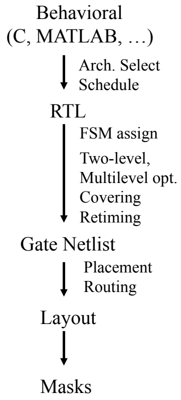
ESE535: Electronic Design Automation

Day 17: March 20, 2013
Modern SAT Solvers
({z}Chaff, GRASP, miniSAT)

Today

- SAT
- Pruning Search
- Davis-Putnam
- Data Structures
- Optimizations
  - Watch2
  - VSIDS
  - ?restarts
- Learning



Problem (almost)

- SAT: Boolean Satisfiability
- **Given:** logical formula g
- Find a set of variable assignments that makes g **true**
- Or conclude no such assignment exists

Example Uses

- Provisioning/Scheduling from last time
- Partitioning, Placement, Routing
- Can I find an assignment that causes this output to become true, false?
  - Automatic Test Pattern Generation (ATPG)
  - Static Timing Analysis (false paths)
- Verification
  - Is this optimized logic the same as the specification logic?
- FSM Encoding

Problem (more precise)

- SAT: Boolean Satisfiability
- **Given:** logical formula g in CNF
- Find a set of variable assignments that makes g **true**
- Or conclude no such assignment exists

CNF

- Conjunctive Normal Form
- Logical AND of a set of **clauses**
  - Product of sums
- **Clauses:** logical OR of a set of literals
- **Literal:** a variable or its complement
- *E.g.*

(A+B+/C)*(/B+D)*(C+/A+/E)

CNF

- Conjunctive Normal Form
- Logical AND of a set of **clauses**
- To be satisfied:
  - Every clause must be made **true**
- (A+B+/C)*(/B+D)*(C+/A+/E)
  - If know D=**false**
    - ➔ B must be **false**

3-SAT Universal

- Can express any set of boolean constraints in CNF with at most 3 literals per clause
- Canonical NP-complete problem

Convert to 3-SAT

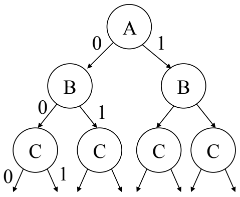
- A=/B*/C=/(B+C) → universal primitive
  - We know can build any logic expression from nor2
- 3-CNF for A=/B*/C
  - (A+B+C)*(/A+/B)*(/A+/C)
    - If (B==0 && C==0) then A=1
    - If (B==1 || C==1) then A=0
- To convert any boolean formula to 3-CNF:
  1. Convert to nor2's
     - Or norX if not limited to 3-CNF formulas
  2. Then use above to convert nor2 expressions to set of clauses
  3. Combine (conjunct=AND) the clauses resulting from all the nor's

Brute Force Exhaustive

- How could we find satisfying assignment?
- How long would it take?
  - With N binary variables

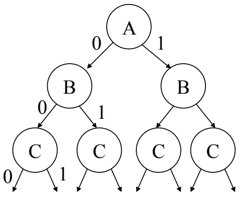
Search Formulation

- Think of as search tree on variables
- Each variable can be true or false
  - Branch on values
- All variables determined at leaves of tree



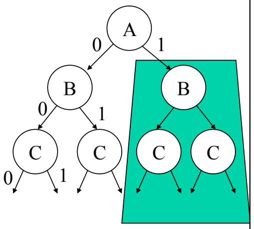
Key Trick

- Avoid searching down to leaf on all subtrees
- "Prune" away branches of tree



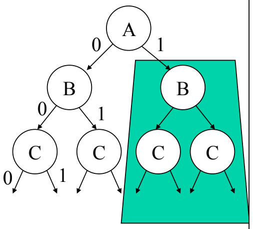
Key Trick

- (A+B+C)*(/A+/B)*(/A+/C)
- Consider A=1



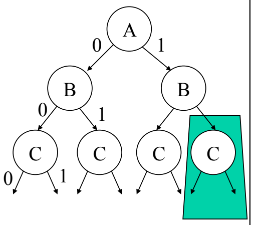
Key Trick

- (A+B+C)*(/A+/B)*(/A+/C)
- Consider A=1
- In this subtree becomes
  /B*/C



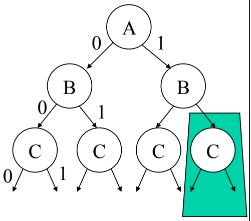
Key Trick

- (A+B+C)*(/A+/B)*(/A+/C)
- Consider A=1
- In this subtree becomes
  /B*/C
- Consider B=1



