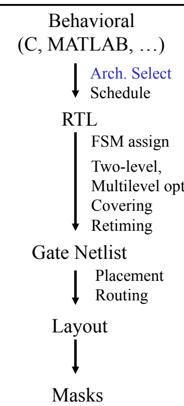
ESE535: Electronic Design Automation

Day 15: March 16, 2011
Architecture Synthesis
(Provisioning, Allocation)

Today

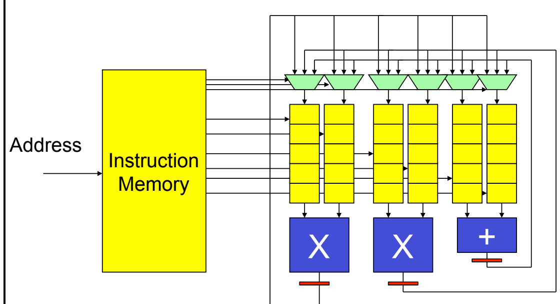
- Problem
- Brute-Force/Exhaustive
- Greedy
- Estimators
- Analytical Provisioning
- ILP Schedule and Provision



Previously

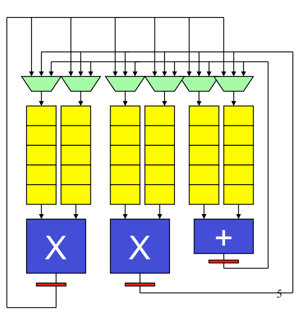
- General formulation for scheduled operator sharing
  - VLIW
- Fast algorithms for scheduling onto fixed resource set
  - List Scheduling

VLIW



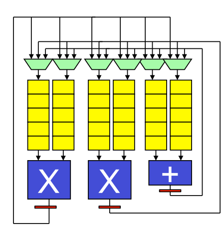
Today

- How do we determine the set of resources?



Today: Provisioning

- Given
  - An area budget
  - A graph to schedule
  - A library of operators
- Determine:
  - Delay minimizing set of operators
    - Or delay-achieving set of operators
  - i.e. select the operator set



Exhaustive

1. Identify all area-feasible operator sets
   – E.g. preclass exercise
2. Schedule for each
3. Select best

- → optimal
- Drawbacks?

Exhaustive

- How large is space of feasible operator sets?
  – As function of
    - operator types – N
      – Types: add, multiply, divide, ….
    - Maximum number of operators of type M

$$M^N$$

Size of Feasible Space

- Consider 10 operators
  – For simplicity all of unit area
- Total area of 100

- How many cases?

$$\binom{100}{9} \approx 2 \times 10^{12}$$

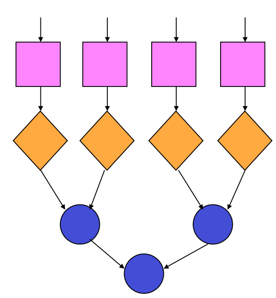
Implication

- Feasible operator space can be too large to explore exhaustively

Greedy Incremental

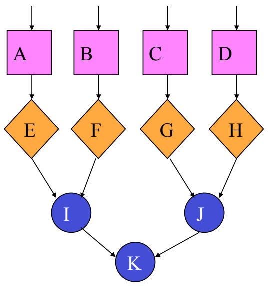
- Start with one of each operator
- While (there is area to hold an operator)
  – Which single operator
    - Can be added without exceeding area limit?
    - And provides largest benefit?
  – Add one operator of that type
- How long does this run?
  – $T_{schedule}(E)$* O(N*M)
  – [M = # types, N=final # operators]
- Weakness?

Example



Find best 5 operator solution.

Example

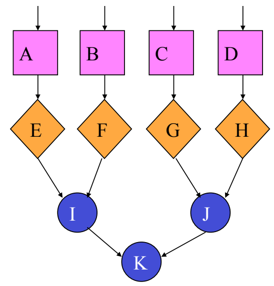


Find best 5 operator solution.

Example

One of each.

Sq	Dia	Circ
A		
B	E	
C	F	
D	G	I
	H	
		J
		K

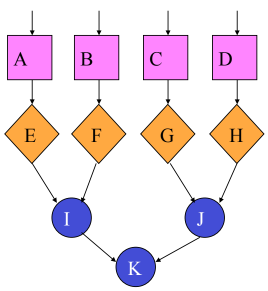


Find best 5 operator solution.

