ESE535: Electronic Design Automation

Day 18: March 25, 2013

Two-Level Logic-Synthesis

Today

- Two-Level Logic Optimization
  - Problem
  - Definitions
  - Basic Algorithm: Quine-McCluskey
  - Improvements

Behavioral (C, MATLAB, …)
↓ Arch. Select, Schedule
RTL
↓ FSM assign, Two-level, Multilevel opt., Covering, Retiming
Gate Netlist
↓ Placement, Routing
Layout
↓
Masks

Problem

- **Given**: Expression in combinational logic
- **Find**: Minimum (cost) sum-of-products expression
- Ex.
  - Y=a*b*c + a*b*/c + a*/b*c
  - Y=a*b + a*c

EDA Use

- Minimum size PLA, PAL, …
  - Programmable Logic Array
  - Programmable Array Logic
- Minimum number of gates for two-level implementation
- Starting point for multi-level optimization

Programmable Logic Arrays (PLAs)

PLA

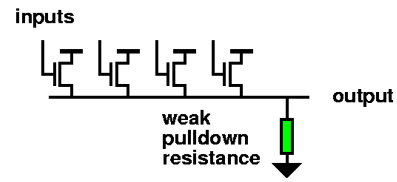
- Directly implement flat (two-level) logic
  - O=a*b*c*d + !a*b*!d + b*!c*d
- Exploit substrate properties allow wired-OR

Wired-**or**

- Connect series of inputs to wire
- Any of the inputs can drive the wire high

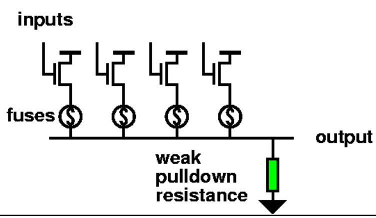
Wired-**or**

- Obvious Implementation with Transistors



Programmable Wired-**or**

- Use some memory function to programmable connect (disconnect) wires to OR
- Fuse:



Programmable Wired-**or**

- Memory configurable PLA model

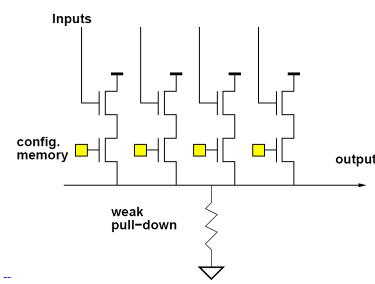
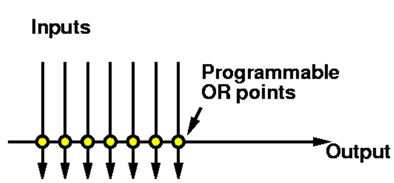
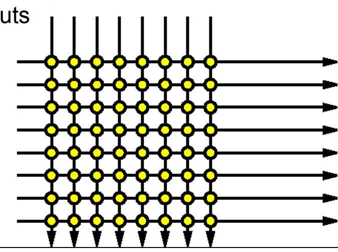


Diagram Wired-**or**



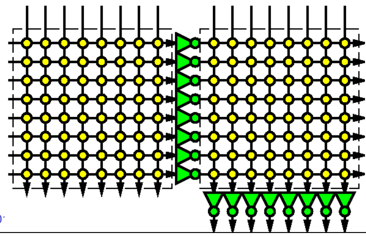
Wired-**or** array

- Build into array
  - Compute many different **or** functions from set of inputs



Combined **or**-arrays to PLA

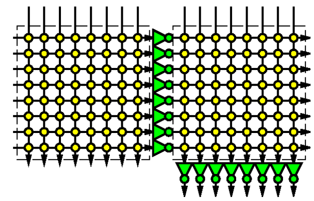
- Combine two or (**nor**) arrays to produce PLA (**or-and** / **and-or** array)



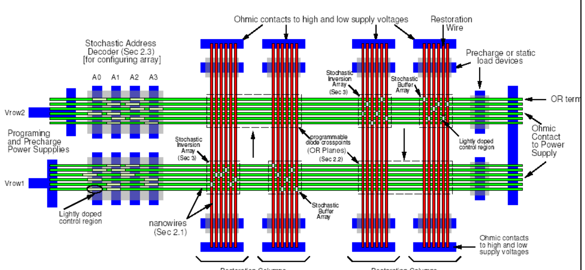
PLA

- Can implement each **and** on single line in first array
- Can implement each **or** on single line in second array

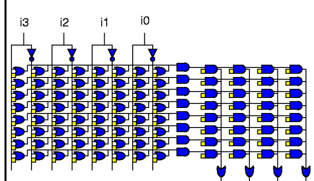
Strictly speaking:
or in first term **and** in second, but with both polarities of inputs, can invert so is **and-or**.

