ESE532:
System-on-a-Chip Architecture

Day 7: September 27, 2021
Pipelining

Penn

Previously

- Pipelining in the large
  - Not just for gate-level circuits
- Throughput and Latency
- Pipelining as a form of parallelism

Today

Pipelining details (for gates, primitive ops)
- Systematic Approach (Part 1)
- Justify Operator and Interconnect Pipelining (Part 2)
- Loop Bodies
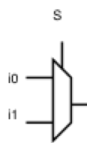- Cycles in the Dataflow Graph (Part 3)
- C-slow [online-only recording] (Part 4)

Message

- Pipelining is an efficient way to reuse hardware to perform the **same** set of operations at high throughput

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Multiplexer Gate

- MUX
  - When S=0, output=i0
  - When S=1, output=i1

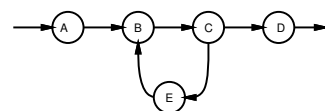| S	i0	i1	Mux2(S,i0,i1)
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Cycle

Two uses of term in this lecture:
- Repetitive waveform
  - E.g. sine wave or square wave
- Graph cycle

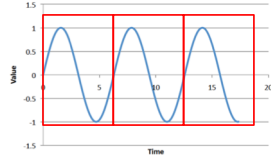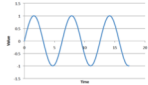## Waveform Cycles

- How many cycles showing of sine wave?

- How many cycles of square wave?

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Waveform Cycles

- How many cycles showing of sine wave?

- How many cycles of square wave?
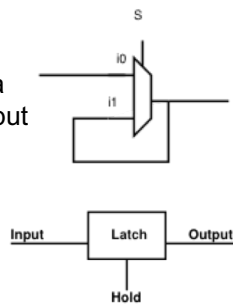
- Note: clock on which we pipeline is a square wave
  - Talk about what happens in a clock cycle
  - Talk about number of clock cycles

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Latch

S

i0

i1

- Element that can hold a previous value of an input

Input | Latch | Output

Hold
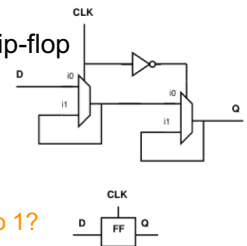
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Register

CLK

- Use a pair to create a flip-flop
  - Also call register
- What happens when
  - CLK is low (0) ?
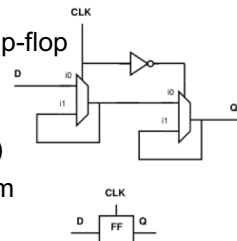 - CLK is high (1) ?
  - CLK transitions from 0 to 1?

D i0 i0 Q
i1 i1

CLK
D FF Q

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Register

CLK

- Use a pair to create a flip-flop
  - Also call register
- Sample D input on 0→1 transition of clock (CLK)
- Never an open path from D→Q
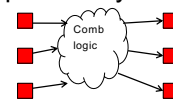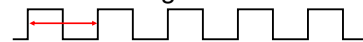 - One of the mux latches always in hold state

D i0 i0 Q
i1 i1

CLK
D FF Q

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Synchronous Circuit Discipline

- Registers that sample inputs at clock edge and hold value throughout clock period
- Compute from registers-to-registers
- Clock Cycle time large enough for longest logic path between registers
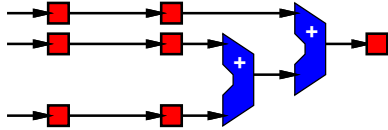- Min cycle = Max path delay between registers

Comb logic

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Preclass 1

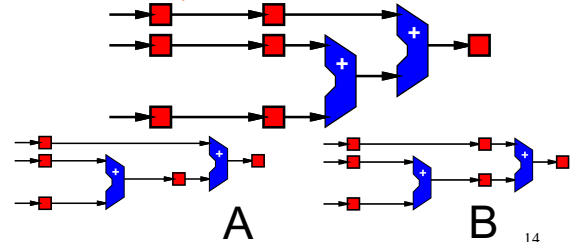• Delay between registers as shown?