Example

Two Squares

Sq	Dia	Circ
A,B		
C,D	E	
	F	
	G	I
	H	
		J
		K

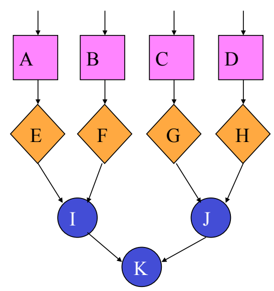


Find best 5 operator solution.

Example

Two Diamonds

Sq	Dia	Circ
A		
B	E	
C	F	
D	G	I
	H	
		J
		K

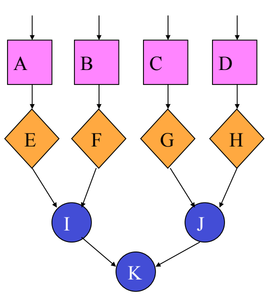


Find best 5 operator solution.

Example

Two Circles

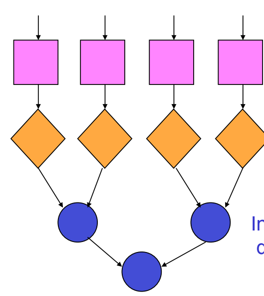
Sq	Dia	Circ
A		
B	E	
C	F	
D	G	I
	H	
		J
		K



Find best 5 operator solution.

Example

Which should greedy add?



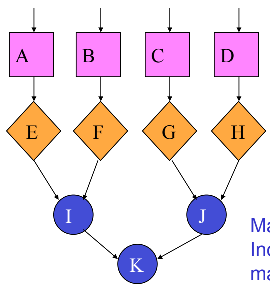
Find best 5 operator solution.

Incremental addition does not accelerate.

Example

Two sqs
+ Two diamonds

Sq	Dia	Circ
A,B		
C,D	E,F	
	G,H	I
		J
		K



Find best 5 operator solution.

Max effect: Incremental may not suggest next single addition.

Analytic Formulation

Challenge

- Scheduling expensive
  - O(|E|) or O(|E|*log(|V|)) using list-schedule
- Results not analytic
  - Cannot write an equation around them
- Bounds are sometimes useful
  - No precedence → is resource bound
  - Often one bound dominates
    - Latency bound unaffected by operator count

Estimations

- Step 1: estimate with resource bound
  - O(|E|) vs. O(N) evaluation
- Step 2: use estimate in equations
  - $T=\max(N_1/R_1,N_2/R_2,\ldots.)$
- Most useful when RB>>CP

Constraints

- Let A_i be area of operator type i
- Let x_i by number of operators of type i

$$\sum A_i \times x_i \leq Area$$

Achieve Time Target

- Want to achieve a schedule in T cycles
- Each resource bound must be less than T cycles:
  - $N_i/x_i \leq T$

Algebraic Solve

- Set of equations
  - $N_i/x_i \leq T$
  - $\Sigma A_i x_i \leq Area$
- Assume equality for time bound
- $N_i/x_i=T$ ➔ $x_i=N_i/T$

$$\frac{\sum A_i \times N_i}{T} \leq Area$$

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Rearranging

$$\frac{\sum A_i \times N_i}{T} \leq Area$$

$$\frac{\sum A_i \times N_i}{Area} \leq T$$

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Bounding T

- Gives Lower Bound on T

$$\frac{\sum A_i \times N_i}{Area} \leq T$$

Intuition: N of each is right balance given unbounded area; Scale to area available.

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27

Preclass

- What is T_{lower} for preclass?

$$\frac{\sum A_i \times N_i}{Area} \leq T$$

$$T \geq \frac{1 \times 8 + 2 \times 4}{7} = \frac{16}{7} \approx 2.3 \qquad T \geq 3$$

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28

Back Substitute from T to x

- $x_i=N_i/T$

$$\frac{\sum A_i \times N_i}{Area} \leq T$$

- x_i won't necessarily be integer
  - Round down definitely feasible solution
  - May have room to move a few up by 1

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Preclass

- $x_i=N_i/T$
- T>=3
- X_{add}, X_{mpy} ?
- X_{add} = 8/3 ➔ 2 or 3
- X_{mpy} = 4/3 ➔ 1 or 2

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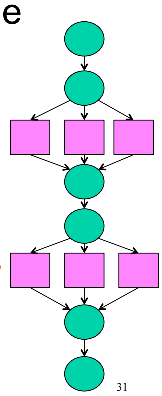
30

Counter Example

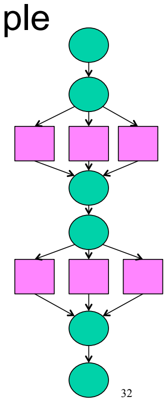
- 1 Unit each
- Area = 4 Units
- What would analytic predict?
- What is best?
- How does CP compare to RB?

