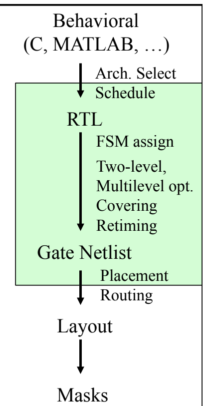
ESE535: Electronic Design Automation

Day 11: February 2, 2011
Partitioning
(Intro, KLFM)

Today

- Partitioning
  - why important
    - Can be used as tool at many levels
  - practical attack
  - variations and issues

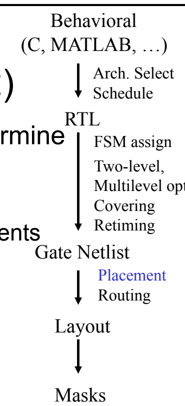


Motivation (1)

- Divide-and-conquer
  - trivial case: decomposition
  - smaller problems easier to solve
    - net win, if super linear
    - Part(n) + 2×T(n/2) < T(n)
  - problems with sparse connections or interactions
  - Exploit structure
    - limited cutsize is a common structural property
    - random graphs would **not** have as small cuts

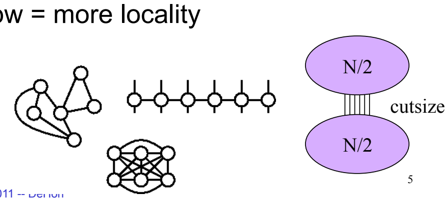
Motivation (2)

- Cut size (bandwidth) can determine
  - Area, energy
- Minimizing cuts
  - minimize interconnect requirements
  - increases signal locality
- Chip (board) partitioning
  - minimize IO
- Direct basis for placement



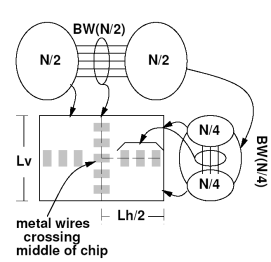
Bisection Width

- Partition design into two equal size halves
  - Minimize wires (nets) with ends in both halves
- Number of wires crossing is **bisection width**
- lower bw = more locality



Interconnect Area

- Bisection width is lower-bound on IC width
  - When wire dominated, may be tight bound
- (recursively)



Classic Partitioning Problem

- **Given:** netlist of interconnect cells
- Partition into two (roughly) equal halves (A,B)
- minimize the number of nets shared by halves
- "Roughly Equal"
  - balance condition: $(0.5-\delta)N \leq |A| \leq (0.5+\delta)N$

Balanced Partitioning

- NP-complete for general graphs
  - [ND17: Minimum Cut into Bounded Sets, Garey and Johnson]
  - Reduce SIMPLE MAX CUT
  - Reduce MAXIMUM 2-SAT to SMC
  - Unbalanced partitioning poly time
- Many heuristics/attacks

KL FM Partitioning Heuristic

- Greedy, iterative
  - pick cell that decreases cut and move it
  - repeat
- small amount of non-greediness:
  - look past moves that make locally worse
  - randomization

Fiduccia-Mattheyses (Kernighan-Lin refinement)

- Start with two halves (random split?)
- Repeat until no updates
  - Start with all cells free
  - Repeat until no cells free
    - Move cell with largest gain (balance allows)
    - Update costs of neighbors
    - Lock cell in place (record current cost)
  - Pick least cost point in previous sequence and use as next starting position
- Repeat for different random starting points

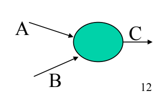
Efficiency

Tricks to make efficient:

- Expend *little* work picking move candidate
  - Constant work ≡ O(1)
  - Means amount of work not dependent on problem size
- Update costs on move cheaply [O(1)]
- Efficient data structure
  - update costs cheap
  - cheap to find next move

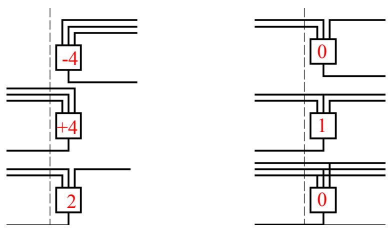
Ordering and Cheap Update

- Keep track of Net gain on node == delta net crossings to move a node
  - cut cost after move = cost - gain
- Calculate node gain as Σ net gains for all nets at that node
  - Each node involved in several nets
- Sort nodes by gain
  - Avoid full resort every move



FM Cell Gains

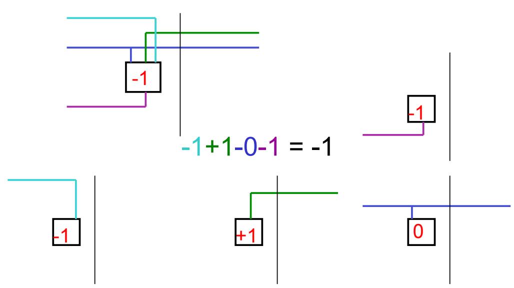
Gain = Delta in number of nets crossing between partitions
= Sum of net deltas for nets on the node



After move node?

