ESE535:
Electronic Design Automation

Day 17: April 2, 2008
Scheduling Introduction

Today

- Scheduling
  - Basic problem
  - Variants
  - List scheduling approximation

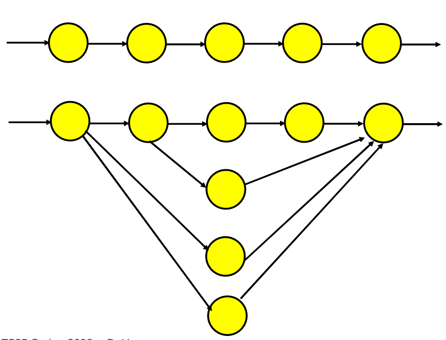
General Problem

- Resources are not free
  - wires, io ports
  - functional units
    - LUTs, ALUs, Multipliers, ….
  - memory locations
  - memory access ports

Trick/Technique

- Resources can be shared (reused) in time
- Sharing resources can reduce
  - instantaneous resource requirements
  - total costs (area)
- **Pattern:** scheduled operator sharing

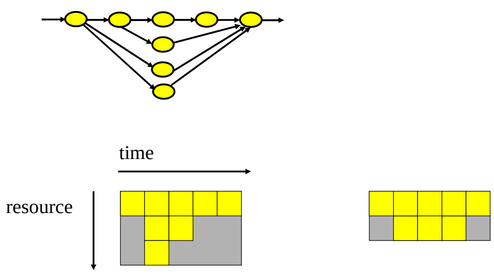
Example



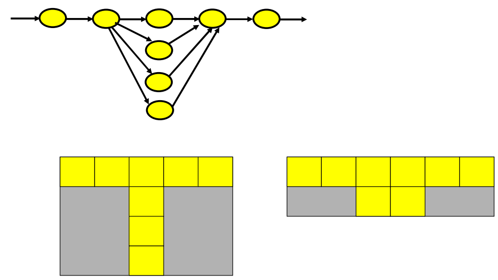
Sharing

- Does not have to increase delay
  - w/ careful time assignment
  - can often reduce peak resource requirements
  - while obtaining original (unshared) delay
- Alternately: Minimize delay given fixed resources

Schedule Examples



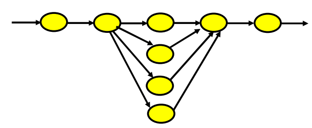
More Schedule Examples

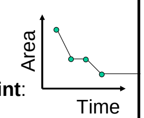


Scheduling

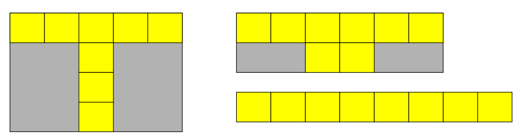
- **Task**: assign time slots (and resources) to operations
  - **time-constrained**: minimizing peak resource requirements
    - *n.b.* time-constrained, not always constrained to minimum execution time
  - **resource-constrained**: minimizing execution time

Resource-Time Example





Time Constraint:
<5 → --
5 → 4
6,7 → 2
>7 → 1



Scheduling Use

- Very general problem formulation
  - HDL/Behavioral → RTL
  - Register/Memory allocation/scheduling
  - Instruction/Functional Unit scheduling
  - Processor tasks
  - Time-Switched Routing
    - TDMA, bus scheduling, static routing
  - Routing (share channel)

Two Types (1)

- **Data independent**
  - graph static
  - resource requirements and execution time
    - independent of data
  - schedule staticly
  - maybe bounded-time guarantees
  - typical ECAD problem

Two Types (2)

- **Data Dependent**
  - execution time of operators variable
    - depend on data
  - flow/requirement of operators data dependent
  - if cannot bound range of variation
    - must schedule online/dynamically
    - cannot guarantee bounded-time
    - general case (*I.e.* halting problem)
  - typical “General-Purpose” (non-real-time) OS problem

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Unbounded Problem

- Easy:
  - compute ASAP schedule
    - *I.e.* schedule everything as soon as predecessors allow
  - will achieve minimum time
  - won’t achieve minimum area
    - (meet resource bounds)

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ASAP Schedule

- For each input
  - mark input on successor
  - if successor has all inputs marked, put in visit queue
- While visit queue not empty
  - pick node
  - update time-slot based on latest input
  - mark inputs of all successors, adding to visit queue when all inputs marked
- Used for timing analysis (Day 6)