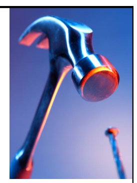
- Analytic Resource Estimate
  - Most useful when RB>>CP



Analytic Counter Example

- How would greedy incremental work on this one?





ILP

Maybe we can do exhaustive,
if we formulate properly.

ILP

- Integer Linear Programming
- Formulate set of linear equation constraints (inequalities)
  - $Ax_0+Bx_1+Cx_2 \leq D$
  - $x_0+x_1=1$
  - A,B,C,D – constants
  - x_i – variables to satisfy
  - No products on variables, just linear weighted sums
- Can constrain variables to integers
- No polynomial time guarantee
  - But often practical
  - Solvers exist (significant piece next lecture)

ILP Provision and Schedule

Now to make it look like an ILP nail…

- Formulate operator selection and scheduling as ILP problem

Formulation

- Integer variables M_i
  - number of operators of type i
- 0-1 (binary) variables $x_{i,j}$
  - 1 if node i is scheduled into timestep j
  - 0 otherwise

- Variable assignment completely specifies operator selection and schedule
- This formulation for achieving a target time T
  - j ranges 0 to T-1

Target T → Min T

- Formulation targets T
- What if we don't know T?
  - Want to minimize T?
- Do binary search for minimum T
  - How does that impact solution time?

Constraints

What properties must hold true for a solution to be valid?

1. Total area constraints
2. Not assign too many things to a timestep
3. Assign every node to some timestep
4. Maintain precedence

(1) Total Area

- Same as before

$$\sum A_i \times M_i \leq Area$$

(2) Not overload timestep

- For each timestep j
  - For each operator type k

$$\sum_{o_i \in FU_k} x_{i,j} \leq M_k$$

(3) Node is scheduled

- For each node in graph

$$\sum_j x_{i,j} = 1$$

Can narrow to sum over slack window.

(4) Precedence Holds

- For each edge from node *src* to node *snk*

$$\sum_j j \times x_{src,j} - \sum_j j \times x_{snk,j} \leq -1$$

Can narrow to sum over slack windows.

Constraints

Roughly what provided code is checking in sched_main

1. Total area constraints
2. Not assign too many things to a timestep
3. Assign every node to some timestep
4. Maintain precedence

ILP Solver

- ILP Solver can take these constraints and find a solution (satisfying assignment)
- On Monday, will see how to start to make this practical

SAT/ILP Scheduling Variant

(Demonstration)

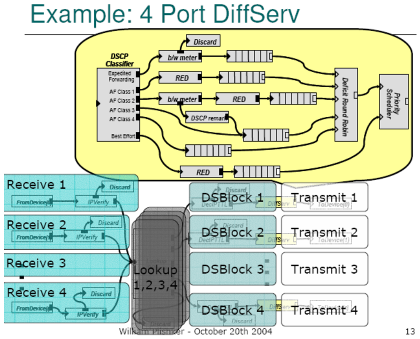
<if time permits>

Two Constraint Challenge

- Processing elements have limited memory
  - Instruction memory (data memory)
- Tasks have different requirements for compute and instruction memory
  - *i.e.* Run length not correlated to code length
- No provisioning, scheduling

Plishker Task Example

Example: 4 Port DiffServ



Task

- **Task:** schedule tasks onto PEs obeying **both** memory and compute capacity limits

Example from DiffServ

Resource	Receive	Look-up	DSBlock	Transmit
Execution Cycles	99	134	320	296
Instructions	462	218	1800	985

Example and ILP solution From Plishker et al. NSCD2004

William Plishker - October 20th 2004 14

Task

- **Task:** schedule tasks onto PEs obeying both memory and compute capacities
- ➔ two capacity assignment problem
- ➔ two capacity bin packing problem
- Task: i <C_i,I_i>

SAT Packing

Variables:

- $A_{i,j}$ – task i assigned to resource j

Constraints

- Coverage constraints
- Uniqueness constraints
- Cardinality constraints
  - PE compute
  - PE memory

$$U_i = \sum_j A_{i,j} = 1$$

$$\sum_i (A_{i,j} \times C_i) \le PE.cap(j)$$

Allow Code Sharing

- Two tasks of same type can share code
- Instead of memory capacity
  - Vector of memory usage
- Compute PE Imem vector
  - As OR of task vectors assigned to it
- Compute mem space as sum of non-zero vector entry weights (dot product)

Allow Code Sharing

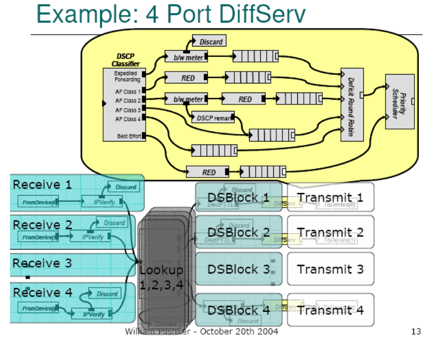
- Two tasks of same type can share code
- Task has vector of memory uage
  - Task i needs set of instructions k: $T_{i,k}$
- Compute PE Imem vector
  - OR (all i): $PE.Imem_{j,k}$+=$A_{i,j}$ * $T_{i,k}$
- PE Mem space
  - $PE.Total\_Imem_j$= Σ($PE.Imem_{j,k}$*Instrs(k))

Symmetries

- Many symmetries
- Speedup with symmetry breaking
  - Tasks in same class are equivalent
  - PEs indistinguishable
  - Total ordering on tasks and PEs
  - Add constraints to force tasks to be assigned to PEs by ordering
  - Plishker claims "significant runtime speedup"
  - Using GALENA [DAC 2003] psuedo-Boolean SAT solver

Plishker Task Example

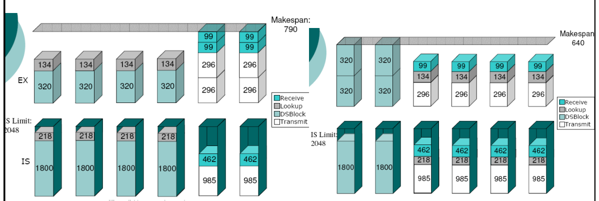
Example: 4 Port DiffServ



Results

Greedy (first-fit) binpack

SAT/ILP Solve



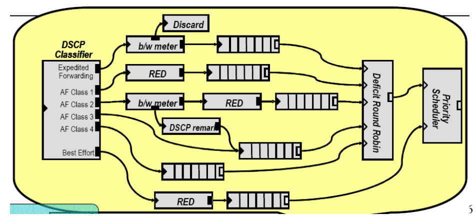
Solutions in < 1 second

55

[skipped over this]

Why can they do this?

- Ignore precedence?
- Ignore Interconnect?



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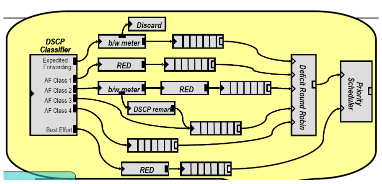
Why can they do this?

- Ignore precedence?
  - feed forward, buffered
- Ignore Interconnect?
  - Through shared memory, not dominant?



57

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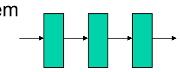
Interconnect Buffers

- Allow "Software Pipelining"

Each data item



Spatial we would pipeline, running all three at once

Think of each schedule instance as one timestep in spatial pipeline.

58

Interconnect Buffer

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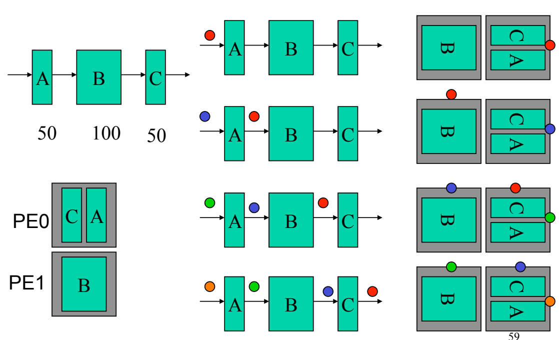

59

Round up Algorithms and Runtimes

- Exhaustive Schedule
- Greedy Schedule
- Analytic Estimates
- ILP formulation

60

Admin

- Assign 5a Monday
- Reading for Monday on web

Big Ideas:

- Estimators
- Dominating Effects
- Reformulate as a problem we already have a solution for
  - ILP
- Technique: Greedy
- Technique: ILP