right)}$$

Also, Σw[n]

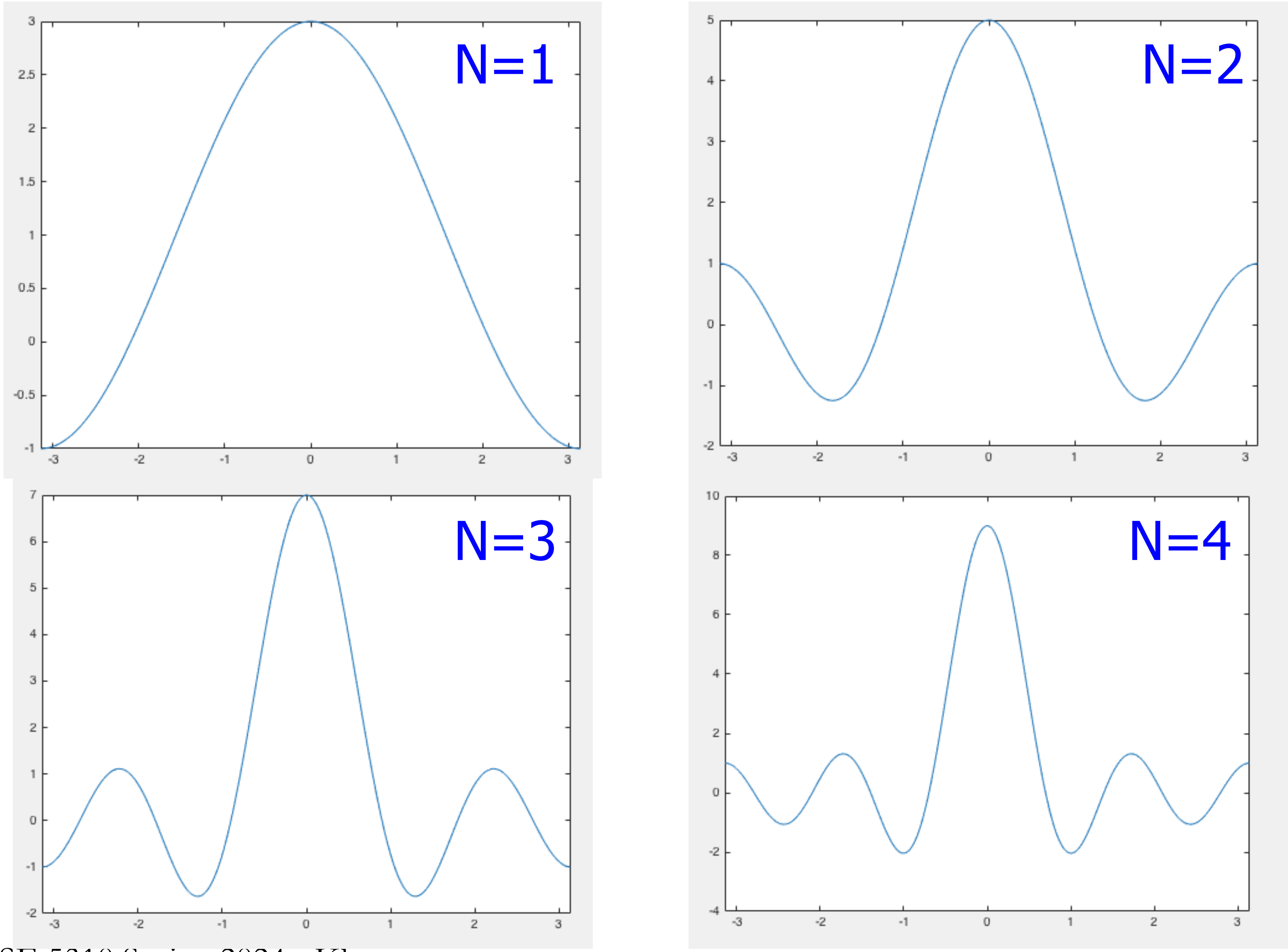


=1 why?

Plot for N=2

Periodic sinc

Periodic Sinc



Fourier Transform Pairs

TABLE 2.3 FOURIER TRANSFORM PAIRS

Sequence	Fourier Transform
1. $\delta[n]$	1
2. $\delta[n-n_0]$	$e^{-j\omega n_0}$
3. $1 \quad (-\infty < n < \infty)$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega+2\pi k)$
4. $a^n u[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{1-ae^{-j\omega}}$
5. $u[n]$	$\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega+2\pi k)$
6. $(n+1)a^n u[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{(1-ae^{-j\omega})^2}$
7. $\frac{r^n \sin\omega_p(n+1)}{\sin\omega_p} u[n] \quad (\lvert r\rvert < 1)$	$\frac{1}{1-2r\cos\omega_p e^{-j\omega}+r^2e^{-j2\omega}}$
8. $\frac{\sin\omega_c n}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1, & \lvert\omega\rvert < \omega_c, \\ 0, & \omega_c < \lvert\omega\rvert \le \pi \end{cases}$
9. $x[n] = \begin{cases} 1, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$	$\frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)} e^{-j\omega M/2}$
10. $e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega-\omega_0+2\pi k)$
11. $\cos(\omega_0 n+\phi)$	$\sum_{k=-\infty}^{\infty} [\pi e^{j\phi}\delta(\omega-\omega_0+2\pi k) + \pi e^{-j\phi}\delta(\omega+\omega_0+2\pi k)]$

Properties of the DTFT

- Linearity:

$$a x_1[n] + b x_2[n] \leftrightarrow a X_1(e^{j\omega}) + b X_2(e^{j\omega})$$

- Periodicity:

$$X(e^{j(\omega+2\pi)}) = X(e^{j\omega})$$

- Conjugate Symmetry:

$$X^*(e^{j\omega}) = X(e^{-j\omega})$$

If x[n] real

$$\mathrm{Re}\{X(e^{-j\omega})\} = \mathrm{Re}\{X(e^{j\omega})\}$$

$$\mathrm{Im}\{X(e^{-j\omega})\} = -\mathrm{Im}\{X(e^{j\omega})\}$$

Properties of the DTFT

- Time Reversal:

$$x[n] \leftrightarrow X(e^{j\omega})$$

$$x[-n] \leftrightarrow X(e^{-j\omega})$$

If x[n] real

$$x[-n] \leftrightarrow X^{*}(e^{j\omega})$$

- Time/Freq Shifting:

$$x[n] \leftrightarrow X(e^{j\omega})$$

$$x[n-n_d] \leftrightarrow e^{-j\omega n_d} X(e^{j\omega})$$

$$e^{j\omega_0 n} x[n] \leftrightarrow X(e^{j(\omega-\omega_0)})$$

Properties of the DTFT

- Differentiation in Frequency:

$$x[n] \leftrightarrow X(e^{j\omega})$$

$$nx[n] \leftrightarrow j\frac{dX(e^{j\omega})}{d\omega}$$

- Convolution in Time:

$$y[n] = x[n] * h[n]$$

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})$$

Fourier Transform Theorems

TABLE 2.2 FOURIER TRANSFORM THEOREMS

Sequence	Fourier Transform
$x[n]$	$X(e^{j\omega})$
$y[n]$	$Y(e^{j\omega})$
1. $ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
2. $x[n - n_d]$ (n_d an integer)	$e^{-j\omega n_d} X(e^{j\omega})$
3. $e^{j\omega_0 n} x[n]$	$X(e^{j(\omega-\omega_0)})$
4. $x[-n]$	$X(e^{-j\omega})$ $X^*(e^{j\omega})$ if $x[n]$ real.
5. $nx[n]$	$j\dfrac{dX(e^{j\omega})}{d\omega}$
6. $x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
7. $x[n]y[n]$	$\dfrac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$