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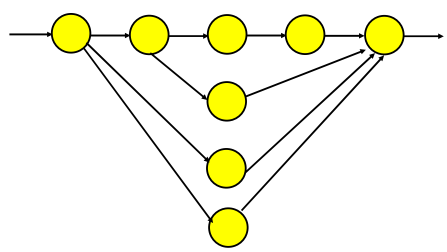
ASAP Example



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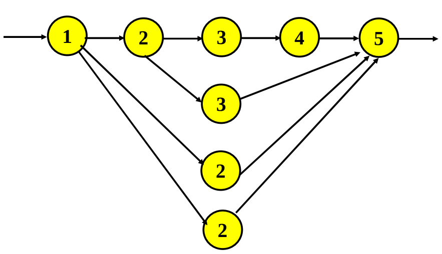
ASAP Example



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Also Useful to Define ALAP

- As Late As Possible
- Work backward from outputs of DAG
- Also achieve minimum time w/ unbounded resources

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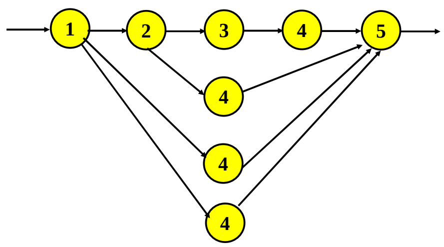
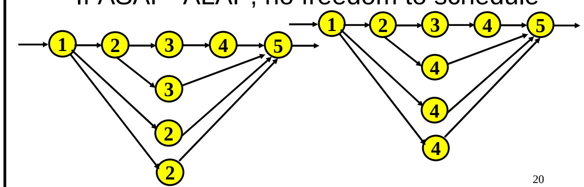
ALAP Example

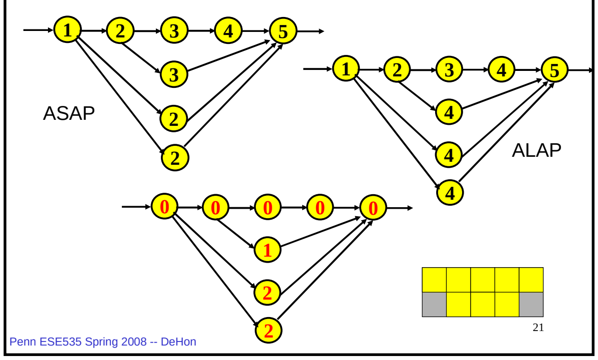


ALAP and ASAP

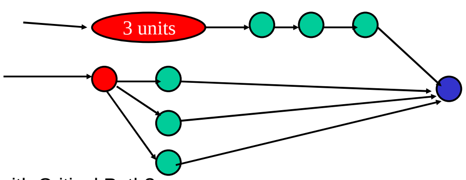
- Difference in labeling between ASAP and ALAP is slack of node
  - Freedom to select timeslot
- If ASAP=ALAP, no freedom to schedule



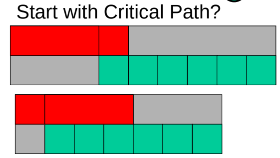
ASAP, ALAP, Difference



Why hard?

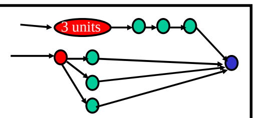


Start with Critical Path?



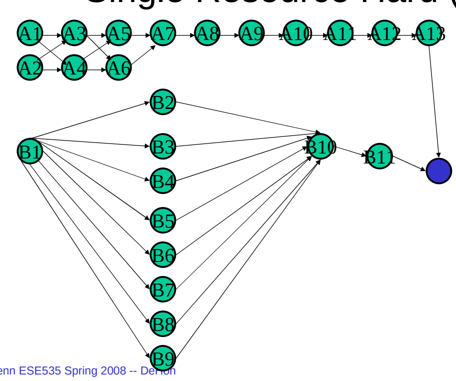
Schedule on:
1 Red Resource
1 Green Resource

General

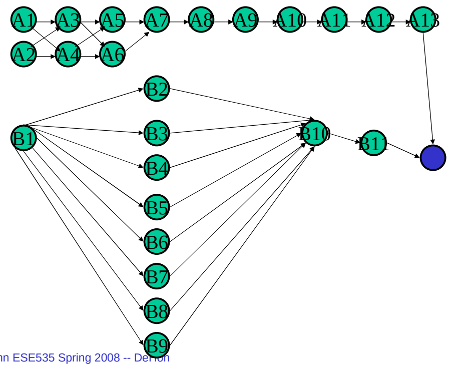


- When selecting, don't know
  - need to tackle **critical path**
  - need to run task to **enable work** (parallelism)
- Can generalize example to single resource case

