

ESE5320: System-on-a-Chip Architecture

Day 2: September 6, 2023
Analysis, Metrics, and Bottlenecks

Day 1 Daily Quiz due.
Work Preclass.
Lecture start 10:20am



Penn ESE5320 Fall 2023 -- DeHon

1

Today: Analysis

- How do we quickly estimate what's possible?
 - Before developing a complete solution
 - less effort than developing complete solution
- How should we attack the problem?
 - Achieve the performance, energy goals?
- When we don't like the performance we're getting, how do we understand it?
- Where should we spend our time?

2

Today: Analysis

- Part 1: Key Terms and Concepts
 - Throughput
 - Latency
 - Bottleneck
- Part 2: Broader view
 - Bottleneck
 - Computation as a Graph, Sequence
 - Critical Path
- Part 3: Time and Space
- Part 4: Limits
 - Resource Bound
 - And Critical Path Bound
 - 90/10 Rule (time permitting)

3

Penn ESE5320 Fall 2023 -- DeHon

Message for Day

- Identify the **Bottleneck**
 - May be in compute, I/O, memory, data movement
- Focus and reduce/remove bottleneck
 - More resources
 - More efficient use of resources
- Repeat

4

4

Latency vs. Throughput

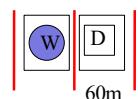
- **Latency:** Delay from inputs to output(s)
- **Throughput:** Rate at which can produce new set of outputs
 - (alternately, can introduce new set of inputs)

Penn ESE5320 Fall 2023 -- DeHon

5

Preclass Washer/Dryer Example

- 10 shirt capacity
- 1 Washer Takes 30 minutes
- 1 Dryer Takes 60 minutes
- **How long to do one load of wash?**
 - → Wash latency
- **Cleaning Throughput?**



6

6

Pipeline Concurrency



- Break up the computation graph into stages
 - Allowing us to
 - reuse resources for new inputs (data),
 - while older data is still working its way through the graph
 - Before it has exited graph
 - Throughput > (1/Latency)
- Relate liquid in pipe
 - Doesn't wait for first drop of liquid to exit far end of pipe before accepting second drop

Penn ESE5320 Fall 2023 -- DeHon

7

Escalator



Image Source: https://commons.wikimedia.org/wiki/File:Tanforan_Target_escalator_1.JPG

Penn ESE5320 Fall 2023 -- DeHon

8

Escalator



- Moves 2 ft/second
- Assume for simplicity one person can step on escalator each second
- Escalator travels 30 feet (vertical and horizontal)
- **Latency of escalator trip?**
- **Throughput of escalator: people/hour ?**

Penn ESE5320 Fall 2023 -- DeHon

9

Bottleneck

- What is the rate limiting item?
 - Resource, computation,

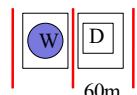
Penn ESE5320 Fall 2023 -- DeHon

10

Preclass Washer/Dryer Example



- 1 Washer Takes 30 minutes
 - Isolated throughput 20 shirts/hour
- 1 Dryer Takes 60 minutes
 - Isolated throughput 10 shirts/hour
- **Where is bottleneck in our cleaning cycle?**



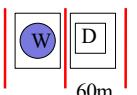
11

Penn ESE5320 Fall 2023 -- DeHon

Preclass Washer/Dryer Example



- 1 Washer \$500
 - Isolated throughput 20 shirts/hour
- 1 Dryer \$500
 - Isolated throughput 10 shirts/hour
- **How do we increase throughput with \$500 investment**



12

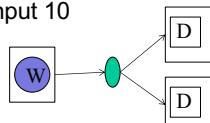
Penn ESE5320 Fall 2023 -- DeHon

11

2

Preclass Washer/Dryer Example

- 1 Washer \$500
 - Isolated throughput 20 shirts/hour
- 2 Dryers \$500
 - Isolated single dryer throughput 10 shirts/hour
- Latency?
- Throughput?