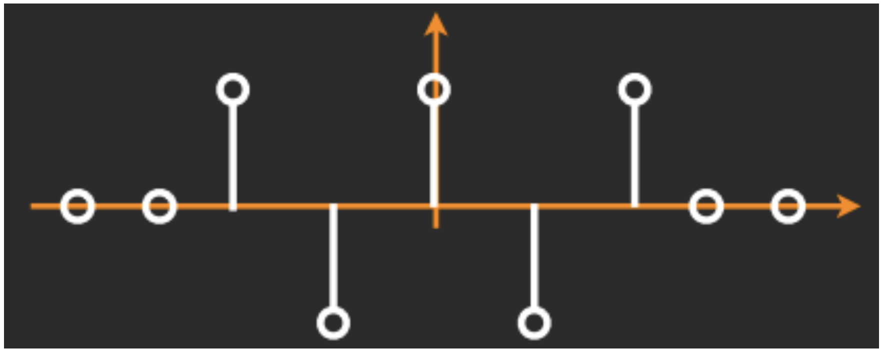
Parseval's theorem:

8. $$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$$

9. $$\sum_{n=-\infty}^{\infty} x[n]y^*[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})Y^*(e^{j\omega}) d\omega$$

Example:

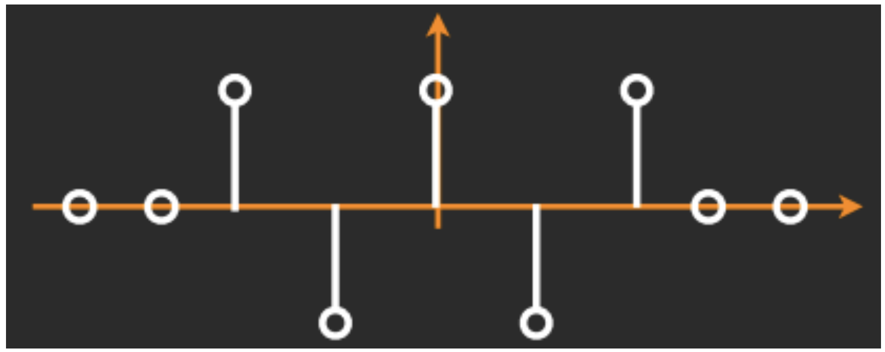
- What is DTFT of:

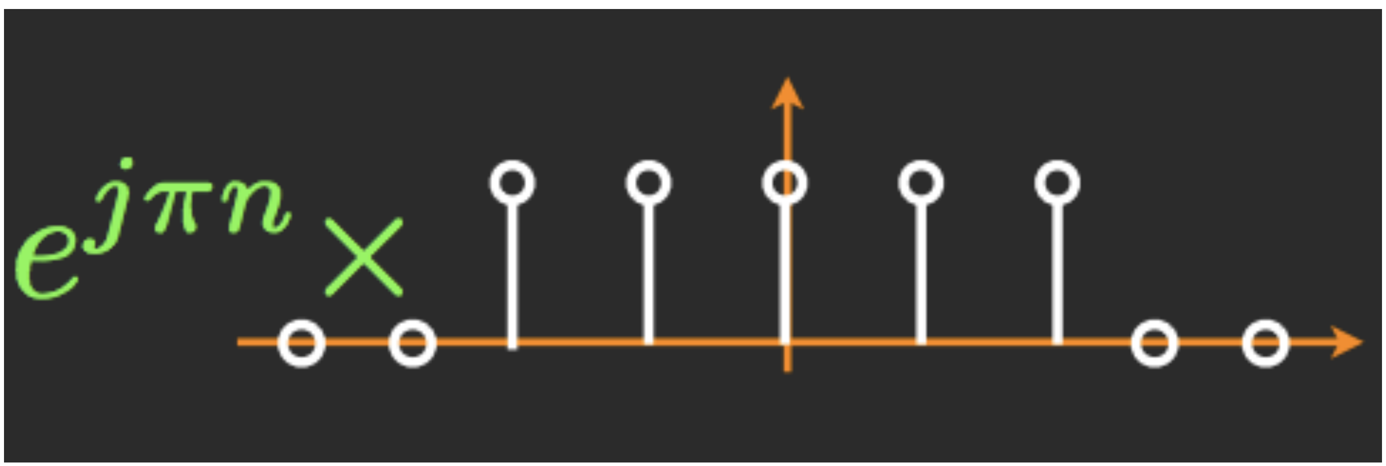


$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} x[k]e^{-j\omega k}$$

Example: Windowed cos(πn)

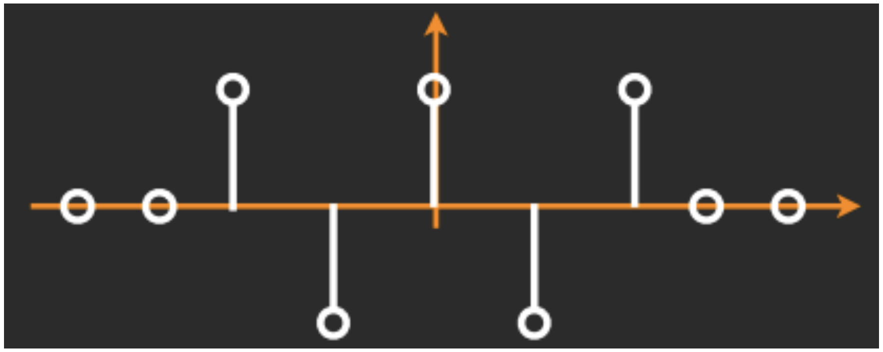
- What is DTFT of:

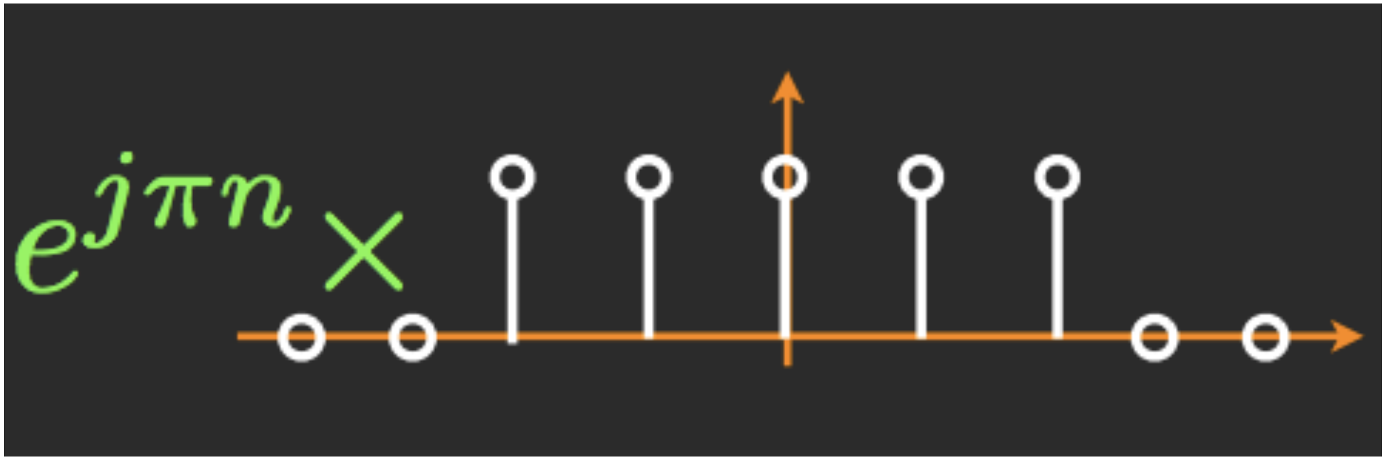




Example: Windowed cos(πn)