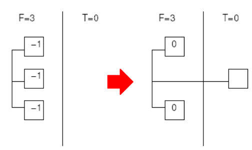
- Update cost
  - Newcost=cost-gain
- Also need to update gains
  - on all nets attached to moved node
  - but moves are nodes, so push to
    - all nodes affected by those nets

Composability of Net Gains



FM Recompute Cell Gain

- For each net, keep track of number of cells in each partition [F(net), T(net)]
- Move update:(for each net on moved cell)
  - if T(net)==0, increment gain on F side of net
    - (think -1 => 0)



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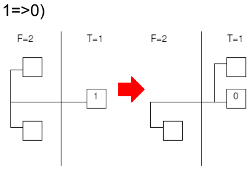

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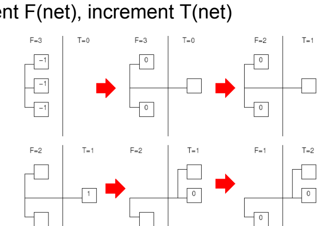

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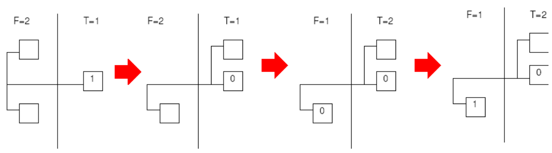
  - if F(net)==1, increment gain on F cell



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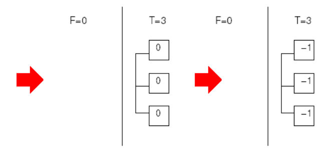
  - decrement F(net), increment T(net)
  - if F(net)==1, increment gain on F cell
  - if F(net)==0, decrement gain on all cells (T)



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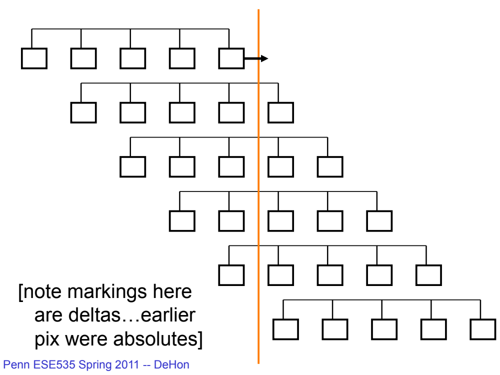
FM Recompute (example)



[note markings here are deltas…earlier pix were absolutes]

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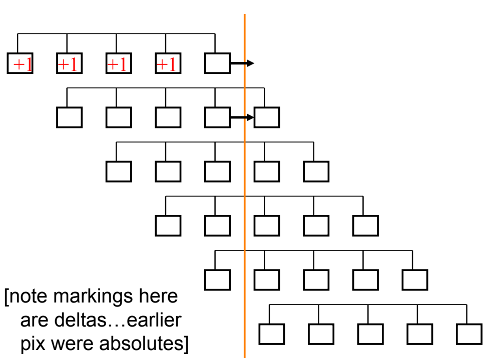

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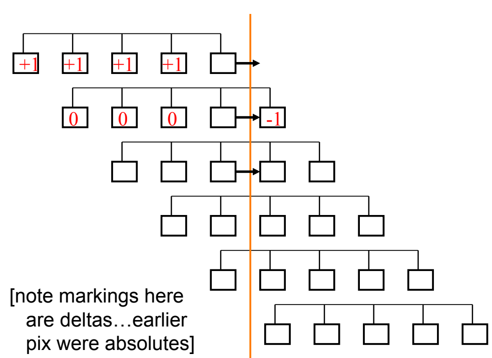

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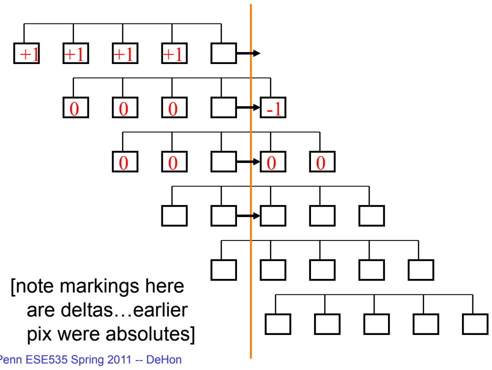