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Preclass 1

• Move registers so can clock at adder delay?
  (which is of choices at bottom is correct = same behavior)

A   B

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Pipeline Reuse

• Lower delay between clocks
  – Higher clock rate
  – Higher potential throughput
  – Faster we reuse our logic
  – More capacity get out of design
    • Assuming registers cheap in area and time overhead
      – T_{setup}, $T_{clk->q}$ ~ 20ps, T_{add} ~ 500ps
      – Registers ~ 10 transistors/bit
      – Adder ~ 40—50 transistors/bit

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Preclass 2: What Happens?

• What would be wrong with this pipelining?
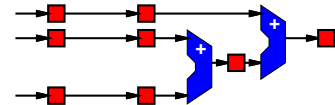
• For this initial design:
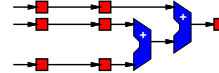
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Behavior

Equations

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3
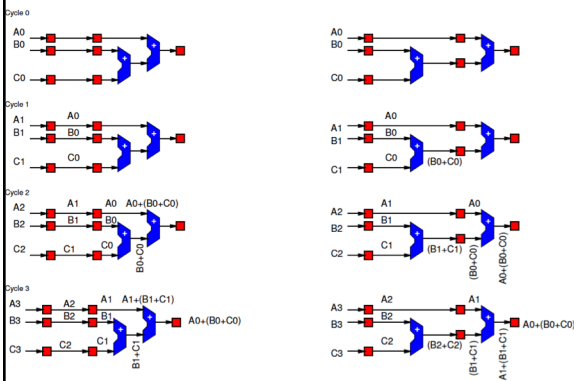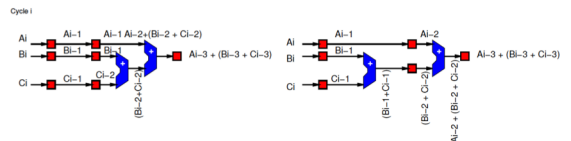
Behavior Add Register

Equations

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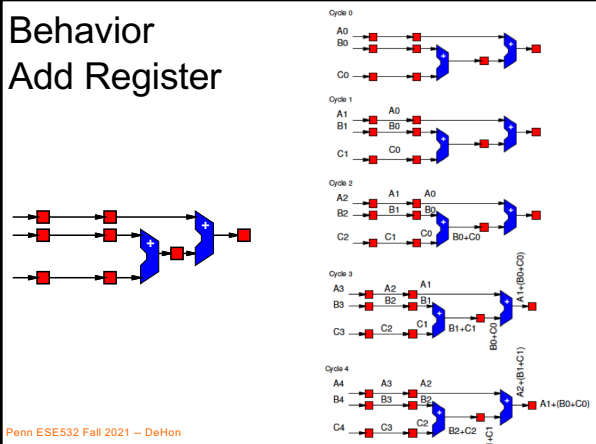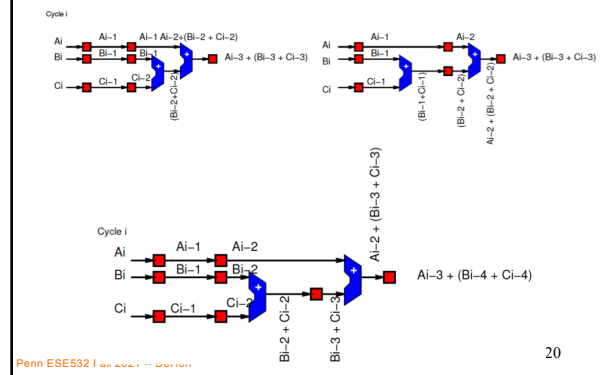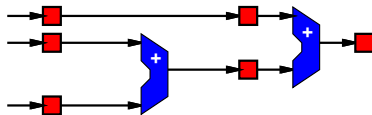
Note Registers on Links

- Some links end up with multiple registers.
- Why?