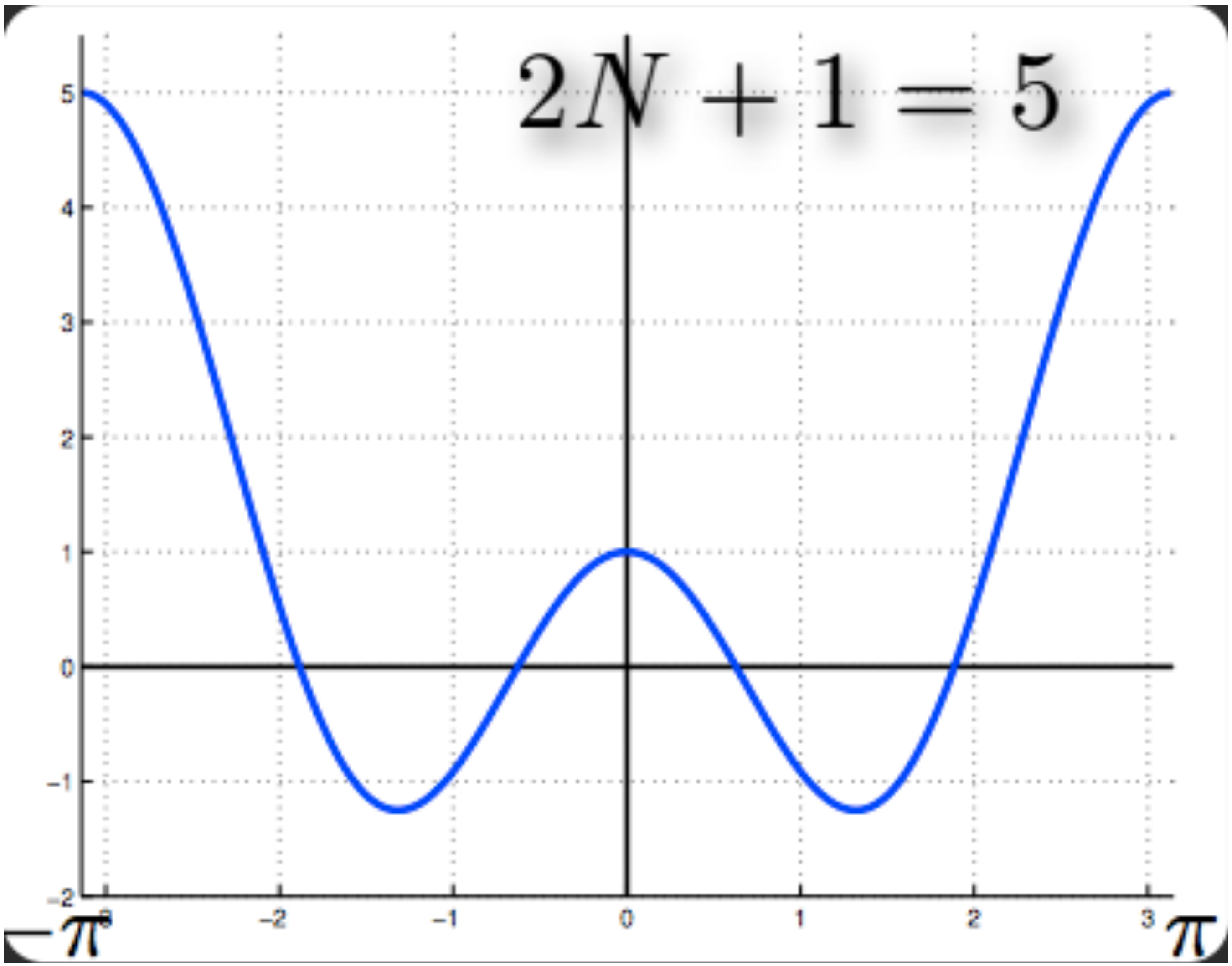
- What is DTFT of:





$$\boxed{e^{j\omega_0 n} x[n] \leftrightarrow X(e^{j(\omega-\omega_0)})}$$

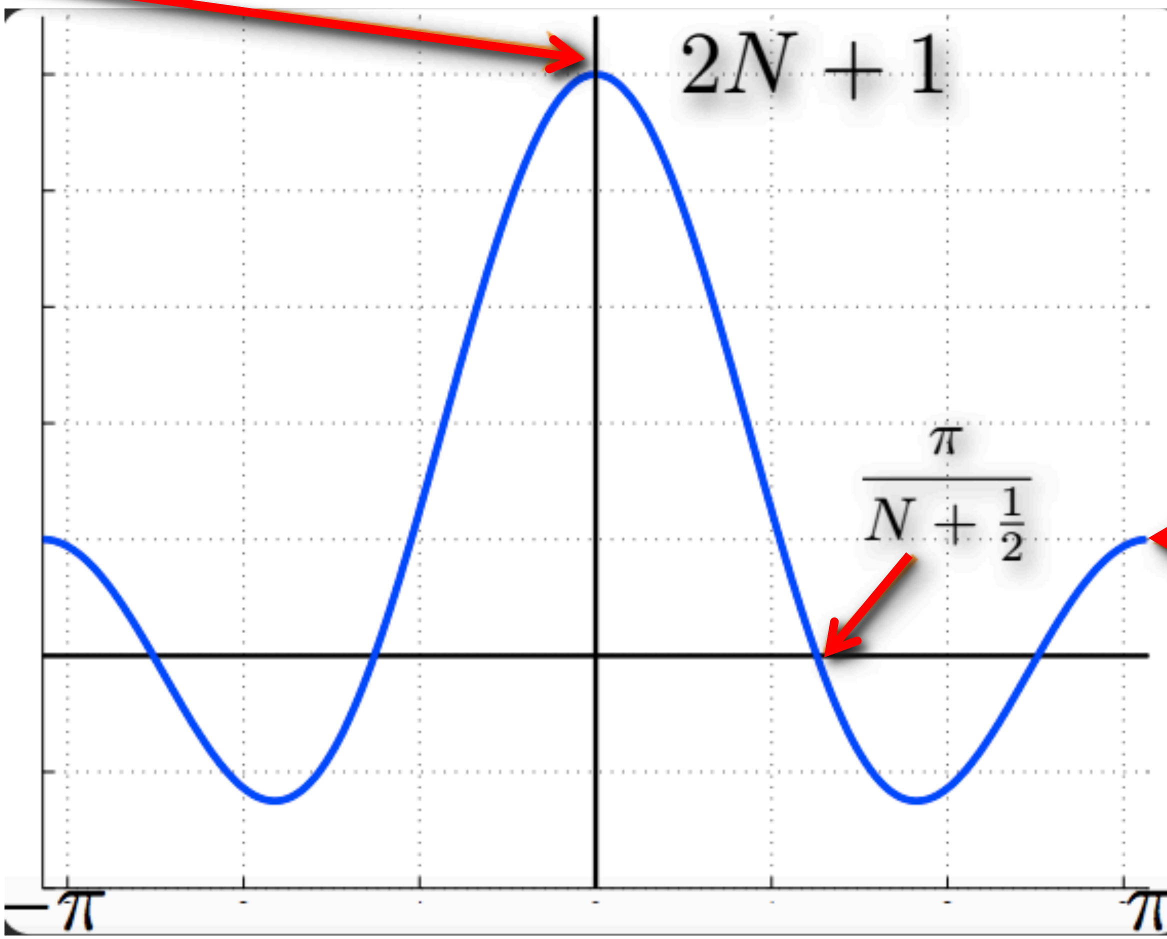
Example: Windowed cos(πn)



Example: Window DTFT

$$W(e^{j\omega}) = \frac{\sin\left((N+1/2)\omega\right)}{\sin\left(\omega/2\right)}$$

Also, Σw[n]



=1 why?

Plot for N=2

Frequency Response of LTI Systems

LTI Systems

> **DEFINITION**
> A system $\mathcal{H}$ is **linear time-invariant** (LTI) if it is both linear and time-invariant

- LTI system ca[...]l by its impulse response

$$\boxed{H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} h[k] e^{-j\omega k}}$$

- Then the output for an arbitrary input is a sum of weighted, delay impulse responses

$$x \longrightarrow \boxed{h} \longrightarrow y \qquad y[n] = \sum_{m=-\infty}^{\infty} h[n-m]\, x[m]$$

$$y[n] = x[n] * h[n]$$

LTI System Frequency Response

- (DT)Fourier Transform of impulse response

$$x[n]=e^{j\omega n} \longrightarrow \boxed{\text{LTI System}} \longrightarrow y[n]=H(e^{j\omega})e^{j\omega n}$$

$$H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} h[k]e^{-j\omega k}$$

Example: Moving Average

- Moving Average Filter
  - Causal: $M_1=0$, $M_2=M$

$$y[n] = \frac{x[n-M] + \ldots + x[n]}{M+1}$$

Example: Moving Average

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  - Causal: $M_1=0$, $M_2=M$

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Impulse response

Freq response?