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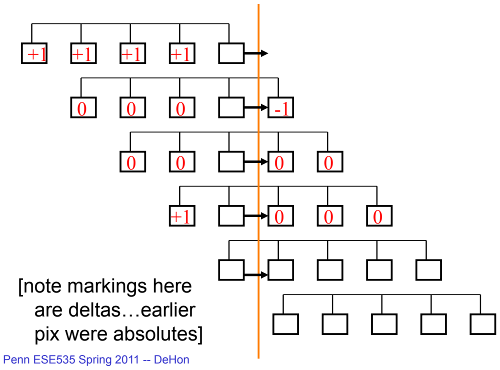

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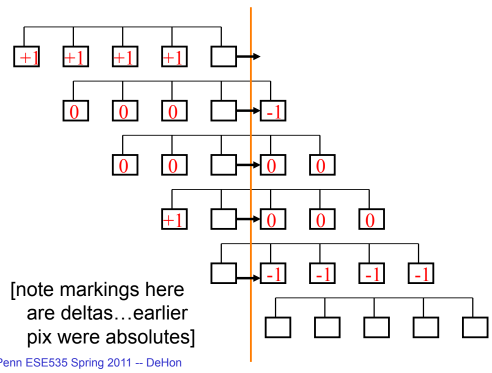

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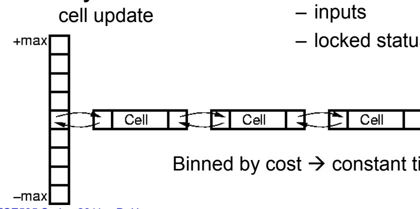
FM Data Structures

- Partition Counts A,B
- Two gain arrays
  - One per partition
  - **Key:** constant time cell update
- Cells
  - successors (consumers)
  - inputs
  - locked status



Binned by cost → constant time update

FM Optimization Sequence (ex)

+3	+2	-1
+3	+1	-1
+2	0	-1
+2	-1	0
+1	0	+1
0	-1	-1
+1	-1	-1
0	-2	-2
0	-2	-2
-1	-1	-2
+1	0	-1
-1	-1	-2
-2	-2	-1
-2	-3	-3
-3	-3	-3
-3	-3	-3
+12	**+3**	**0**

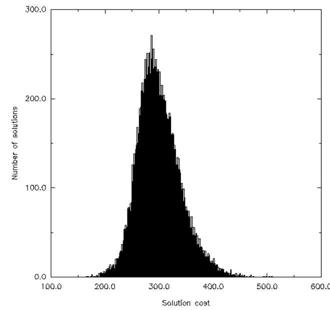
FM Running Time?

- Randomly partition into two halves
- Repeat until no updates
  - Start with all cells free
  - Repeat until no cells free
    - Move cell with largest gain
    - Update costs of neighbors
    - Lock cell in place (record current cost)
  - Pick least cost point in previous sequence and use as next starting position
- Repeat for different random starting points

FM Running Time

- **Claim:** small number of passes to converge
  - Constant passes?
- Small (constant?) number of random starts
- N cell updates each round (swap)
- Updates K + fanout work (avg. fanout K)
  - assume at most K inputs to each node
  - For every net attached (K+1)
    - For every node attached to those nets (O(K))
- Maintain ordered list O(1) per move
  - every io move up/down by 1
- Running time: $O(K^2N)$
  - Algorithm significant for its speed
    - (more than quality)

FM Starts?

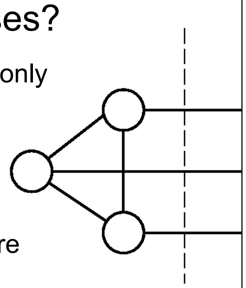


So, FM gives a not bad solution quickly

21K random starts, 3K network -- Alpert/Kahng

Weaknesses?

- Local, incremental moves only
  - hard to move clusters
  - no lookahead
  - Stuck in local minima?
- Looks only at local structure



Improving FM

- Clustering
- Initial partitions
- Runs
- Partition size freedom
- Replication

Following comparisons from Hauck and Boriello '96

Clustering

- Group together several leaf cells into cluster
- Run partition on clusters
- Uncluster (keep partitions)
  - iteratively
- Run partition again
  - using prior result as starting point
    - instead of random start

Clustering Benefits

- Catch local connectivity which FM might miss
  - moving one element at a time, hard to see move whole connected groups across partition
- Faster (smaller N)
  - METIS -- fastest research partitioner exploits heavily

How Cluster?