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Consistent Pipelining
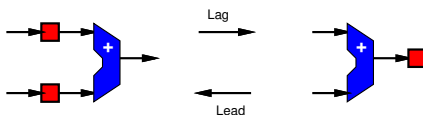
- Makes sure a consistent input set arrives at each gate/operator
  - Don't get mixing between input sets

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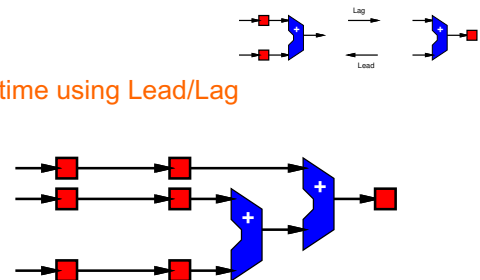# Legal Register Moves

- Retiming Lag/Lead

- Lag: remove register every input add register every output
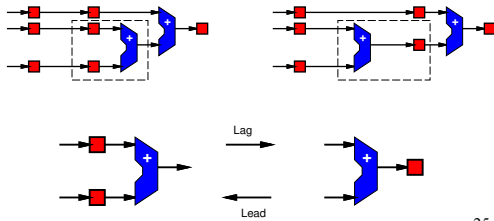- Lead: remove register every output add register every input

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Preclass 1

- Retime using Lead/Lag
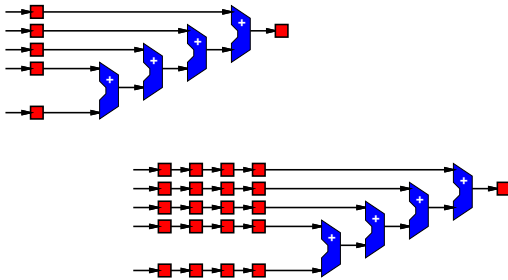
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Preclass 1 (revisited)

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Add Registers and Move

- If we're willing to add pipeline delay
  - Add any number of pipeline registers at input
  - Move registers into circuit to reduce cycle time
    - Reduce max delay between registers

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Add Registers at Input

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Add Register and Retime

- Add chain of registers on every input
- Retime registers into circuit
  - Minimizing delay between registers

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Add Registers and Retime

- Lets us think about behavior
  - What the pipelining is doing to cycles of delay
- Separate from details of how redistribute registers
- Behavioral equivalence between the registers-at-front and properly retimed version of circuit

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Justify Pipelining

(or composing pipelined operators)

Part 2

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Handling Pipelined Operators

- Given a pipelined operator
  - (or a pipelined interconnect)
- Discipline of picking a frequency target and designing everything for that
  - May be necessary to pipeline operator since its delay is too high
- Due to hierarchy
  - Pipelined this operator and now want to use it as a building block

Examples

- Run at 500MHz
- Floating-point unit that takes 9ns
  - Can pipeline into 5, 2ns stages
- Multiplier that takes 6ns
- Memory can access in 2ns
  - Only if registers on address/inputs and output
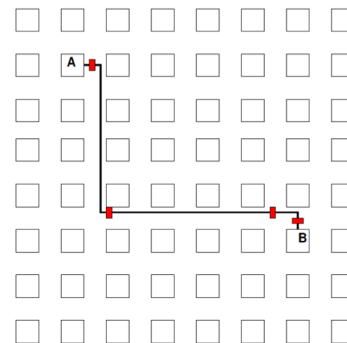 - i.e. exist in own clock stage

Interconnect Delay

- Chips >> Clock Cycles
- May have chip 100s of Operators wide
- May only be able to reach across 10 operators in a 2ns cycle
- Must pipeline long interconnect links

Interconnect Example

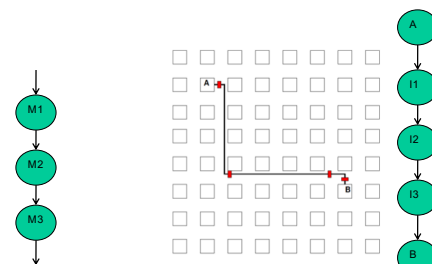## Methodology: Pipelined Operator Graph

- Start with logical, unpipelined graph
- Treat each pipelined operator as a set of unit-delay operators of mandatory depth
- Treat each interconnect pipeline stage as a unit-delay buffer
- Add registers at input
- Retime into graph

Model

- 3-stage Multiplier
- Interconnect Delay

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Pipeline Loop

(and use for justify pipeline example)

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Preclass 4

- Logical (unpipelined) dataflow graph for loop body
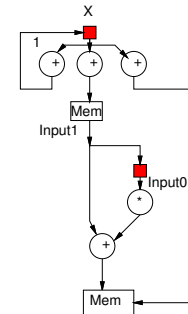
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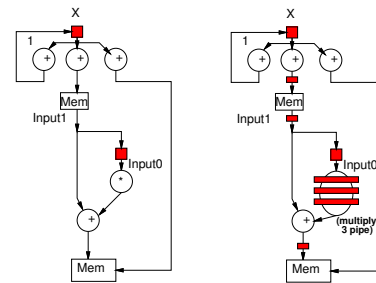
Example Operators

- Operator and Interconnect delays
  - Multiplier 3 cycles
  - Reading from Input array
    - Memory op is cycle after computing address
    - Takes one cycle delay bring data back to multiplier (or adder)

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Illustrate Need

- What happens if just use graph as is (with operators pipelined as required)?