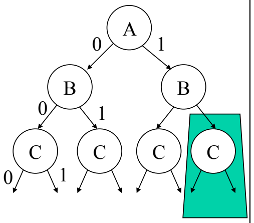
Key Trick

- (A+B+C)*(/A+/B)*(/A+/C)
- Consider A=1
- In this subtree becomes
  /B*/C
- Consider B=1
  - Becomes false
  - Regardless of C
  - Don't need to explore tree further



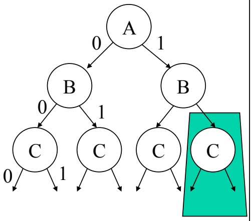
Key Trick

- (A+B+C)*(/A+/B)*(/A+/C)
- Consider A=1
- In this subtree becomes
  /B*/C
- **Implication**
  - When there is only one literal left in a clause
  - Can conclude it must be true
  - → Select it and prune other branch



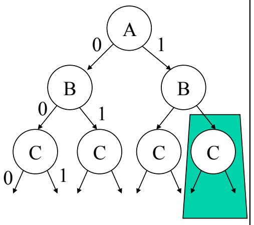
Key Trick

- (…)*B*/B*(…)
- **Contradiction**
  - If implications lead to a conflicting assignments
  - Can conclude this subtree is unsatisfiable
  - Prune branch



Prospect

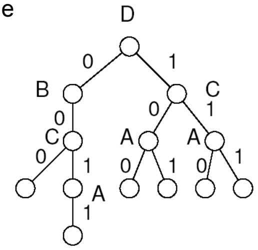
- Use **implications** and **contradictions** to prune subtrees and avoid visiting full space



Pruning Search

(A+B+/C)*(/B+D)*(C+/A+/E)

- Solve with pruning search
  - Pick an unassigned variable
  - Branch on true/false
  - Compute implications



Davis-Putnam

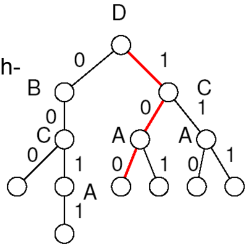
```
while (true) {
  if (!decide()) // no unassigned vars
     return(satisfiable);
  while ( !bcp()) { // constraint propagation
     if (!resolveConflict()) // backtrack
        return(not satisfiable);
   }
}
```

decide()

- Picks an unassigned variable
- Gives it a value
- Push on *decision stack*
  - Efficient structure for depth-first search tree

(A+B+/C)*(/B+D)*(C+/A+/E)

A=0
C=0
D=1



Data Structures

- Decision “stack”
- Variable “array”
- Clause “DB”
  - Each clause is a set of variables

(A+B+/C)*(/B+D)*(C+/A+/E)

A=0
C=0
D=1

A
B
C
D
E

A	B	/C
/B	D	
/A	C	/E

bcp
(boolean constraint propagation)

- What do we need to do on each variable assignment?
  - Find implications
    - Implication when all other literals in a clause are **false**
    - Look through all clauses this assignment effects
    - See if any now have all **false** and one unassigned
  - Assign implied values
  - Propagate that assignment
  - Conflict if get implications for **true** and **false**

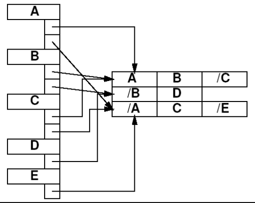
bcp()

- Q=new queue();
- Q.insert(top of decision stack);
- while (!Q.empty())
  - V=Q.pop();
  - For each clause C in DB with V
    - If C now satisfied, mark as such (remove from DB)
    - If C has one unassigned literal, rest **false**
      - Vnew=unassigned literal in C
      - val=value Vnew must take
      - If (Vnew assigned to value other than val)
        - » return (**false**); // conflict
      - Q.add(Vnew=val);
- return(**true**)

Variable Array

- Each variable has a list pointing to all clauses in which it appears?
  - Avoid need to look at every clause