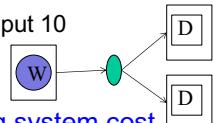
13

Penn ESE5320 Fall 2023 -- DeHon

13

Preclass Washer/Dryer Example

- 1 Washer \$500
 - Isolated throughput 20 shirts/hour
- 2 Dryers \$500
 - Isolated single dryer throughput 10 shirts/hour
- Able to double the throughput without doubling system cost



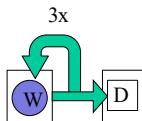
14

Penn ESE5320 Fall 2023 -- DeHon

14

Preclass Stain Example

- 1 Washer Takes 30 minutes
 - Isolated throughput 20 shirts/hour
- 1 Dryer Takes 60 minutes
 - Isolated throughput 10 shirts/hour
- Shirt need 3 wash cycles
- Latency?
- Throughput?
 - (assuming reuse single washer)



15

Penn ESE5320 Fall 2023 -- DeHon

15

Beyond Computation

(Part 2: Broader View)

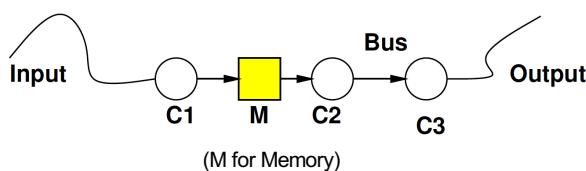
16

Penn ESE5320 Fall 2023 -- DeHon

16

Bottleneck

- May be anywhere in path
 - I/O, compute, memory, data movement



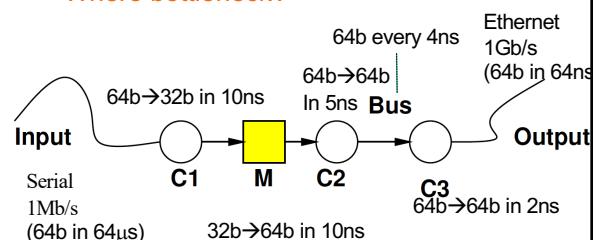
17

Penn ESE5320 Fall 2023 -- DeHon

17

Bottleneck

- Where bottleneck?



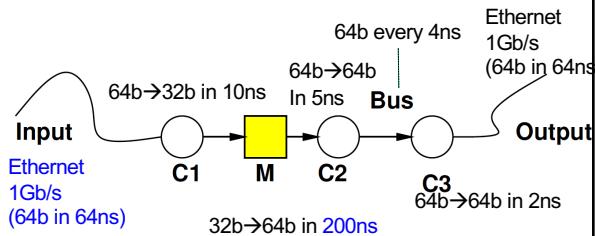
18

Penn ESE5320 Fall 2023 -- DeHon

18

Bottleneck

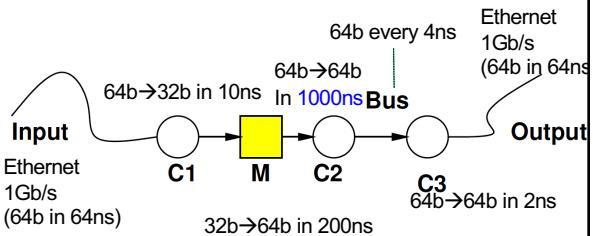
- Where bottleneck?



19

Bottleneck

- Where bottleneck?



20

Feasibility / Limits

- First things to understand
 - Obvious limits in system?
- Impossible?
- Which aspects will demand efficient mapping?
- Where might there be spare capacity

Penn ESE5320 Fall 2023 -- DeHon

21

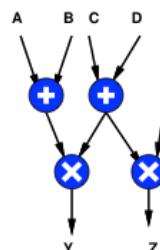
Generalizing

(to more general task graphs)

22

Computation as Graph

- Shown “simple” graphs (pipelines) so far
- $Y = (A+B)*(C+D)$
- $Z = (C+D)*E$



Note: HW2 ask you to draw a dataflow graph.

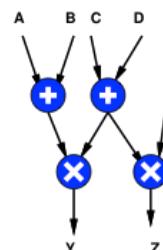
Here's an example...more to come.