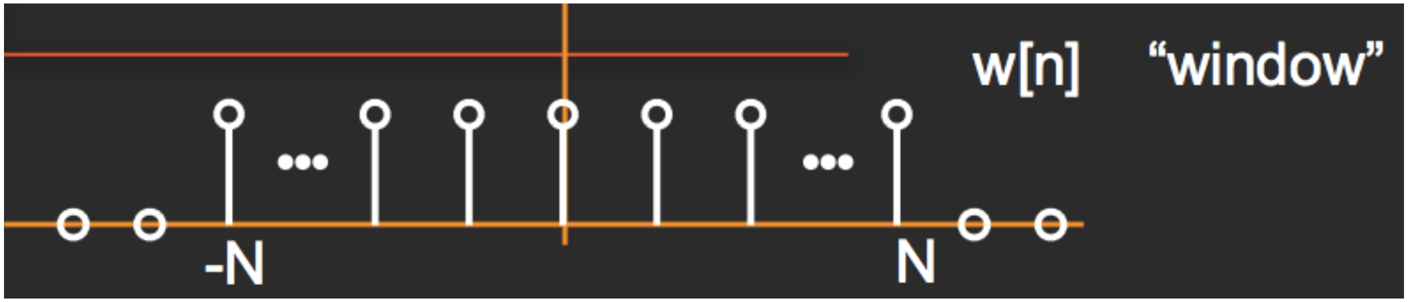
Example: Moving Average

- Moving Average Filter
  - Causal: M_1=0, M_2=M

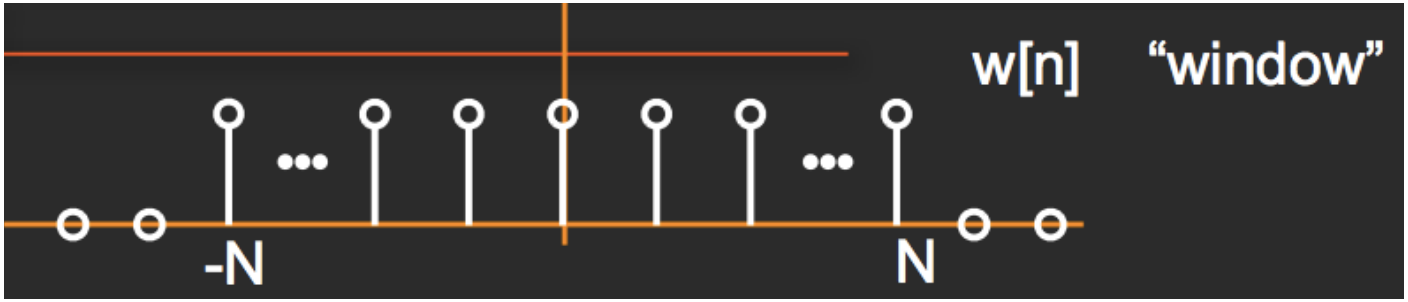
$$y[n] = \frac{x[n-M] + \ldots + x[n]}{M+1}$$

Impulse response

Scaled &Time Shifted window



Example: Moving Average

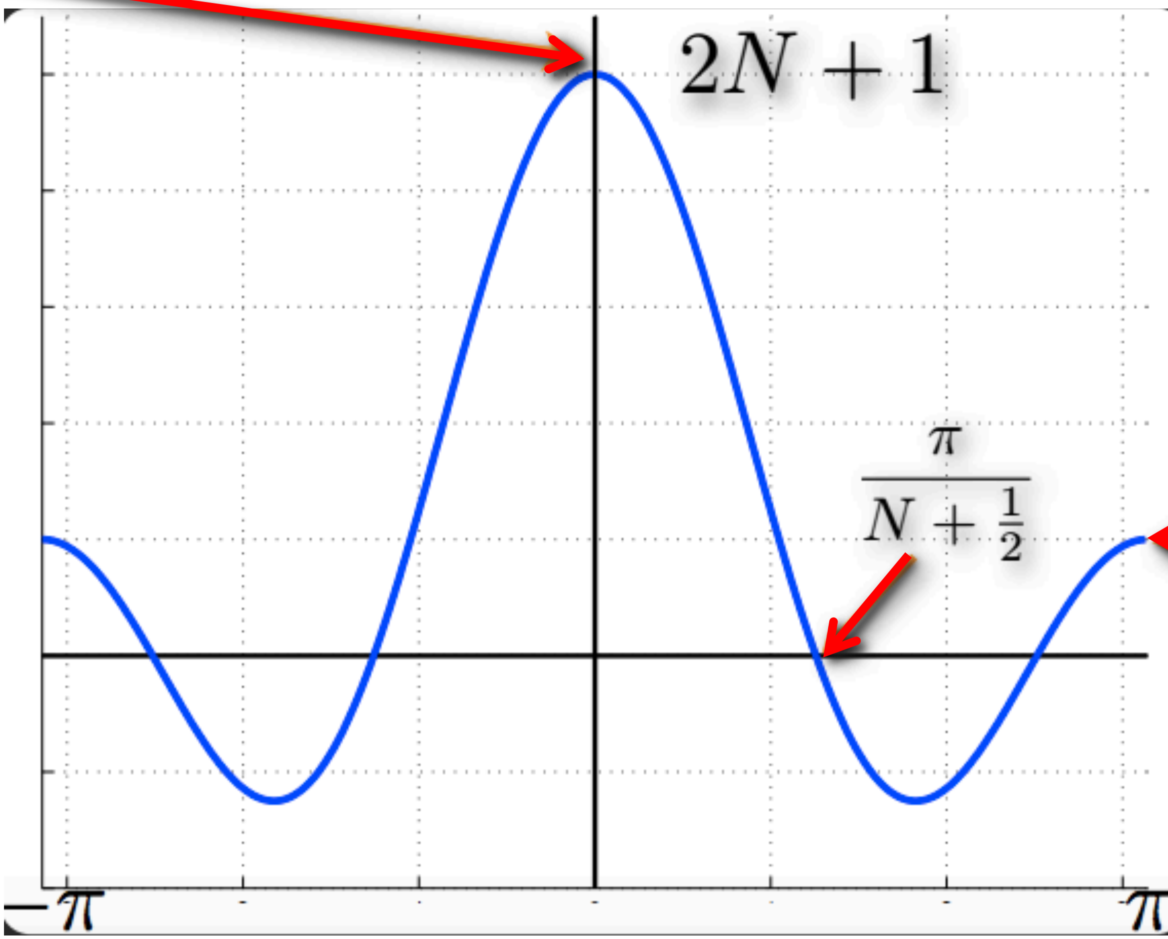


$$w[n] \leftrightarrow W(e^{j\omega}) = \frac{\sin\left((N+1/2)\omega\right)}{\sin\left(\omega/2\right)}$$

Example: Window DTFT

$$W(e^{j\omega}) = \frac{\sin\left((N+1/2)\omega\right)}{\sin\left(\omega/2\right)}$$

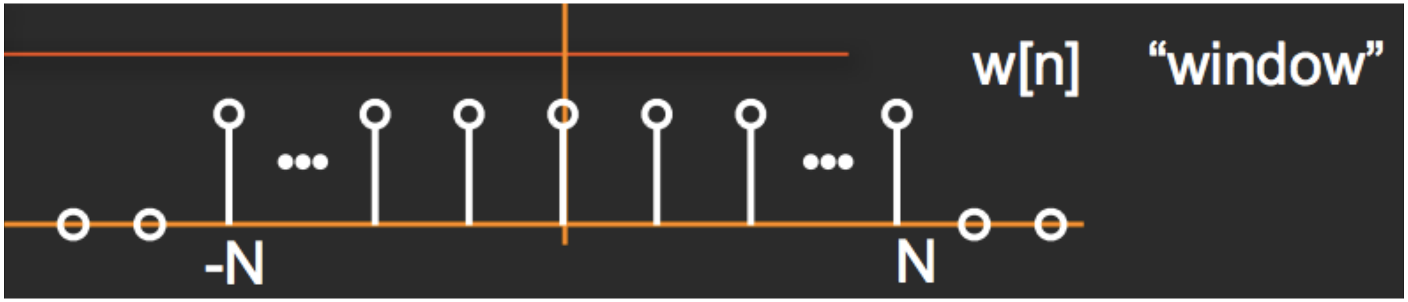
Also, Σx[n]



=1 why?