(A+B+/C)*(/B+D)*(C+/A+/E)

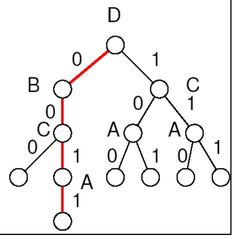


Tracking Implications

(A+B+/C)*(/B+D)*(C+/A+/E)

- Each implication made at some tree level
  - Associated with some entry on decision stack
  - Has associated decision stack height
- On backtrack
  - Unassign implications above changed decision level

A=1 at DL=2 | C=1
B=0 at DL=1 | D=0



Track Variable Assignment

- Each clause has counter
  - Count number of unassigned literals
  - Decrement when assign **false** literal
  - Mark clause as satisfied when assign **true** literal (remove from clause database?)

3	**A**	**B**	**/C**
2	**/B**	**D**	
3	**/A**	**C**	**/E**

Track Variable Assignment

- Each clause has counter
  - Count number of unassigned literals
  - Decrement when assign **false** literal
  - Mark clause as satisfied when assign **true** literal (remove from clause database?)

3	**A**	**B**	**/C**
2	**/B**	**D**	
3	**/A**	**C**	**/E**

E=1

3	**A**	**B**	**/C**
2	**/B**	**D**	
2 ~~3~~	**/A**	**C**	**/E**

Track Variable Assignment

- Each clause has counter
  - Count number of unassigned literals
  - Decrement when assign **false** literal
  - Mark clause as satisfied when assign **true** literal
  - Counter avoids need to check all variable assignments in clause on every assignment
  - Watch for counter decrement 2→1
    - That's when a literal is implied.

3	**A**	**B**	**/C**
2	**/B**	**D**	
2 ~~3~~	**/A**	**C**	**/E**

resolveConflict()

- What does resolveConflict need to do?
  - Look at most recent decision
  - If can go other way, switch value
    - (clear implications to this depth)
  - Else pop and recurse on previous decision
  - If pop top decision,
    - Unsatisfiable
- Alternates:
  - Treat literals separately
    - Unassign and pick another literal
  - Learning (later in lecture)
    - May allow more direct backtracking

Chaff Optimizations

How will this perform?

- 10,000's of variables
- 100,000's of clauses (millions)
- Every assignment walks to the clause database
- Cache performance?
- How big is L1 cache? L2 cache?
- Ratio of main-memory speed to L1 cache speed?

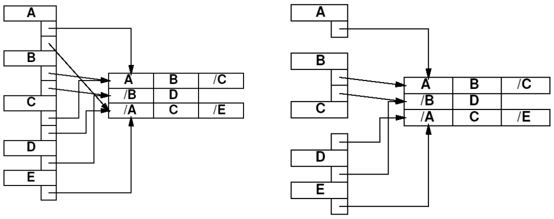
Challenge 1

- Currently, visit every clause on each assignment
  - Clause with K variables
  - Visited K-1 times
  - K-2 of which just to discover it's not the last
- Can we avoid visiting every clause on every assignment?
  - Every clause in which a variable appears?

Avoiding Clause Visits

- **Idea:** watch only 2 variables in each clause
- Only care about final set of next to last variable
- If set other k-2, won't force an implication
- When set one of these (and everything else set)
  - Then we have an implication

Watch 2 Data Structure



Avoiding Clause Visits

- **Idea:** watch only 2 variables in each clause
- Only care about final set of next to last variable
- What if we set one of these two “watched” variables?
  - If not last, change the watch to one of the unset variables

Watch 2

- If watched literal becomes false
  - Check if all non-watched are set
    - if so, set implication on other watched
    - else, update watch literal

Note

- Watch pair is arbitrary
- Unassigning a variable (during backtrack)
  - Does not require reset of watch set
  - Constant time to “unset” a variable

Challenge 2: Variable Ordering

- How do we decide() which variable to use next?
  - Want to pick one that facilitates lots of pruning

Variable Ordering

- Old Ideas:
  - Random
  - (DLIS) Dynamic largest individual sum
    - Used most frequently in unresolved clauses
    - Potential weakness:
      - Must re-sort with every variable assignment?
  - …none clearly superior
    - DLIS competitive
    - Rand good on CAD benchmarks?