Penn ESE5320 Fall 2023 -- DeHon

23

Computation as Graph

- Nodes have multiple input/output edges
- Edges may fanout
 - Results go to multiple successors



Penn ESE5320 Fall 2023 -- DeHon

24

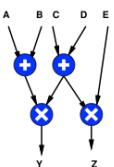
23

24

Computation as Sequence

- Shown “simple” graphs (pipelines) so far
 - $Y = (A+B)*(C+D)$
 - $Z = (C+D)*E$

T1=A+B
T2=C+D
Y=T1*T2
Z=T2*E



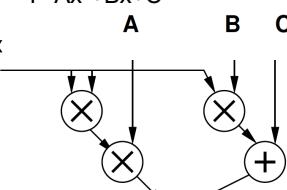
Penn ESE5320 Fall 2023 -- DeHon

25

25

Computation as Graph

- $Y = Ax^2 + Bx + C$

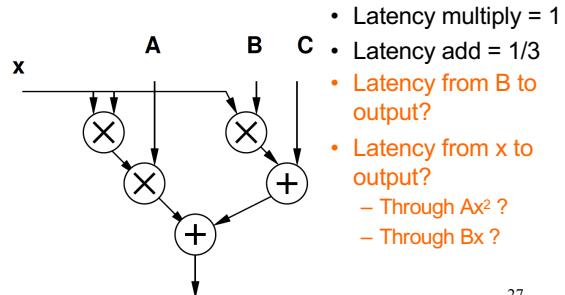


Penn ESE5320 Fall 2023 -- DeHon

26

26

Computation as Graph



Penn ESE5320 Fall 2023 -- DeHon

27

- Latency multiply = 1
- Latency add = 1/3
- Latency from B to output?
- Latency from x to output?
 - Through Ax^2 ?
 - Through Bx ?

Delay in Graphs

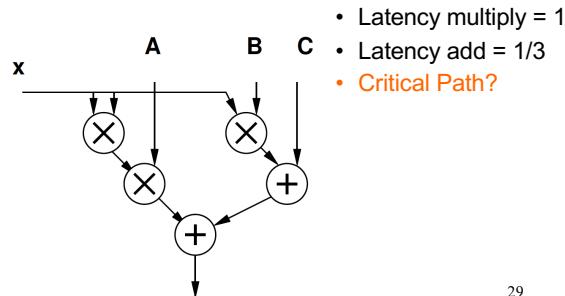
- **Observe:** There are multiple paths from inputs to outputs
 - Need to complete all of them to produce outputs
 - Limited by longest path
 - **Critical path:** longest path in the graph

28

28

28

Computation as Graph



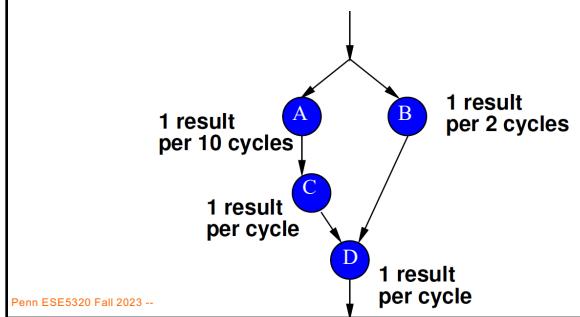
Penn ESE5320 Fall 2023 -- DeHon

29

- Latency multiply = 1
- Latency add = 1/3
- Critical Path?

Bottleneck

- Where is the bottleneck?



Penn ESE5320 Fall 2023 --

30

Time and Space

(Part 3)

Penn ESE5320 Fall 2023 -- DeHon

31

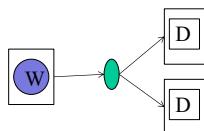
Space

- “Space” is an abstract term for physical resources
 - On VLSI chip: Area – mm² of silicon
 - On our FPGA: # of LUTs used
 - LUT = Lookup Table = Programmable Gate
 - More abstractly: # of Adders, multipliers
 - Laundry example
 - \$\$ to spend on laundry equipment
 - Physical space (sq. ft) in laundry room

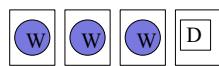
32

Space-Time

- In general, we can spend resources to reduce time
 - Increase throughput



Three wash stain removal case



33

Penn ESE5320 Fall 2023 -- DeHon

Space Time

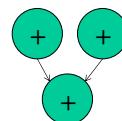
- Computation
 - $A=x_0+x_1$
 - $B=x_2+x_3$
 - $C=A+B$
- Adder takes one cycle
- Could perform on one adder
 - (like one washer)
 - Reuse adder in time
 - Let cycle time be one adder delay
- Latency on one adder?