Single Resource Hard (1)

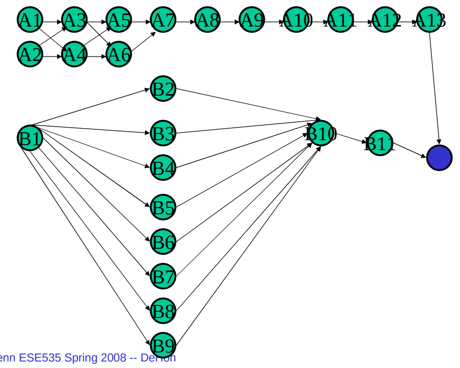


Single Resource Hard (2)



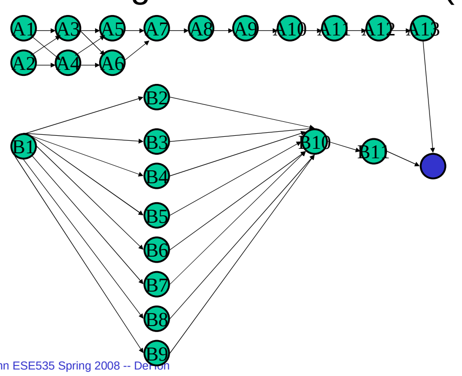
Crit. Path:
A1 A2
A3 A4
A5 A6
A7 B1
A8 B2
A9 B3
A10 B4
A11 B5
A12 B6
A13 B7
B8 B9
B10
B11

Single Resource Hard (3)



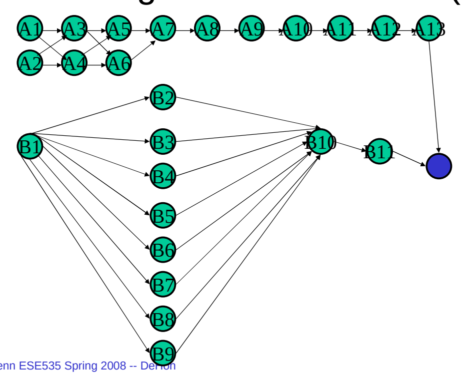
PFirst
A1 B1
B2 B3
B4 B5
B6 B7
B8 B9
A2 B10
A3 A4
A5 A6
A7 B11
A8
A9
A10
A11
A12
A13

Single Resource Hard (4)



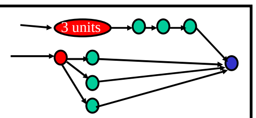
Balance1
A1 B1
A2 B2
A3 B3
A4 B4
A5 B5
A6 B6
A7 B7
A8 B8
A9 B9
A10 B10
B11 B11
A12
A13

Single Resource Hard (5)



???
A1 B1
A2 B2
A3 A4
A5 B3
A6 B4
A7 B5
A8 B6
A9 B7
A10 B8
A11 B9
A12 B10
A13 B11

General: Why Hard



- When selecting, don't know
  - need to tackle **critical path**
  - need to run task to **enable work** (parallelism)

Two Bounds

Bounds

- Useful to have bounds on solution
- Two:
  - CP: Critical Path
  - RB: Resource Bound

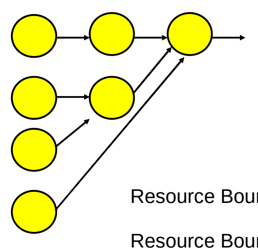
Critical Path Lower Bound

- ASAP schedule ignoring resource constraints
  - (look at length of remaining critical path)
- Certainly cannot finish any faster than that

Resource Capacity Lower Bound

- Sum up all capacity required per resource
- Divide by total resource (for type)
- Lower bound on remaining schedule time
  - (best can do is pack all use densely)
  - Ignores schedule constraints

Example

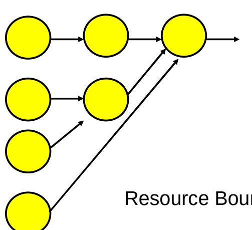


Critical Path

Resource Bound (2 resources)

Resource Bound (4 resources)

Example



Critical Path 3

Resource Bound (2 resources) 7/2=4

Resource Bound (4 resources) 7/4=2

List Scheduling

Greedy Algorithm → Approximation

List Scheduling (basic algorithm flow)

- Keep a ready list of “available” nodes
  - (one whose predecessors have already been scheduled)
- Pick an unscheduled task and schedule on next available resource
- Put any tasks enabled by this one on ready list