New: VSIDS

- Variable State Independent Decaying Sum
  - Each literal has a counter
  - When clause added to DB, increment counter for each literal
  - Select unassigned literal with highest count
  - Periodically, all counters are divided by a constant

New: VSIDS

- Variable State Independent Decaying Sum
  - Each literal has a counter
  - When clause added to DB, increment counter for each literal
    - Remove clauses when satisfied?
    - Reinsert on backtrack
  - Select unassigned literal with highest count
  - Periodically, all counters are divided by a constant

New: VSIDS

- Variable State Independent Decaying Sum
  - Each literal has a counter
  - When clause added to DB, increment counter for each literal
  - Select unassigned literal with highest count
    - Don't need to re-sort each selection
    - Only re-sort on backtrack
    - Maybe priority queue insert?
  - Periodically, all counters are divided by a constant

VSIDS

- **Goal:** satisfy *recent* conflict clauses
- Decaying sum weights things being added
  - Clauses not conflicting for a while, have values reduced
    - (? Avoid walking through them by increasing weight on new stuff rather than decreasing all old?)
- **Impact:** order of magnitude speedup

Restarts

- Periodically restart
  - Clearing the state of all variables
    - i.e. clear decision stack
  - Leave clauses in clause database
    - ? Keep ordering based on recent costs
    - ? Re-insert clauses must reinsert on restart?
  - State of clause database drives variable ordering
    - Benefit: new variable ordering based on lessons of previous search

Overall

- Two orders of magnitude benefit on unsatisfiable instances
- One order of magnitude on satisfiable instances

Learning

Learning

- When encounter a conflict
  - Determine variable assignment contributing to conflict
  - Add new clause to database
- New clause allows pruning

Davis-Putnam w/ Learning

```
while (true) {
  if (!decide()) // no unassigned vars
     return(satisfiable);
  while ( !bcp()) { // constraint propagation
     analyzeConflicts(); // learning
     if (!resolveConflict()) // backtrack
        return(not satisfiable);
   }
}
```

Implication Graph

- As perform bcp propagation
  - When set variable, insert back link to previous variable set forcing this variable set
  - Graph captures what this implication depends upon
- When encounter a conflict
  - Identify what variable values caused

Example

Current Truth Assignment: $\{x_9 = 0@1, x_{10} = 0@3, x_{11} = 0@3, x_{12} = 1@2, x_{13} = 1@2, \ldots\}$

Current Decision Assignment: $\{x_1 = 1@6\}$

$\omega_1 = (\neg x_1 + x_2)$

$\omega_2 = (\neg x_1 + x_3 + x_9)$

$\omega_3 = (\neg x_2 + \neg x_3 + x_4)$

$\omega_4 = (\neg x_4 + x_5 + x_{10})$

$\omega_5 = (\neg x_4 + x_6 + x_{11})$

$\omega_6 = (\neg x_5 + \neg x_6)$

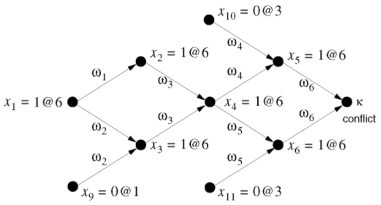
$\omega_7 = (x_1 + x_7 + \neg x_{12})$

$\omega_8 = (x_1 + x_8)$

$\omega_9 = (\neg x_7 + \neg x_8 + \neg x_{13})$

...

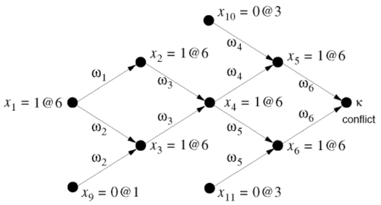
Clause Database



Implication Graph for Current Decision Assignment

Marques-Silva/Sakallah TRCOMP v48n5p506 1999

Conflict Resolution



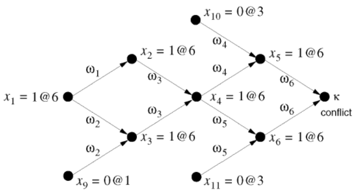
- x1 & /x9 & /x10 & /x11 lead to conflict
- /(x1 & /x9 & /x10 & /x11)
- /x1+x9+x10+x11 🡐 new clause for DB