35

Penn ESE5320 Fall 2023 -- DeHon

Space Time

- Computation
 - $A=x_0+x_1$
 - $B=x_2+x_3$
 - $C=A+B$
- Adder takes one cycle
- Latency on 3 adders?



34

Penn ESE5320 Fall 2023 -- DeHon

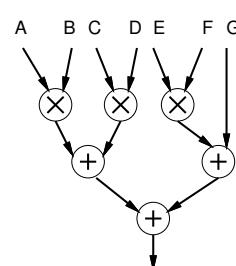
Space Time

- Computation
 - $A=x_0+x_1$
 - $B=x_2+x_3$
 - $C=A+B$
- Adder takes one cycle
- Could perform on one adder
 - (like one washer)
 - Reuse adder in time
 - Let cycle time be one adder delay
- Latency on one adder?

35

Penn ESE5320 Fall 2023 -- DeHon

Computation as Graph



- Latency multiply = 1
- Space multiply = 3
- Latency add = 1
- Space add = 1
- (can perform add or multiply in one cycle)
- Latency and Space
 - 3 mul, 2 add

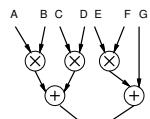
36

Penn ESE5320 Fall 2023 -- DeHon

36

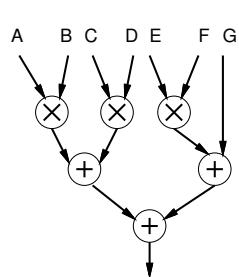
Schedule 3 mul, 2 add

| Cycle | Mul | Mul | Mul | Add | Add |
|-------|-----|-----|-----|-----------|---------|
| 0 | A*B | C*D | E*F | | |
| 1 | | | | A*B+C*D | E*F+G |
| 2 | | | | (A*B+C*D) | (E*F+G) |



37

Computation as Graph



- Latency multiply = 1
- Space multiply = 3
- Latency add = 1
- Space add = 1
- **Latency and Space**
– 1 mul, 1 add

38

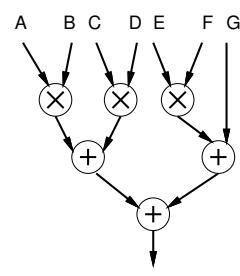
Schedule 1 mul, 1 add

| Cycle | Mul | Add |
|-------|-----|-------------------|
| 0 | A*B | |
| 1 | C*D | |
| 2 | E*F | A*B+C*D |
| 3 | | E*F+G |
| 4 | | (A*B+C*D)+(E*F+G) |

Penn ESE5320 Fall 2023 -- DeHon

39

Computation as Graph



- Latency multiply = 1
- Space multiply = 3
- Latency add = 1
- Space add = 1
- **Latency and Space**
– 2 mul, 1 add

40

Schedule 2 mul, 1 add

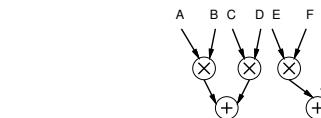
| Cycle | Mul | Mul | Add |
|-------|-----|-----|-------------------|
| 0 | A*B | C*D | |
| 1 | E*F | | (A*B+C*D) |
| 2 | | | E*F+G |
| 3 | | | (A*B+C*D)+(E*F+G) |

Penn ESE5320 Fall 2023 -- DeHon

41

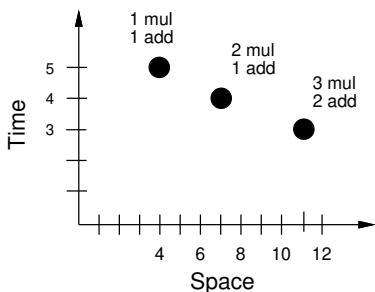
$A*B+C*D+E*F+G$ Design Points

| mul | add | space | latency |
|-----|-----|--------------|---------|
| 3 | 2 | $3*3+2*1=11$ | 3 |
| 2 | 1 | $2*3+1*1=7$ | 4 |
| 1 | 1 | $1*3+1*1=4$ | 5 |