Plot for N=2

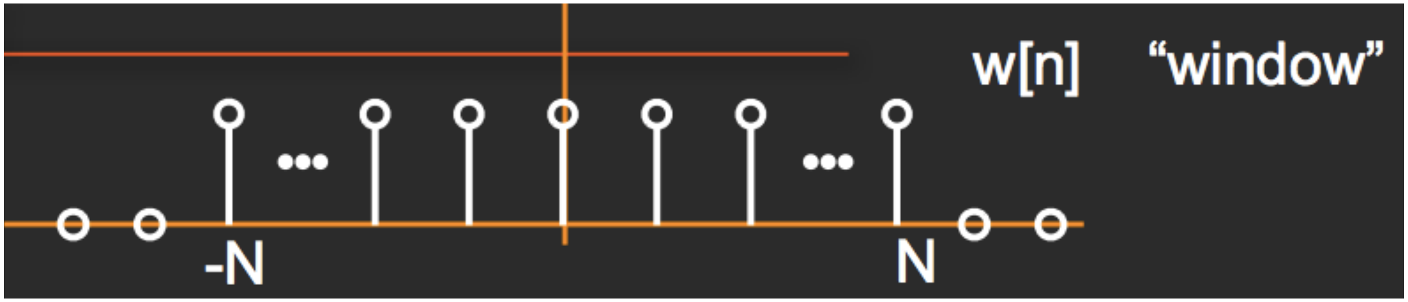
Example: Moving Average



$$w[n] \leftrightarrow W(e^{j\omega}) = \frac{\sin\left((N+1/2)\omega\right)}{\sin\left(\omega/2\right)}$$

$h[n] =$

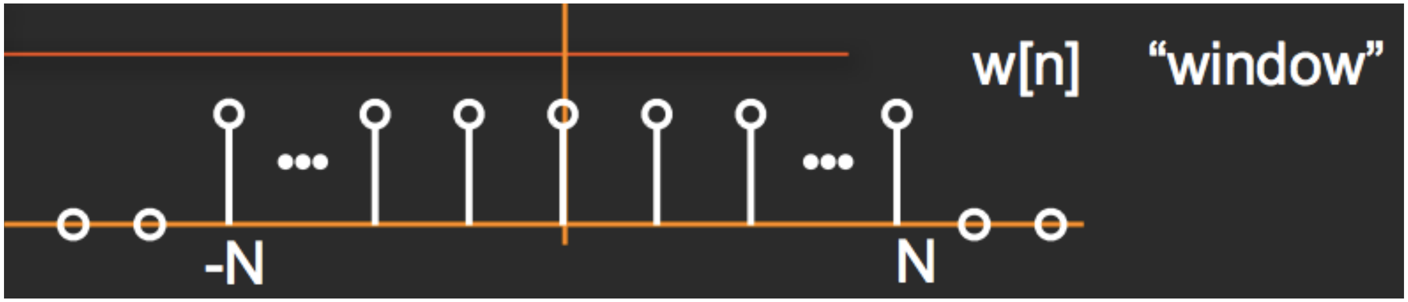
Example: Moving Average



$$w[n] \leftrightarrow W(e^{j\omega}) = \frac{\sin\left((N+1/2)\omega\right)}{\sin\left(\omega/2\right)}$$

$$h[n] = \frac{1}{M+1} w[n - M/2] \leftrightarrow H(e^{j\omega}) =$$

Example: Moving Average



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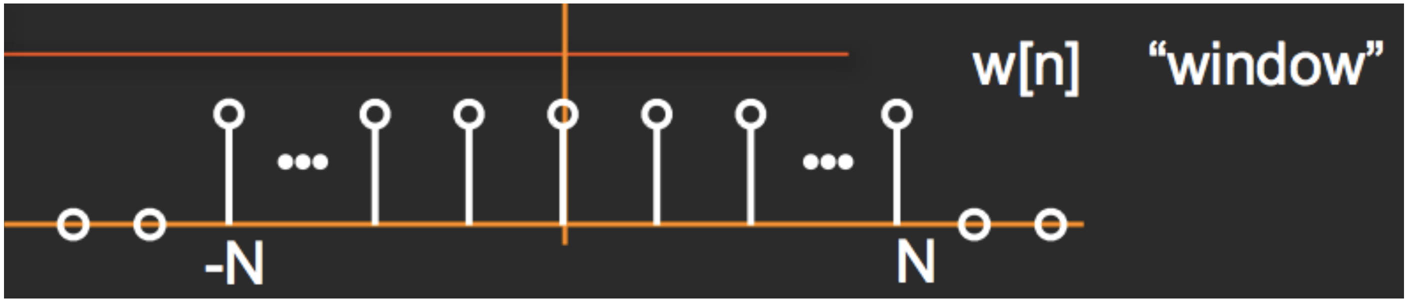
$$\boxed{x[n-n_d] \leftrightarrow e^{-j\omega n_d} X(e^{j\omega})}$$

Example: Moving Average



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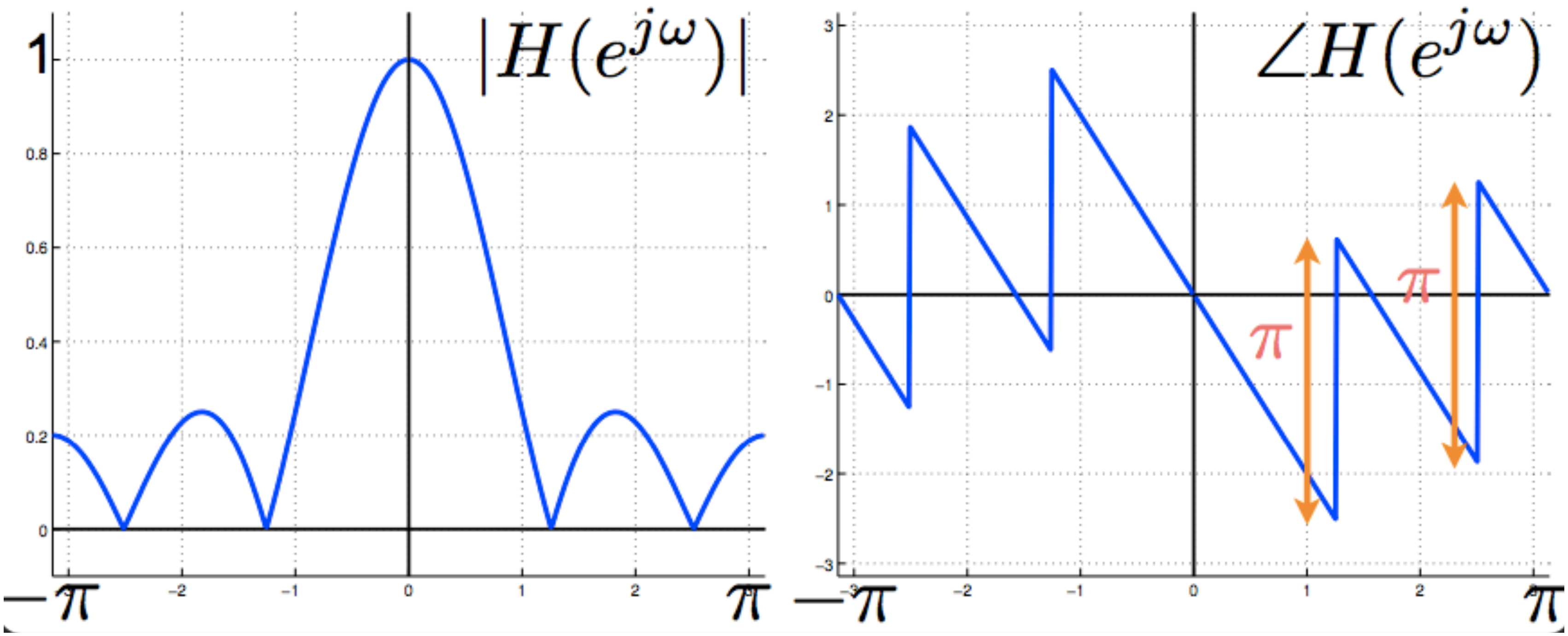
$$h[n] = \frac{1}{M+1} w[n - M/2] \leftrightarrow H(e^{j\omega}) = \frac{e^{-j\omega M/2}}{M+1} \frac{\sin\left((M/2+1/2)\omega\right)}{\sin\left(\omega/2\right)}$$

Freq response plot?

