# ESE535: Electronic Design Automation

Day 4: January 26, 2011
Scheduling Introduction



## Today

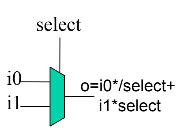
- Scheduling
  - Basic problem
  - Variants
  - List scheduling approximation

Behavioral (C, MATLAB, …)
↓ Arch. Select, Schedule
RTL
↓ FSM assign, Two-level, Multilevel opt., Covering, Retiming
Gate Netlist
↓ Placement, Routing
Layout
↓
Masks



## General Problem

- Resources are not free
  - Wires, io ports
  - Functional units
    - LUTs, ALUs, Multipliers, ….
  - Memory access ports
  - State elements
    - memory locations
    - Registers
      - Flip-flop
      - loadable master-slave latch
  - Multiplexers (mux)



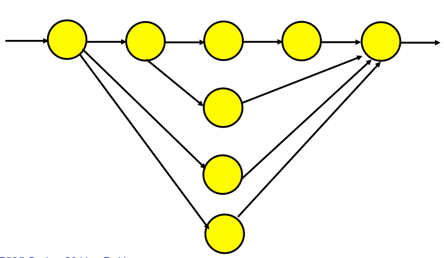



## Trick/Technique

- Resources can be shared (reused) in time
- Sharing resources can reduce
  - instantaneous resource requirements
  - total costs (area)
- **Pattern:** scheduled operator sharing



## Example

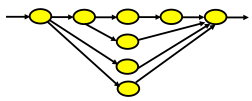


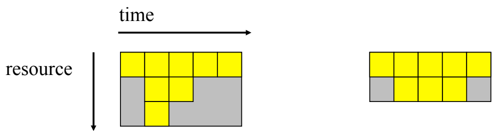


## Sharing

- Does not have to increase delay
  - w/ careful time assignment
  - can often reduce peak resource requirements
  - while obtaining original (unshared) delay
- Alternately: Minimize delay given fixed resources

## Schedule Examples



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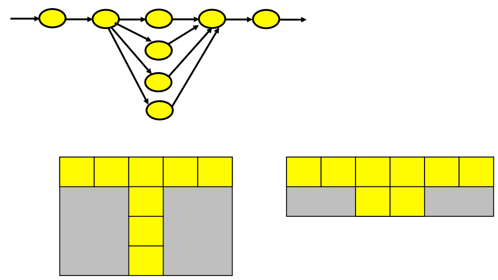
## More Schedule Examples



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## Scheduling

- **Task**: assign time slots (and resources) to operations
  - **time-constrained**: minimizing peak resource requirements
    - *n.b.* time-constrained, not always constrained to minimum execution time
  - **resource-constrained**: minimizing execution time

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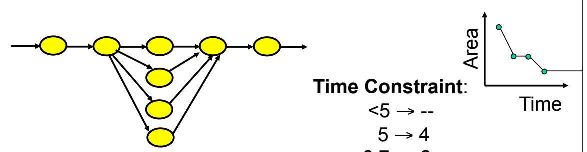
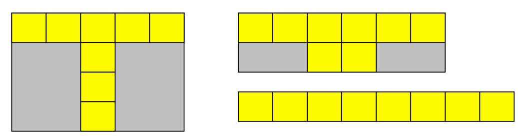
## Resource-Time Example




**Time Constraint**:
<5 → --
5 → 4
6,7 → 2
>7 → 1



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## Scheduling Use

- Very general problem formulation
  - HDL/Behavioral → RTL
  - Register/Memory allocation/scheduling
  - Instruction/Functional Unit scheduling
  - Processor tasks
  - Time-Switched Routing
    - TDMA, bus scheduling, static routing
  - Routing (share channel)

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## Two Types (1)

- **Data independent**
  - graph static
  - resource requirements and execution time
    - independent of data
  - schedule staticly
  - maybe bounded-time guarantees
  - typical ECAD problem

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# Two Types (2)

- **Data Dependent**
  - execution time of operators variable
    - depend on data
  - flow/requirement of operators data dependent
  - if cannot bound range of variation
    - must schedule online/dynamically
    - cannot guarantee bounded-time
    - general case (*I.e.* halting problem)
  - typical “General-Purpose” (non-real-time) OS problem



# Unbounded Resource Problem

- Easy:
  - compute ASAP schedule (next slide)
    - *I.e.* schedule everything as soon as predecessors allow
  - will achieve minimum time
  - won’t achieve minimum area
    - (meet resource bounds)