42

Space-Time Graph

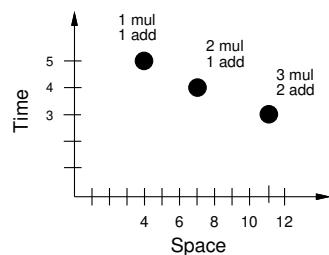


Penn ESE5320 Fall 2023 -- DeHon

43

Space-Time Graph

Depending on goals,
time could be
throughput
or latency
(may need
to look at
both)



44

Two Bounds

Part 4: Limits
(still in Time and Space)

Penn ESE5320 Fall 2023 -- DeHon

45

Problem

- Coming up with an exact time count can be hard (human/computer time consuming)
 - Technically a hard problem
 - NP-Complete: no known non-exponential solution
- Requires reasoning about structure of graph
- Would be nice to have a quick (easy) answer on what is feasible
 - ...and what is not feasible → impossible.

Penn ESE5320 Fall 2023 -- DeHon

46

Bounds

- Establish the feasible range
 - Must be larger (or equal) than LB (lower bound)
 - Must be smaller (or equal) than UB (upper bound)
 - Solution will be between LB and UB
 - $LB \leq ActualTime \leq UB$
- Bounds in sports
 - Ball landing in-bounds or out-of bounds

Penn ESE5320 Fall 2023 -- DeHon

47

Bounds

- Quick **lower** bounds (LB) can estimate
 - $LB \leq ActualTime$
- Two:
 - CP: Critical Path
 - Sometimes call it "Latency Bound"
 - RB: Resource Capacity Bound
 - Sometimes call it "Throughput Bound" or "Compute Bound"

Penn ESE5320 Fall 2023 -- DeHon

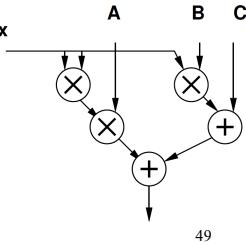
48

47

48

Critical Path Lower Bound

- Critical path assuming infinite resources
- Certainly cannot finish any faster than that
- $CP \leq ActualTime$
- Ignores resource limits



49

Penn ESE5320 Fall 2023 -- DeHon

49

Resource Bound: Single resource

- Ignore precedence (graph)
- If adds take one cycle,
 - How many additions can perform on 3 adders in 3 cycles?
- How many additions can perform in C cycles on M adders?
- If need to perform N operations, and have M adders, how many cycles?

50

Penn ESE5320 Fall 2023 -- DeHon

50

Resource Bound: Single resource

- Ignore precedence (graph)
- N operations (calculations to make)
- M operators (resource can perform calculation)
- Perform operation in one time step (cycle)
- Need at least $\lceil N/M \rceil$ time steps

51

Penn ESE5320 Fall 2023 -- DeHon

51

Resource Capacity Lower Bound

- Sum up all capacity required per resource: $TotalOps = \sum Ops$
 - E.g. number of multiplications, additions, memory lookups
- Divide by total resource (for type)
 - E.g., number of multipliers, adders, memory ports
 - $RB = [TotalOps/Operators] \leq ActualTime$
- Lower bound on compute
 - (best can do is pack all use densely)
 - Ignores data dependency constraints

52

Penn ESE5320 Fall 2023 -- DeHon

52

RB: Multiple Resource Types

- $RB = Max([TotalOps_1/Operators_1], [TotalOps_2/Operators_2], \dots) \leq ActualTime$
- Combine Critical Path Lower Bound
 $Max(CP, [TotalOps_1/Operators_1], [TotalOps_2/Operators_2], \dots) \leq ActualTime$

53

Penn ESE5320 Fall 2023 -- DeHon

53

For Single Resource Type

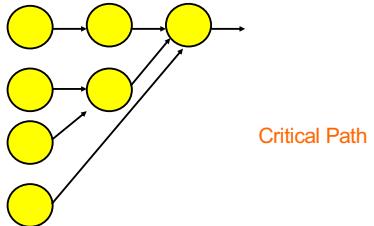
- (and no communication time...)
- Can use to get upper bound:
- $ActualTime \leq CP + RB$
- Together:
- $Max(CP, RB) \leq ActualTime \leq CP + RB$