Example: Moving Average



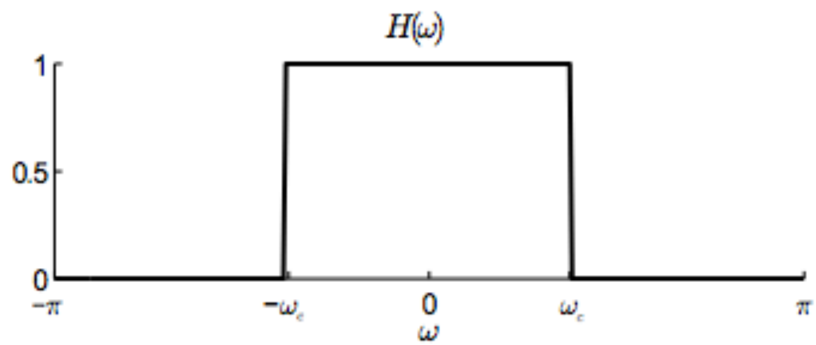
M=4
(N=2)

$$H(e^{j\omega}) = \frac{e^{-j\omega M/2}}{M+1}\frac{\sin\left((M/2+1/2)\omega\right)}{\sin\left(\omega/2\right)}$$

Example: Ideal Low-Pass Filter

- The frequency response $H(\omega)$ of the ideal low-pass filter passes low frequencies (near $\omega = 0$) but blocks high frequencies (near $\omega = \pm\pi$)

$$H(\omega) = \begin{cases} 1 & -\omega_c \le \omega \le \omega_c \\ 0 & \text{otherwise} \end{cases}$$



Example: Ideal Low-Pass Filter

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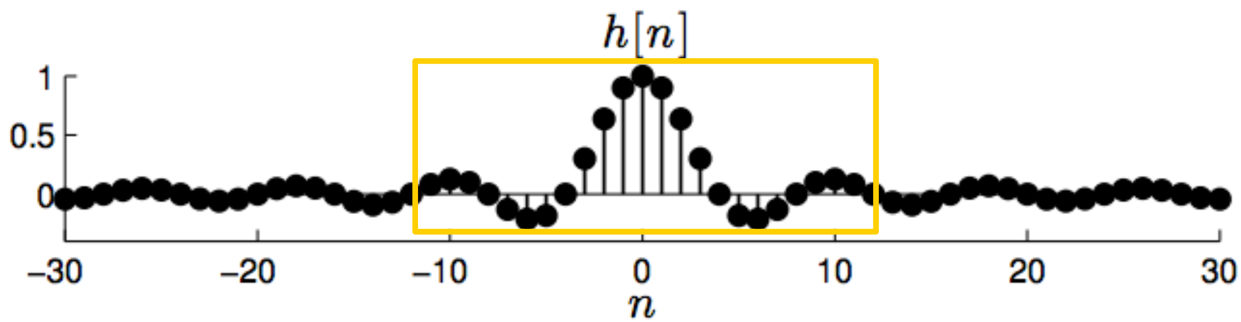
$$H(\omega) = \begin{cases} 1 & -\omega_c \leq |\omega| \leq \omega_c \\ 0 & \text{otherwise} \end{cases}$$

$$h[n] = 2\omega_c \frac{\sin(\omega_c n)}{\omega_c n}$$



Truncate and shift

$$h_{LP}[n] = w_N[n-N] \cdot h[n-N]$$

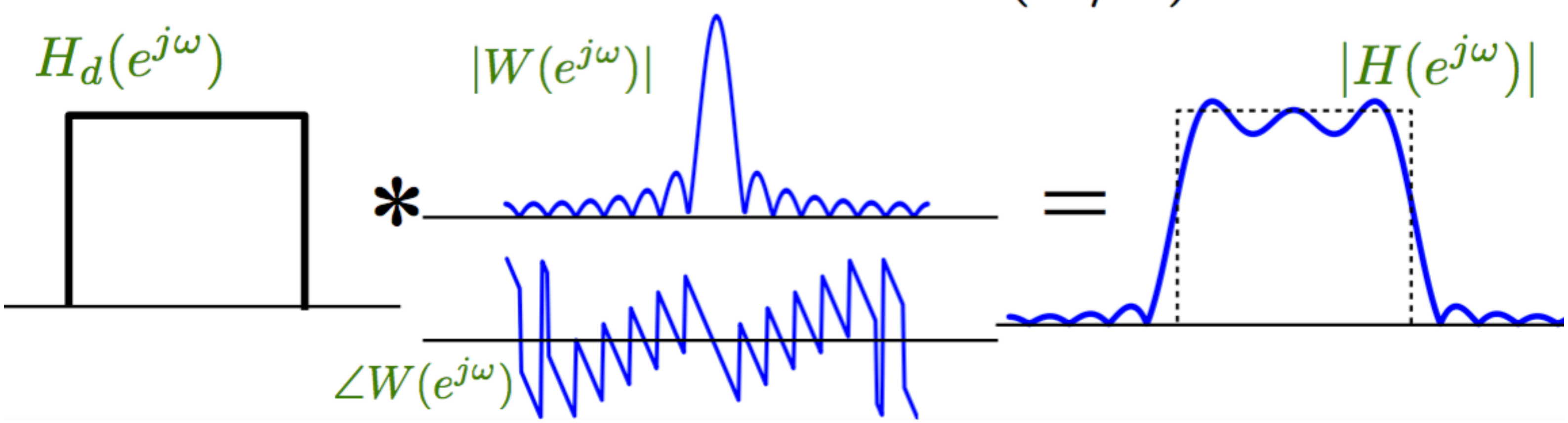
FIR Design by Windowing

- Desired filter,

$$H(e^{j\omega}) = H_d(e^{j\omega}) * W(e^{j\omega})$$

- For Boxcar (rectangular) window

$$W(e^{j\omega}) = e^{-j\omega\frac{M}{2}} \frac{\sin(w(M+1)/2)}{\sin(w/2)}$$



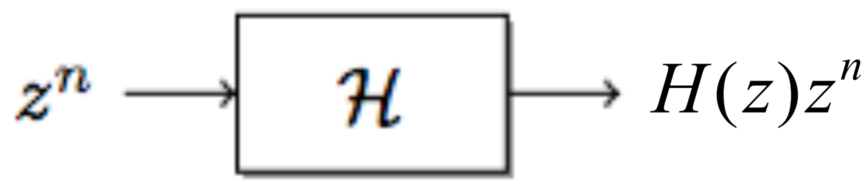
z-Transform

z-Transform

- The z-transform generalizes the Discrete-Time Fourier Transform (DTFT) for analyzing infinite-length signals and systems
- Very useful for designing and analyzing signal processing systems
- Properties are very similar to the DTFT with a few caveats

Complex Exponentials as Eigenfunctions

- Fact: A more general set of eigenfunctions of an LTI system are the complex exponentials z^n, $z \in C$