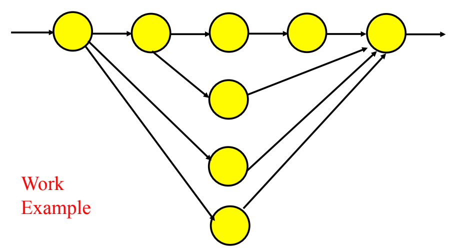


# ASAP Schedule
# As Soon As Possible (ASAP)

- For each input
  - mark input on successor
  - if successor has all inputs marked, put in visit queue
- While visit queue not empty
  - pick node
  - update time-slot based on latest input
  - mark inputs of all successors, adding to visit queue when all inputs marked



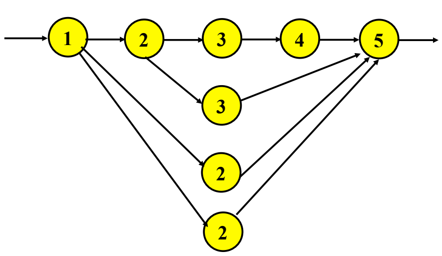
# ASAP Example



Work
Example



# ASAP Example



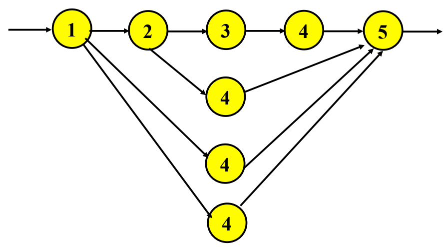



# Also Useful to Define ALAP

- As Late As Possible
- Work backward from outputs of DAG
- Also achieve minimum time w/ unbounded resources

Rework
Example

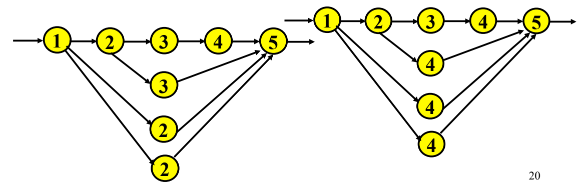
# ALAP Example





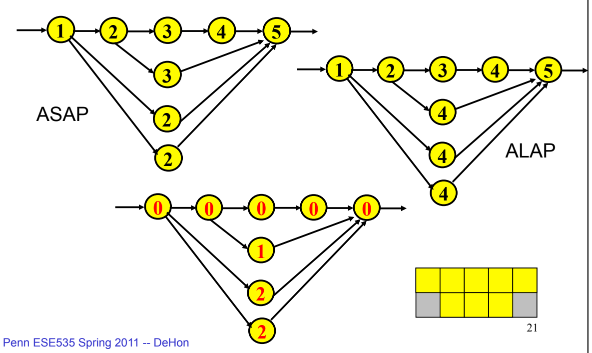

# ALAP and ASAP

- Difference in labeling between ASAP and ALAP is slack of node
  - Freedom to select timeslot
  - **Class theme:** exploit freedom to reduce costs
- If ASAP=ALAP, no freedom to schedule



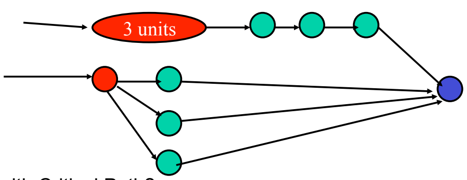


# ASAP, ALAP, Difference

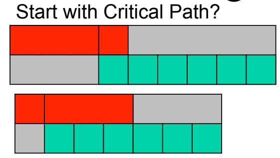





# Why hard?



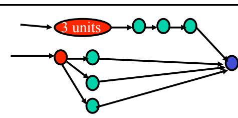

Start with Critical Path?



Schedule on:
1 Red Resource
1 Green Resource



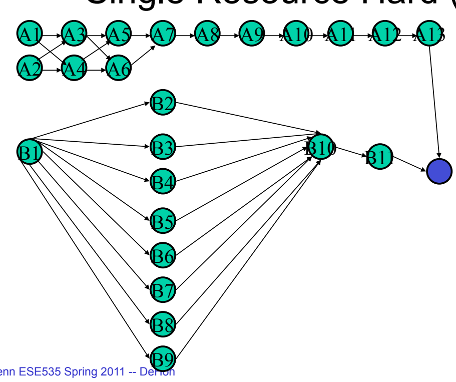
# General



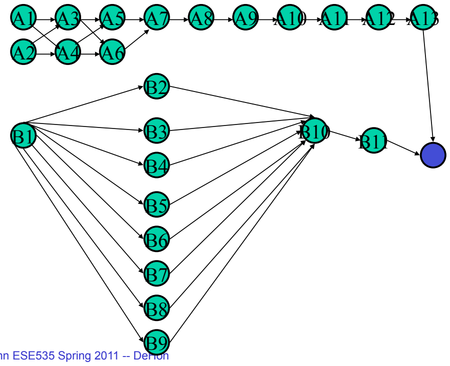

- When selecting, don't know
  - need to tackle **critical path**
  - need to run task to **enable work** (parallelism)
- Can generalize example to single resource case