- Random
  - cheap, some benefits for speed
- Greedy "connectivity"
  - examine in random order
  - cluster to most highly connected
  - 30% better cut, 16% faster than random
- Spectral (next week)
  - look for clusters in placement
  - (ratio-cut like)
- Brute-force connectivity (can be $O(N^2)$)

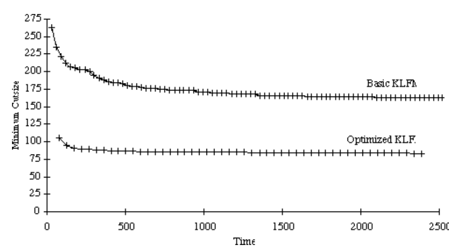
Initial Partitions?

- Random
- Pick Random node for one side
  - start imbalanced
  - run FM from there
- Pick random node and Breadth-first search to fill one half
- Pick random node and Depth-first search to fill half
- Start with Spectral partition

Initial Partitions

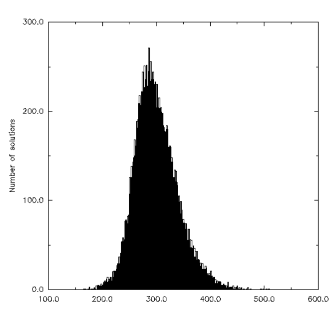
- If run several times
  - pure random tends to win out
  - more freedom / variety of starts
  - more variation from run to run
  - others trapped in local minima

Number of Runs



Number of Runs

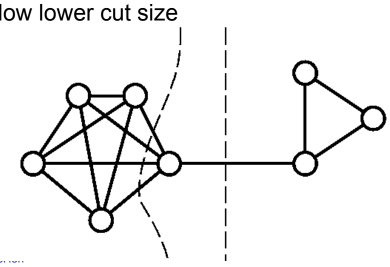
- 2 - 10%
- 10 - 18%
- 20 <20%
- 50 < 22%
- …but?



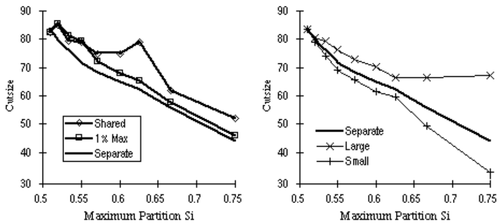
21K random starts, 3K network
Alpert/Kahng

Unbalanced Cuts

- Increasing slack in partitions
  - may allow lower cut size



Unbalanced Partitions

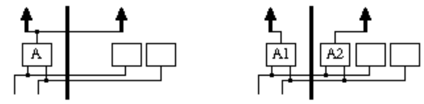


Small/large is benchmark size not small/large partition IO.

Following comparisons from Hauck and Boriello '96

Replication

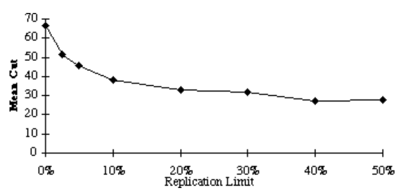
- Trade some additional logic area for smaller cut size
  - Net win if wire dominated



Replication data from: Enos, Hauck, Sarrafzadeh '97

Replication

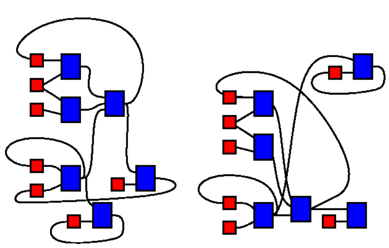
- 5% ➔ 38% cut size reduction
- 50% ➔ 50+% cut size reduction



What Bisection doesn't tell us

- Bisection bandwidth purely geometrical
- No constraint for delay
  - *I.e.* a partition may leave critical path weaving between halves

Critical Path and Bisection



Minimum cut may cross critical path multiple times.
Minimizing long wires in critical path => increase cut size.

So...

- Minimizing bisection
  - good for area
  - oblivious to delay/critical path

Partitioning Summary

- Decompose problem
- Find locality
- NP-complete problem
- linear heuristic (KLFM)
- many ways to tweak
  - Hauck/Boriello, Karypis
- even better with replication
- only address cut size, not critical path delay

Admin

- Reading for Wed. online
- Assignment 2A due on Monday

Today's Big Ideas:

- Divide-and-Conquer
- Exploit Structure
  - Look for sparsity/locality of interaction
- Techniques:
  - greedy
  - incremental improvement
  - randomness avoid bad cases, local minima
  - incremental cost updates (time cost)
  - efficient data structures