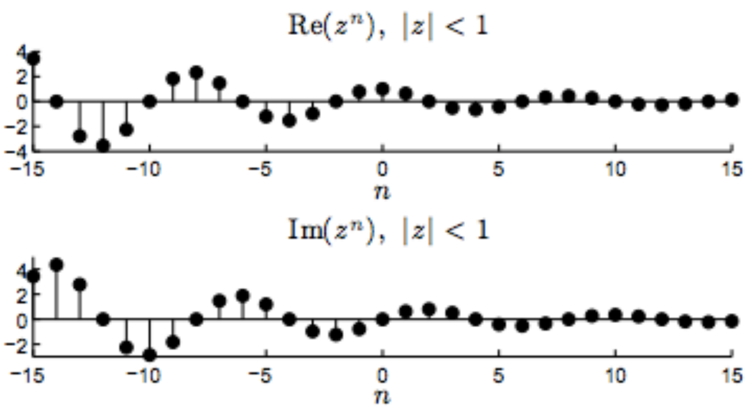
Reminder: Complex Exponentials

$$z^n = \left(|z|\, e^{j\omega n}\right)^n = |z|^n\, e^{j\omega n}$$

$|z|^n$ is a **real exponential** envelope $(a^n$ with $a = |z|)$

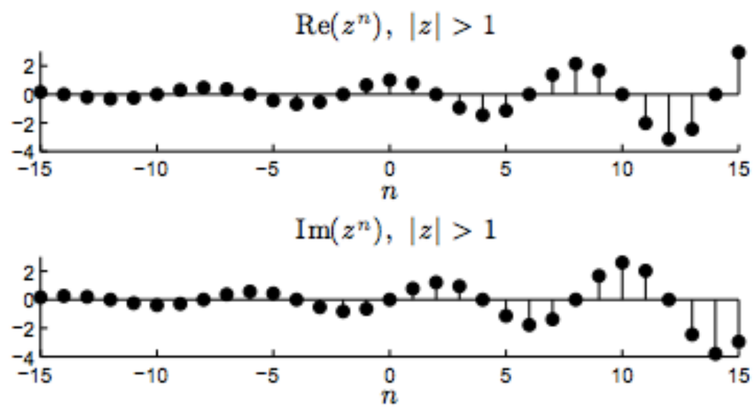
$e^{j\omega n}$ is a **complex sinusoid**

$|z| < 1$



Bounded

$|z| > 1$



Unbounded

Proof: Complex Exponentials as Eigenfunctions

$$z^n \longrightarrow \boxed{\mathcal{H}} \longrightarrow H(z)z^n$$

- Prove by computing the convolution with input $x[n] = z^n$

Proof: Complex Exponentials as Eigenfunctions

$$z^n \longrightarrow \boxed{\mathcal{H}} \longrightarrow H(z)z^n$$

- Prove by computing the convolution with input x[n] = z^n

$$
\begin{aligned}
z^n * h[n] &= \sum_{m=-\infty}^{\infty} z^{n-m}\, h[m] = \sum_{m=-\infty}^{\infty} z^n\, z^{-m}\, h[m] \\
&= \left(\sum_{m=-\infty}^{\infty} h[m]\, z^{-m} \right) z^n \\
&= H(z)z^n \checkmark
\end{aligned}
$$

z-Transform

- Define the **forward z-transform** of x[n] as

$$X(z) \;=\; \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$$

- The core "basis functions" of the z-transform are the complex exponentials z^n with arbitrary $z \in C$; these are the eigenfunctions of LTI systems for infinite-length signals

- **Notation abuse alert:** We use X(•) to represent both the DTFT $X(e^{j\omega})$ and the z-transform X(z); they are, in fact, intimately related

$$X_z(z)\big|_{z=e^{j\omega}} \;=\; X_z(e^{j\omega})$$

Transfer Function of LTI System

- We can use the z-Transform to characterize an LTI system

$$x \longrightarrow \boxed{\mathcal{H}} \longrightarrow y$$

$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} h[n-m]\, x[m]$$

- and relate the z-transforms of the input and output

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}, \quad H(z) = \sum_{n=-\infty}^{\infty} h[n]\, z^{-n}$$

$$Y(z) = X(z)\, H(z)$$

Z-transform

What are we missing?

$$\boxed{X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}}$$

Z-Transform

$$z^n = \left(|z|\, e^{j\omega n}\right)^n = |z|^n\, e^{j\omega n}$$

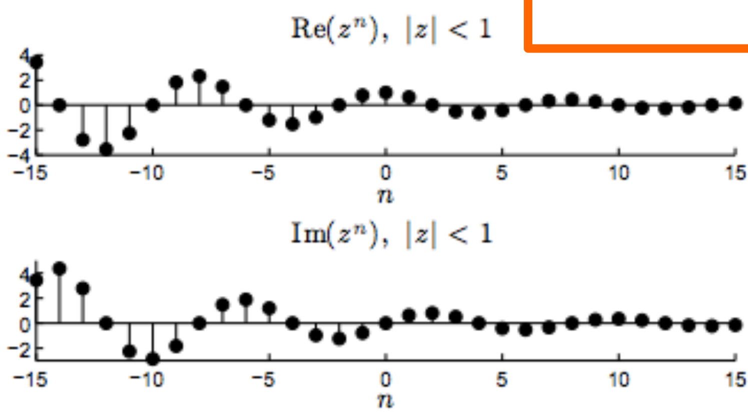
$|z|^n$ is a **real exponential** envelope $\left(a^n \text{ with } a = |z|\right)$

$e^{j\omega n}$ is a **complex sinusoid**

What are we missing?

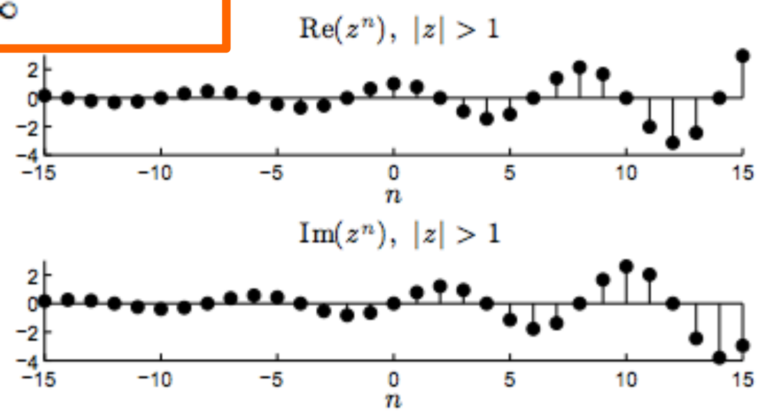
$$\boxed{X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}}$$

$|z| < 1$



Bounded

$|z| > 1$



Unbounded

Region of Convergence (ROC)

Region of Convergence (ROC)

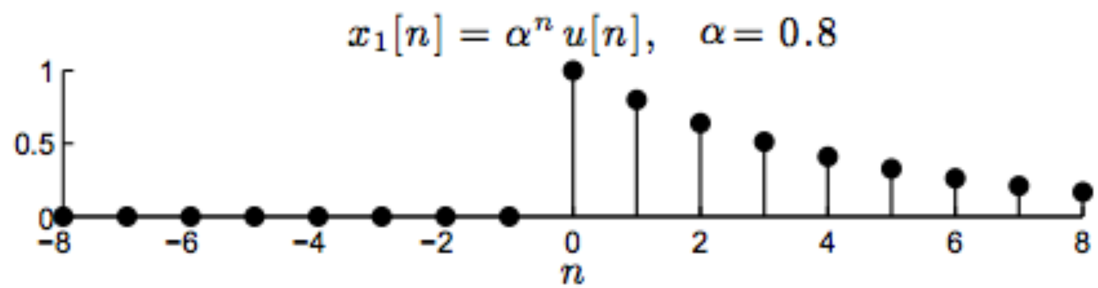
DEFINITION

Given a time signal $x[n]$, the **region of convergence** (ROC) of its z-transform $X(z)$ is the set of $z \in \mathbb{C}$ such that $X(z)$ converges, that is, the set of $z \in \mathbb{C}$ such that $x[n]\, z^{-n}$ is absolutely summable

$$\sum_{n=-\infty}^{\infty} |x[n]\, z^{-n}| \; < \; \infty$$

ROC Example 1

- Signal $x_1[n] = \alpha^n u[n]$, $\alpha \in \mathbb{C}$ (causal signal) Right-sided sequence
- Example for $\alpha = 0.8$



ROC Example 1

- Signal $x_1[n] = \alpha^n u[n]$, $\alpha \in \mathbb{C}$ (causal signal) Right-sided sequence
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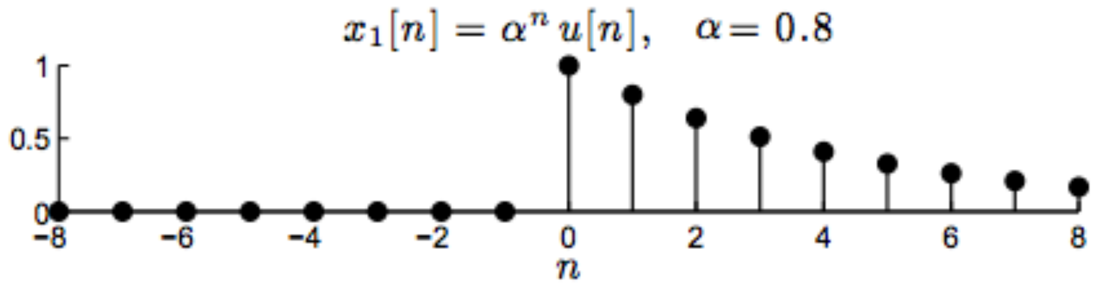