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Model Graph
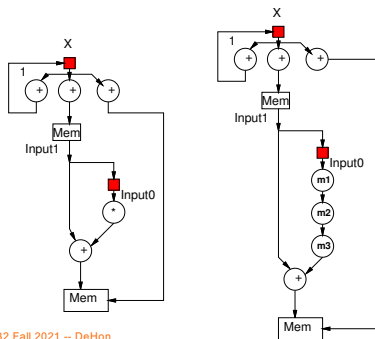
- Revised graph for modeling

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Pipeline Graph
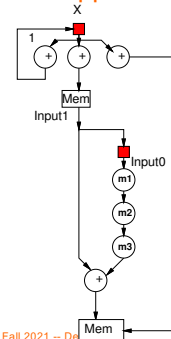
- What happens after pipeline and retiming?

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Pipelining Lesson

- Can always pipeline an **acyclic** graph (no graph cycles)
  to fixed frequency target
  – fixed pipelining of primitive operators
  – Pipeline interconnect delays
- Need to keep track of registers to balance paths
  – So see consistent delays to operators

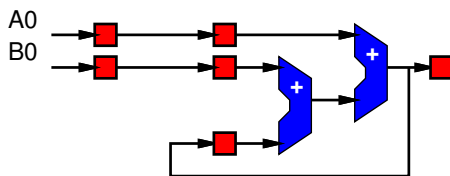## Graph Cycles

Watch: Clock cycle
Cycle time
Cycle in Graph
Part 3
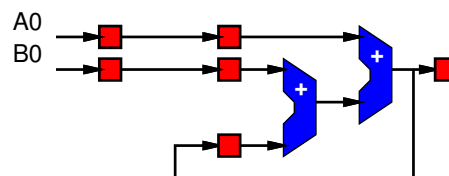
Preclass 3

- Can we retime to reduce clock cycle time?

A0
B0

Retiming Limits?

- What prevents us from retiming?

A0
B0

(Graph) Cycle Observation

- Retiming does not allow us to change the *number of registers inside a graph cycle*.
- Limit to clock cycle time
  – Max delay in graph cycle / Registers in graph cycle
- Pipelining doesn't help inside graph cycle
  – Cannot push registers into graph cycle

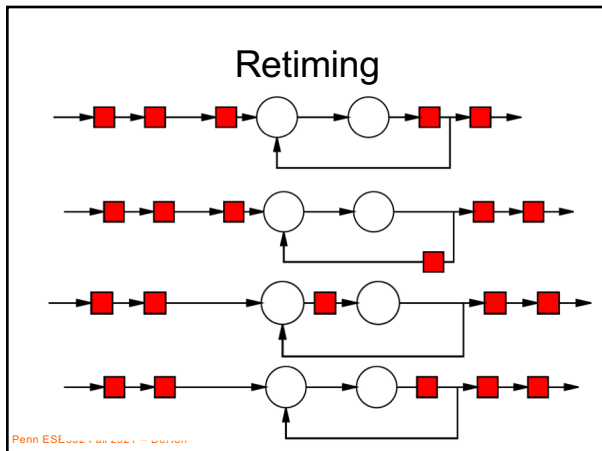## Simple Graph Cycle

- Delay of graph cycle?
- Registers in graph cycle?
- What happens to graph cycle if try to apply lead/lag?