# Single Resource Hard (1)



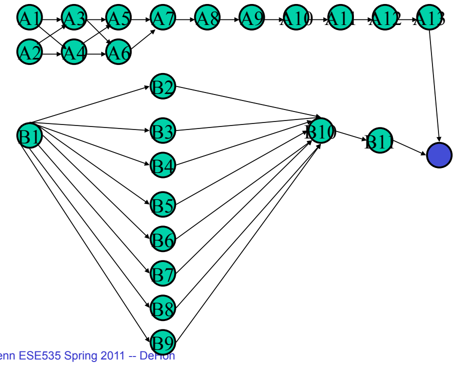
## Single Resource Hard (2)



Crit. Path:
A1 A2
A3 A4
A5 A6
A7 B1
A8 B2
A9 B3
A10 B4
A11 B5
A12 B6
A13 B7
B8 B9
B10
B11



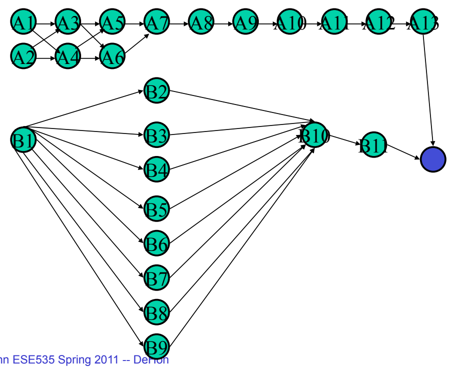
## Single Resource Hard (3)



PFirst
A1 B1
B2 B3
B4 B5
B6 B7
B8 B9
A2 B10
A3 A4
A5 A6
A7 B11
A8
A9
A10
A11
A12
A13



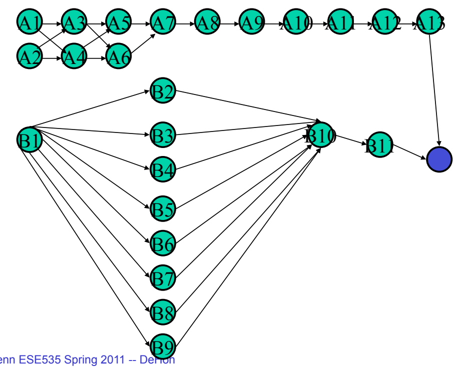
## Single Resource Hard (4)



Balance1
A1 B1
A2 B2
A3 B3
A4 B4
A5 B5
A6 B6
A7 B7
A8 B8
A9 B9
A10 B10
B11 B11
A12
A13



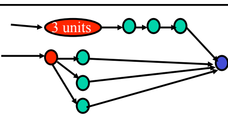
## Single Resource Hard (5)



???
A1 B1
A2 B2
A3 A4
A5 B3
A6 B4
A7 B5
A8 B6
A9 B7
A10 B8
A11 B9
A12 B10
A13 B11



## General: Why Hard




- When selecting, don't know
  - need to tackle **critical path**
  - need to run task to **enable work** (parallelism)



## Two Bounds

## Bounds

- Useful to have bounds on solution
- Two:
  - CP: Critical Path
    - Sometimes call it "Latency Bound"
  - RB: Resource Bound
    - Sometimes call it "Throughput Bound" or "Compute Bound"



## Critical Path Lower Bound

- ASAP schedule ignoring resource constraints
  - (look at length of remaining critical path)
- Certainly cannot finish any faster than that

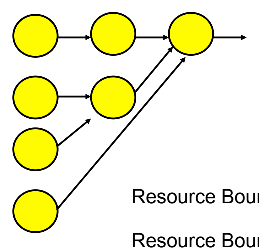


## Resource Capacity Lower Bound

- Sum up all capacity required per resource
- Divide by total resource (for type)
- Lower bound on remaining schedule time
  - (best can do is pack all use densely)
  - Ignores schedule constraints



## Example



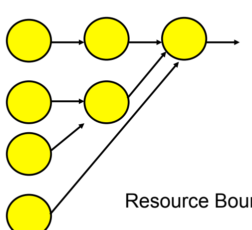
Critical Path

Resource Bound (2 resources)

Resource Bound (4 resources)



## Example



Critical Path 3

Resource Bound (2 resources) 7/2=4

Resource Bound (4 resources) 7/4=2



## List Scheduling

Greedy Algorithm → Approximation

## List Scheduling (basic algorithm flow)

- Keep a ready list of “available” nodes
  - (one whose predecessors have already been scheduled)
  - Like ASAP queue
    - But won’t necessary process in FIFO order
- While there are unscheduled tasks
  - Pick an unscheduled task and schedule on next available resource
  - Put any tasks enabled by this one on ready list



## List Scheduling

- Greedy heuristic
- **Key Question:** How prioritize ready list?
  - What is dominant constraint?
    - least slack (worst critical path) → LPT
      - LPT = Longest Processing Time first
    - enables work
    - utilize most precious (limited) resource
- So far:
  - seen that no single priority scheme would be optimal



## List Scheduling

- Use for
  - resource constrained
  - time-constrained
    - give resource target and search for minimum resource set
- Fast: O(N) →O(Nlog(N)) depending on prioritization
- Simple, general
- How good?