List Scheduling

- Greedy heuristic
- **Key Question:** How prioritize ready list?
  - What is dominant constraint?
    - least slack (worst critical path)
    - enables work
    - utilize most precious resource
- So far:
  - seen that no single priority scheme would be optimal

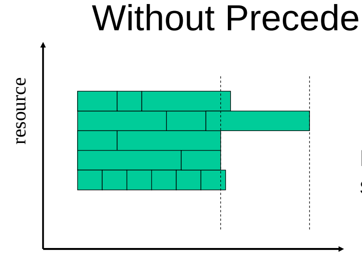
List Scheduling

- Use for
  - resource constrained
  - time-constrained
    - give resource target and search for minimum resource set
- Fast: O(N) →O(Nlog(N)) depending on prioritization
- Simple, general
- How good?

Approximation

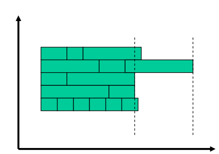
- Can we say how close an algorithm comes to achieving the optimal result?
- Technically:
  - **If** can show
    - Heuristic(Prob)/Optimal(Prob)$\leq\alpha$ $\forall$ prob
  - **Then** the Heuristic is an α-approximation

Scheduled Example Without Precedence



How bad is this schedule?

Observe



- ∃ optimal length L
- No idle time up to start of last job to finish
- start time of last job ≤ L
- last job length ≤ L
- Total LS length ≤ 2L

➢ Algorithm is within factor of 2 of optimum

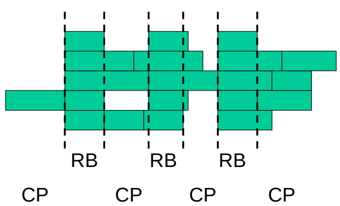
Results

- Scheduling of identical parallel machines has a 2-approximation
  - *i.e.* we have a polynomial time algorithm which is guaranteed to achieve a result within a factor of two of the optimal solution.
- In fact, for precedence unconstrained there is a 4/3-approximation
  - *i.e.* schedule Longest Processing Time first

Recover Precedence

- With precedence we may have idle times, so need to generalize
- Work back from last completed job
  - two cases:
    - entire machine busy
    - some predecessor in critical path is running
- Divide into two sets
  - whole machine busy times
  - critical path chain for this operator

Precedence



Precedence Constrained

- Optimal Length > All busy times
  - Optimal Length ≥ Resource Bound
  - Resource Bound ≥ All busy
- Optimal Length>This Path
  - Optimal Length ≥ Critical Path
  - Critical Path ≥ This Path
- List Schedule = This path + All busy times
- List Schedule ≤ 2 *(Optimal Length)

Conclude

- Scheduling of identical parallel machines with precedence constraints has a 2-approximation.

Tighten

- LS schedule ≤ Critical Path+Resource Bound
- LS schedule ≤ Min(CP,RB)+Max(CP,RB)
- Optimal schedule ≥ Max(CP,RB)
- LS/Opt ≤ 1+Min(CP,RB)/Max(CP,RB)
- The more one constraint dominates
  - ➔the closer the approximate solution to optimal
  - ⇘(EEs think about 3dB point in frequency response)

Tightening

- Example of
  - More information about problem
  - More internal variables
  - …allow us to state a tighter result
- 2-approx for any graph
  - Since CP may = RB
- Tighter approx as CP and RB diverge

Multiple Resource

- Previous result for homogeneous functional units
- For heterogeneous resources:
  - also a 2-approximation
    - Lenstra+Shmoys+Tardos, Math. Programming v46p259
    - (not online, no precedence constraints)

Bounds

- Precedence case, Identical machines
  - no polynomial approximation algorithm can achieve better than 4/3 bound
    - (unless P=NP)
- Heterogeneous machines (no precedence)
  - no polynomial approximation algorithm can achieve better than 3/2 bound

Summary

- Resource sharing saves area
  - allows us to fit in fixed area
- Requires that we schedule tasks onto resources
- General kind of problem arises
- We can, sometimes, bound the “badness” of a heuristic
  - get a tighter result based on gross properties of the problem
  - approximation algorithms often a viable alternative to finding optimum
  - play role in knowing “goodness” of solution

Admin

- Schedule reorg.
  - Deal with recent slip
  - No class on Monday 4/14

Big Ideas:

- Exploit freedom in problem to reduce costs
  - (slack in schedules)
- Use dominating effects
  - (constrained resources)
  - the more an effect dominates, the “easier” the problem
- Technique: Approximation