- The **forward** z**-transform** of $x_1[n]$

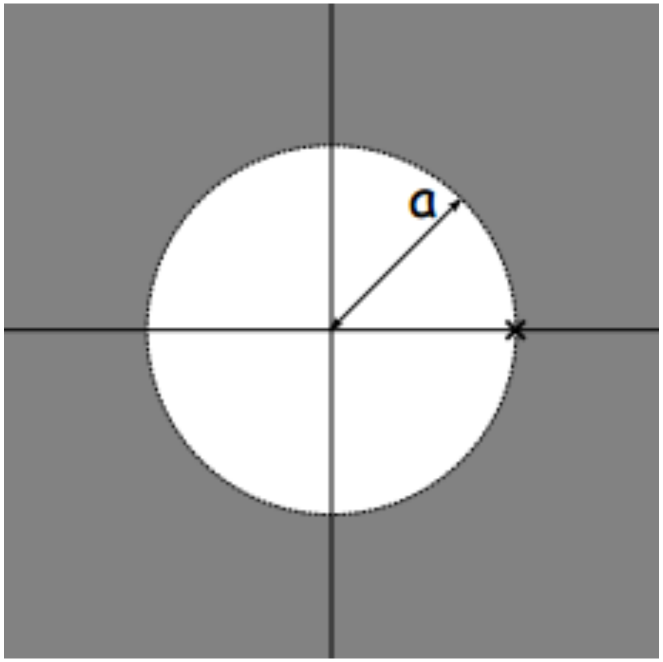
$$X_1(z) = \sum_{n=-\infty}^{\infty} x_1[n]\, z^{-n} = \sum_{n=0}^{\infty} \alpha^n\, z^{-n} = \sum_{n=0}^{\infty} (\alpha\, z^{-1})^n = \frac{1}{1-\alpha\, z^{-1}} = \frac{z}{z-\alpha}$$

- **Important:** We can apply the geometric sum formula only when $|\alpha\, z^{-1}| < 1$ or $|z| > |\alpha|$

ROC Example 1

- Signal $x_1[n] = \alpha^n u[n]$, $\alpha \in \mathbb{C}$ (causal signal)

$$ROC = \{z : \; |z| > |a|\}$$



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- **Important:** We can apply the geometric sum formula only when $|\alpha\, z^{-1}| < 1$ or $|z| > |\alpha|$

Poles and zeros?

ROC Example 1

- What is the DTFT of $x_1[n] = \alpha^n u[n]$?

$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} x[k] e^{-j\omega k}, \qquad -\pi \leq \omega < \pi$$

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega, \qquad -\infty \leq n < \infty$$

ROC Example 1

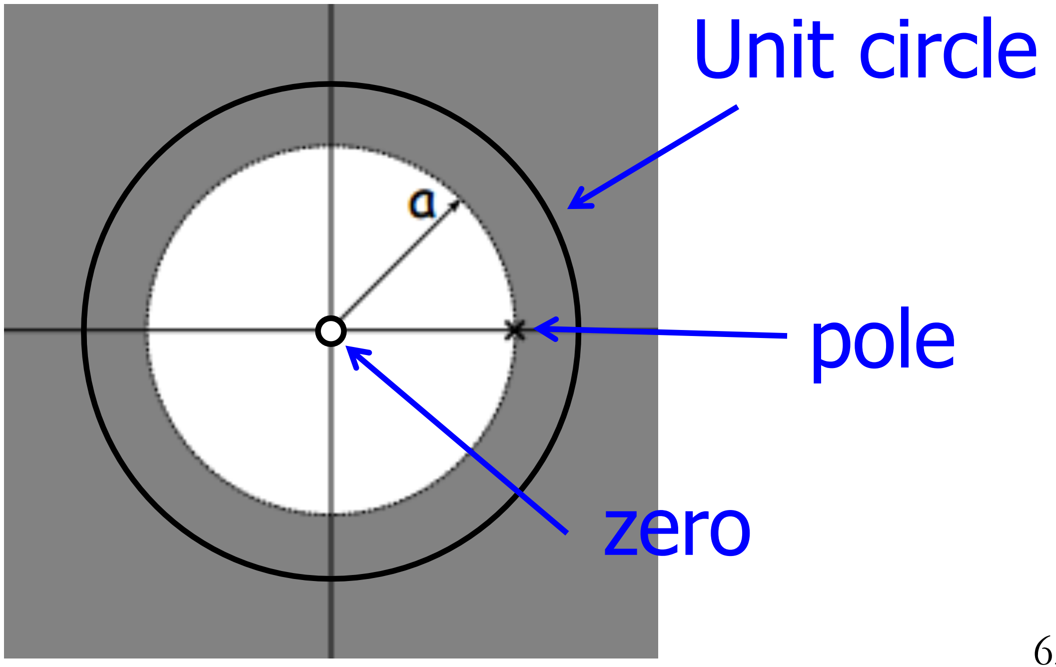
- What is the DTFT of $x_1[n]=a^n u[n]$?

$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} x[k]e^{-j\omega k}, \qquad -\pi \le \omega < \pi$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega, \qquad -\infty \le n < \infty$$

$$X_1(z) = \frac{z}{z-a}$$

$$ROC = \{z: \ |z| > |a|\}$$



ROC Example 2

- What is the z-transform of $x_2[n]$? ROC?

$$x_2[n] = \left(\frac{1}{2}\right)^n u[n] + \left(-\frac{1}{3}\right)^n u[n]$$

ROC Example 2

- What is the z-transform of $x_2[n]$? ROC?

$$x_2[n] = \left(\frac{1}{2}\right)^n u[n] + \left(-\frac{1}{3}\right)^n u[n]$$

- Hint: $x_1[n] = a^n u[n] \xleftrightarrow{Z} \dfrac{1}{1 - az^{-1}}$ $\quad ROC = \{z : \ |z| > |a|\}$

ROC Example 3

- What is the z-transform of $x_3[n]$? ROC?

$$x_3[n] = -a^n u[-n-1]$$

$$\boxed{X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}}$$

Left-sided sequence

ROC Example 3

- What is the z-transform of $x_3[n]$? ROC?

$$x_3[n] = -a^n u[-n-1]$$

$$\boxed{X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}}$$

- The z-transform without ROC does not uniquely define a sequence!

Properties of ROC (so far)

- For right-sided sequences: ROC extends outward from the outermost pole to infinity
  - Examples 1,2
- For left-sided: inwards from inner most pole to zero
  - Example 3

Big Ideas

- Discrete Time Fourier Transform
  - Represent signals as a sum of scaled and phase shifted complex sinusoids (eigenfunctions)
  - Continuous in frequency over 2π
- Frequency Response of LTI Systems
  - Frequency response of impulse response
  - Describes scaling and phase shifting of a pure frequency

$$x[n]=e^{j\omega n} \longrightarrow \boxed{\text{LTI System}} \longrightarrow y[n]=H(e^{j\omega n})e^{j\omega n}$$

$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} x[k]e^{-j\omega k}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}\,d\omega$$

Big Ideas

- z-Transform
  - Uses complex exponential eigenfunctions to represent discrete time sequence
    - DTFT is z-Transform where $z=e^{j\omega}$, $|z|=1$
  - Draw pole-zero plots
  - Must specify region of convergence (ROC)
  - More next lecture…

Admin

- HW 1 out since 1/24
  - Due Sunday 2/4 at midnight
  - Submit in Gradescope (via Canvas)
    - Leave time to submit to avoid late penalty
    - Make sure you link the correct page to the correct question
- HW 2 posted 2/4 due on 2/11
- Diagnostic quiz grades/solutions