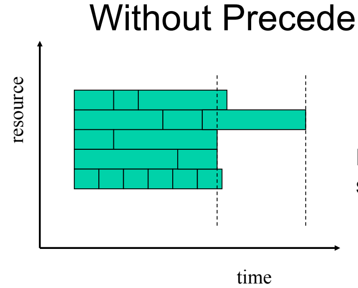


## Approximation

- Can we say how close an algorithm comes to achieving the optimal result?
- Technically:
  - **If** can show
    - Heuristic(Prob)/Optimal(Prob)$\leq\alpha$ $\forall$ prob
  - **Then** the Heuristic is an $\alpha$-approximation



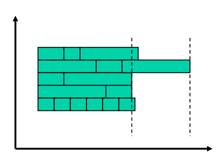
## Scheduled Example Without Precedence




How bad is this schedule?



## Observe



- ∃ optimal length L
- No idle time up to start of last job to finish
- start time of last job ≤ L
- last job length ≤ L
- Total LS length ≤ 2L
- ➢ Algorithm is within factor of 2 of optimum

## Results

- Scheduling of identical parallel machines has a 2-approximation
  - *i.e.* we have a polynomial time algorithm which is guaranteed to achieve a result within a factor of two of the optimal solution.
- In fact, for precedence unconstrained there is a 4/3-approximation
  - *i.e.* schedule Longest Processing Time first

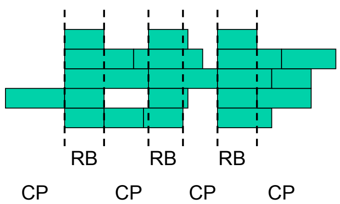


## Recover Precedence

- With precedence we may have idle times, so need to generalize
- Work back from last completed job
  - two cases:
    - entire machine busy
    - some predecessor in critical path is running
- Divide into two sets
  - whole machine busy times
  - critical path chain for this operator



## Precedence






## Precedence Constrained

- Optimal Length > All busy times
  - Optimal Length ≥ Resource Bound
  - Resource Bound ≥ All busy
- Optimal Length>This Path
  - Optimal Length ≥ Critical Path
  - Critical Path ≥ This Path
- List Schedule = This path + All busy times
- List Schedule ≤ 2 *(Optimal Length)



## Conclude

- Scheduling of identical parallel machines with precedence constraints has a 2-approximation.



## Tighten

- LS schedule ≤ Critical Path+Resource Bound
- LS schedule ≤ Min(CP,RB)+Max(CP,RB)
- Optimal schedule ≥ Max(CP,RB)
- LS/Opt ≤ 1+Min(CP,RB)/Max(CP,RB)
- The more one constraint dominates
  - ➔the closer the approximate solution to optimal
  - ➪(EEs think about 3dB point in frequency response)

## Tightening

- Example of
  - More information about problem
  - More internal variables
  - …allow us to state a tighter result
- 2-approx for any graph
  - Since CP may = RB
- Tighter approx as CP and RB diverge



## Multiple Resource

- Previous result for homogeneous functional units
- For heterogeneous resources:
  - also a 2-approximation
    - Lenstra+Shmoys+Tardos, Math. Programming v46p259
    - (not online, no precedence constraints)



## Bounds

- Precedence case, Identical machines
  - no polynomial approximation algorithm can achieve better than 4/3 bound
    - (unless P=NP)
- Heterogeneous machines (no precedence)
  - no polynomial approximation algorithm can achieve better than 3/2 bound



## Summary

- Resource sharing saves area
  - allows us to fit in fixed area
- Requires that we schedule tasks onto resources
- General kind of problem arises
- We can, sometimes, bound the "badness" of a heuristic
  - get a tighter result based on gross properties of the problem
  - approximation algorithms often a viable alternative to finding optimum
  - play role in knowing "goodness" of solution



## Admin

- Reading on web for Monday
  - For scheduling … today's reading
  - New reading for Mon. – architectures relevant to project
- Assignment 1 Due Monday



## Big Ideas:

- Exploit freedom in problem to reduce costs
  - (slack in schedules)
- Use dominating effects
  - (constrained resources)
  - the more an effect dominates, the "easier" the problem
- Technique: Approximation