54

Penn ESE5320 Fall 2023 -- DeHon

54

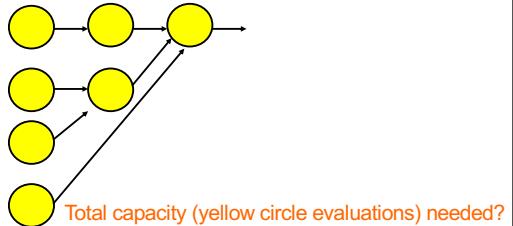
Example



Penn ESE5320 Fall 2023 -- DeHon

55

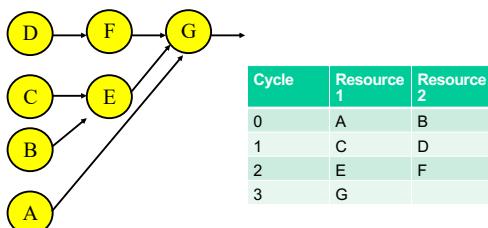
Example



Penn ESE5320 Fall 2023 -- DeHon

56

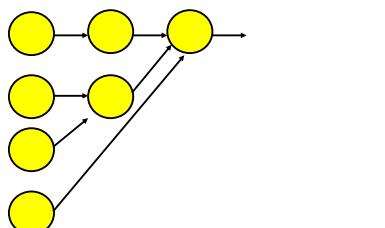
Example



Penn ESE5320 Fall 2023 -- DeHon

57

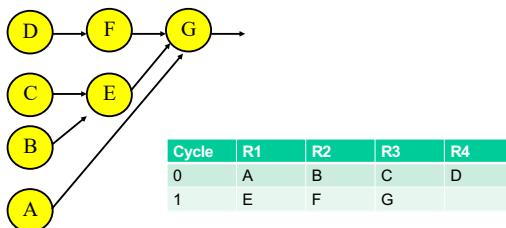
Example



Penn ESE5320 Fall 2023 -- DeHon

58

Example



Penn ESE5320 Fall 2023 -- DeHon

59

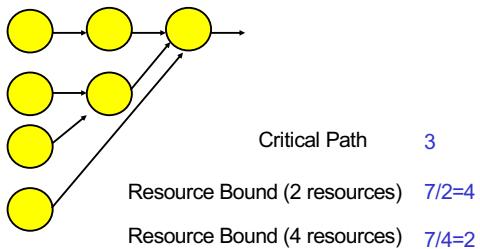
Resource Capacity Lower Bound

- Sum up all capacity required per resource: $\text{TotalOps} = \sum \text{Ops}$
 - E.g. number of multiplications, additions, memory lookups
- Divide by total resource (for type)
 - E.g., number of multipliers, adders, memory ports
 - $RB = [\text{TotalOps}/\text{Operators}] \leq \text{ActualTime}$
- Lower bound on compute
 - (best can do is pack all use densely)
 - Ignores data dependency constraints

Penn ESE5320 Fall 2023 -- DeHon

60

Example



Either one (CP,RB) can be limit. Check both.

In general, independent → relation depends on task.

61

What are the telling us

- If CP<RB
 - Adding resources (space) may be effective at reducing latency
- If RB<CP
 - Adding resources (space) will not reduce latency

62

90/10 Rule (of Thumb)

- Observation that code is not used uniformly
- 90% of the time is spent in 10% of the code
- Knuth: 50% of the time in 2% of the code
- Implications
 - There will typically be a bottleneck
 - We don't need to optimize everything
 - We don't need to uniformly replicate space to achieve speedup
 - Not everything needs to be accelerated

Penn ESE5320 Fall 2023 -- DeHon

63

Big Ideas

- Identify the Bottleneck
 - May be in compute, I/O, memory ,data movement
- Focus and reduce/remove bottleneck
 - More resources
 - More efficient use of resources

64

Admin

- Diagnostic Assessment due today!
- Reading for Day 3 on web
- HW1 due Friday
- HW2 out today
 - Individual assignment
- Remember feedback
- Remaining Questions?

Penn ESE5320 Fall 2023 -- DeHon

65

65