Retiming
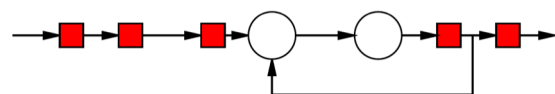
Loop

- Consider
  - [multiply and mod each take 3 cycles]
- For (i=0;i<N;i++)
  C[i]=(C[i-1]*A[i])%N;
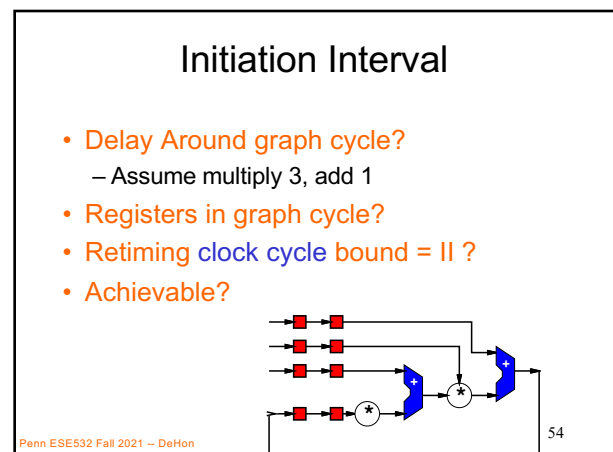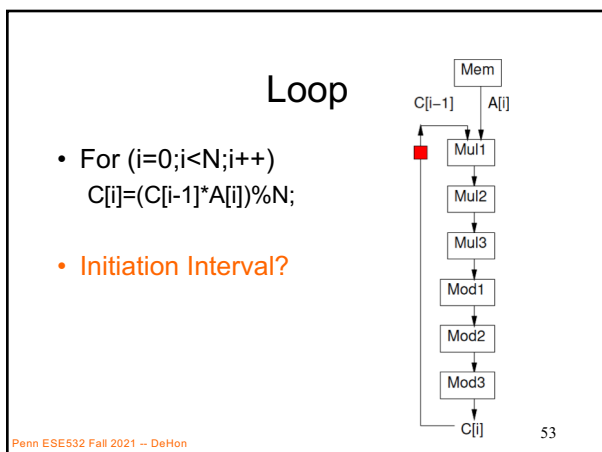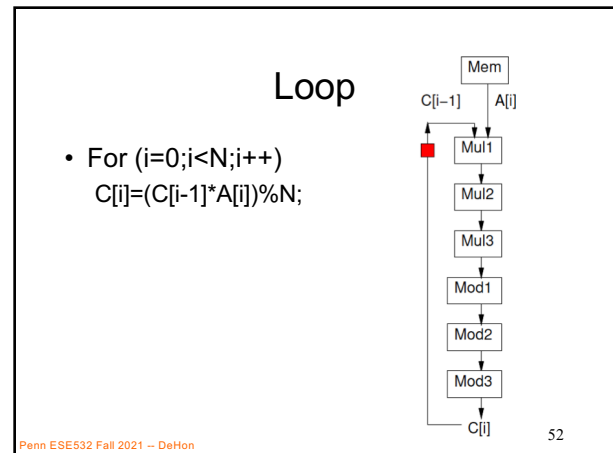
Initiation Interval (II)

- Cyclic dependencies in a dataflow graph can limit throughput
- Due to data-dependent cycles in graph,
  - May not be able to initiate a new computation on every clock cycle
- II – clock cycles (delay) before can initiate
- Throughput = 1/II

Loop

- For (i=0;i<N;i++)
  C[i]=(C[i-1]*A[i])%N;

Loop

- For (i=0;i<N;i++)
  C[i]=(C[i-1]*A[i])%N;

- Initiation Interval?

Initiation Interval

- Delay Around graph cycle?
  - Assume multiply 3, add 1
- Registers in graph cycle?
- Retiming clock cycle bound = II ?
- Achievable?

II and Latency

- Actually is a cycle
  - II?
  - Latency?
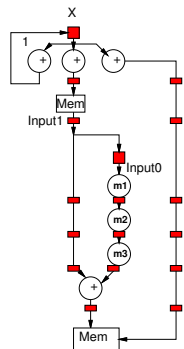
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II and Latency

- II? (assume willing to pipeline inputs)
- Latency?
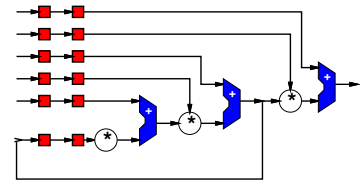
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Lesson