New Clause

Current Truth Assignment: $\{x_9 = 0@1, x_{10} = 0@3, x_{11} = 0@3, x_{12} = 1@2, x_{13} = 1@2, \ldots\}$

Current Decision Assignment: $\{x_1 = 1@6\}$

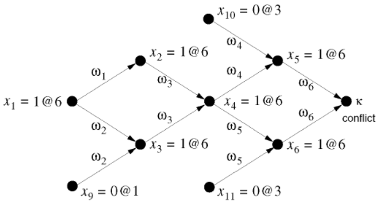
- New clause does not include x12, x13
- May encounter this case again



Implication Graph for Current Decision Assignment

/x1+x9+x10+x11 🡐 new clause for DB

More Implications



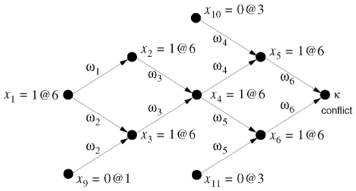
- x4 & /x10 & /x11 lead to conflict
- /x4+x10+x11 ← new clause for DB
- Also (/x1+x9+x4) since x1*/x9 → x4

New Clauses

Current Truth Assignment: $\{x_9 = 0@1, x_{10} = 0@3, x_{11} = 0@3, x_{12} = 1@2, x_{13} = 1@2, \ldots\}$

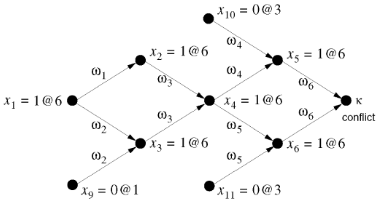
Current Decision Assignment: $\{x_1 = 1@6\}$

- /x4+x10+x11
  - Doesn't depend on x9
- (/x1+x9+x4)
  - x4 not in decision tree
- Will be useful for later pruning



Implication Graph for Current Decision Assignment

Unique Implication Point



- UIP = vetext that dominates verticies leading to conflict
  - x1 is UIP (decision variable causing is always a UIP)
  - x4 is UIP

Clause Tradeoff

- Adding clauses facilitates implications
  - Increases pruning
  - Must make less decisions
- Adding clauses increases size of clause database
  - Increases memory
  - Could add exponential clauses
  - Forces more work to push implications

Learned Clauses

- Runtime = Decisions * ImplicationTime
  - Decisions decreasing
  - Implication Time increasing
- Starting from 0 learned clauses,
  - Net decrease in runtime
- Eventually, Implication Time too large and slows down
- Optimum with limited number of learned clauses

Limiting Learned Clauses

- Filter out dominated clauses
- Keep smaller clauses (fewer literals)
  - Have most relevance
- zChaff study suggest inserting only UIP closest to conflict [Zhang et al., ICCAD2001]
- Treat like cache and evict learned clauses
  - Use activity statistics as with variables so keep most useful clauses [minisat 1.2]

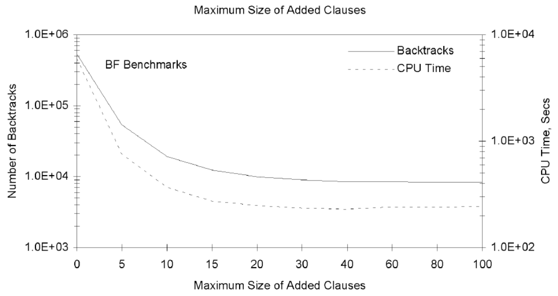
(Recall) Restarts

- Periodically restart
  - Clearing the state of all variables
    - i.e. clear decision stack
  - Leave clauses in clause database
  - State of clause database drives variable ordering
    - Benefit: new variable ordering based on lessons of previous search

Impact of Learning

- zChaff [ICCAD2001] showed 2x improvement based on tuning the learning scheme
- Learning can be orders of magnitude benefit

Impact of Learning



Marques-Silva/Sakallah TRCOMP v48n5p506 1999

Big Ideas

- Technique: SAT
- Exploit Structure
  - Constraint propagation
  - Pruning search technique
  - Learning (discover structure)
- Constants matter
  - Exploit hierarchy in modern memory systems

Admin

- Assign 5a feedback on Blackboard
- Assign 5b Monday
- Reading for Monday on Blackboard