- Cyclic dependencies limit throughput on single task or data stream
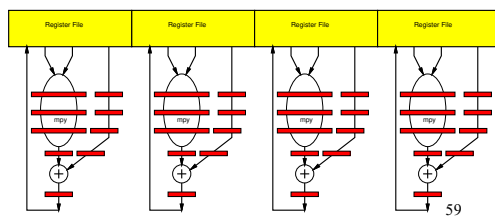 - Cycle-length / registers-in-cycle

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Vector Pipelines

- Data Parallel Vector Operations are interesting even when
    Vector Lanes<Vector Length
- Within Vector operation, data parallel so no cyclic dependencies
  - So get an II=1 issuing Vector Lane operations
  - May have data dependences between Vector operations

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Vector Pipeline Example

for (int i=0;i<32; i++)
    c[i]+=a[i]*b[i]

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Dependence between Vector Operations

for (int i=0;i<32; i++)
    c[i]+=a[i]*b[i]
for (int i=0;i<32; i++)
    e[i]+=c[i]*e[i]
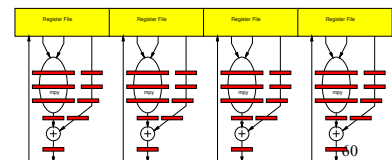
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Big Ideas

- Pipeline computations to reuse hardware and maximize computational capacity
- Can compose pipelined operators and accommodate fixed-frequency target
  - Be careful with data retiming
- Graph cycles limit pipelining on single stream -- II
- C-slow to share hardware among multiple, data-parallel streams (part 4)

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Admin

- Remember Feedback form
  - Including HW3
- Reading for Day 8 on web
- HW4 due Friday
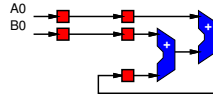
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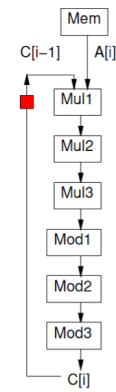## C-Slow

(Probably record separately)

Part 4

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Problem

- Pipelining cannot push registers into a graph cycle
- Graph cycles can prevent running at full pipeline target (maximum clock frequency)
- If not reusing operators at full pipeline target are underutilizing resources
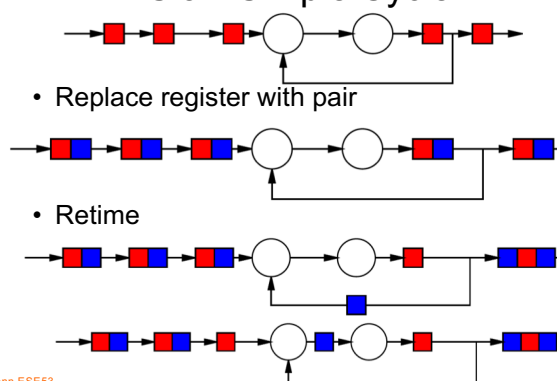- Can we use the resources for something?

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C-Slow

- **Observation:** if we have data-level parallelism, can use to solve independent problems on same hardware
- **Transformation:** make C copies of each register
- **Guarantee:** C computations operate independently
  - Do not interact with each other

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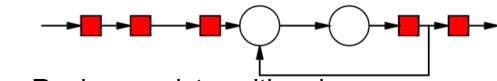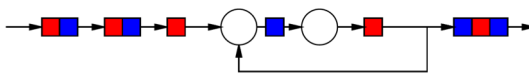## 2-Slow Simple Cycle

- Replace register with pair
- Retime

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2-Slow Simple Cycle

- Replace register with pair

- Retime

- Observe independence of red/blue computations
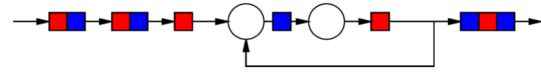
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Equivalence

- The 2-slow operator is equivalent to two data parallel operators running at half the speed
  - E.g. processing separate audio channels

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Automation

- No mainstream tool today will perform C-slow transformation for you automatically
- Synthesis tools will retime registers

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Lesson

- Cyclic dependencies limit throughput on single task or data stream
  - II=Cycle-length / registers-in-cycle

- Can use on C (C<=II) independent (data parallel) tasks

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Big Ideas

- Pipeline computations to reuse hardware and maximize computational capacity
- Can compose pipelined operators and accommodate fixed-frequency target
  - Be careful with data retiming
- Graph cycles limit pipelining on single stream -- II
- C-slow (C<=II) to share hardware among multiple, data-parallel streams (part 4)

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