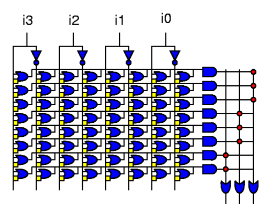


Nanowire PLA



PLA and PAL





PAL = Programmable Array Logic

PAL has **fixed** OR plane.

…back to optimization…

EDA Use for 2-level Logic Min.

- Minimum size PAL, PLA, …
  - Programmable Logic Array
  - Programmable Array Logic
- Minimum number of gates for two-level implementation
- Starting point for multi-level optimization

Complexity

- Set covering problem
  - NP-hard

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Terminology (1)

- Literals -- a, /a, b, /b, ….
  - Qualified, single inputs
- Minterms --
  - full set of literals covering one input case
  - in y=a*b+a*c
    - a*b*c
    - a*/b*c

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Cost

- PLA/PAL – to first order costs is:
  - number of product terms
- Abstract (mis, sis)
  - {multilevel,sequential} interactive synthesis
  - number of literals
    - cost(y=a*b+a*/c)=4
- General (simple, multi-level)
  - ∑cost(product-term)
    - e.g. nand2=4, nand3=5,nand4=6…

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Terminology (2)

- Cube:
  - product covering one or more minterms
  - Y=a*b+a*c
  - cubes:
    - a*b*c abc
    - a*b ab
    - a*c ac

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Terminology (3)

- Cover:
  - set of cubes
  - sum products
  - {abc, a/bc, ab/c}
  - {ab,ac}

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Truth Table

- Also represent function

a b c	y
0 0 0	0
0 0 1	0
0 1 0	0
0 1 1	0
1 0 0	0
1 0 1	1
1 1 0	1
1 1 1	1

Specify on-set only

a b c	y
1 0 1	1
1 1 0	1
1 1 1	1

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Cube/Logic Specification

- Canonical order for variables
- Use {0,1,-} to indicate input appearance in cube
  - 0 ≡ inverted — abc 111
  - 1 ≡ not inverted — a/bc 101
  - - ≡ not present — ac 1-1

a b c	y
1 0 1	1
1 1 0	1
1 1 1	1

1 0 1	1
1 1 0	1
1 1 1	1

1 - 1	1
1 1 -	1

1 - 1
1 1 -

In General

- Three sets:
  - on-set (must be set to one by cover)
  - off-set (must be set to zero by cover)
  - don't care set (can be zero or one)
- Don't Cares
  - allow freedom in covering (reduce cost)
  - arise from cases where value doesn't matter
    - *e.g.* outputs in non-existent FSM state
    - data bus value when not driving bus

Multiple Outputs

Truth Table:

a b	y x
0 0	1 1
0 1	0 0
1 0	0 0
1 1	0 1

Convert to single-output problem

a b	y x	o
0 0	1 -	1
0 0	- 1	1
0 1	0 -	1
0 1	- 0	1
1 0	0 -	1
1 0	- 0	1
1 1	0 -	1
1 1	- 1	1

On-set for result

001-
00-1
010 -
01- 0
100 -
10- 0
110 -
11- 1

Multiple Outputs

- Can reduce to single output case
  - write equations on inputs and each output
    - with onset for relation being true
  - after cover
    - remove literals associated with outputs

Multiple Outputs

- Could Optimize separately
- By optimizing together
  - Maximize sharing of cubes/product-terms

Multiple Outputs

- Consider:
  - X=/a/b+ab+ac
  - Y=/bc
- Trivial solution has 4 product terms

000 10
001 11
010 00
011 00
100 00
101 11
110 10
111 10

000 10
-01 11
01- 00
100 00
11- 10

Multiple Outputs

- Consider:
  - X=/a/b+ab+ac
  - Y=/bc
- Now read off cover:
  - Y=/bc
  - A=/a/b/c+/bc+ab
    - =/a/b+/bc+ab

000 10	000 10
001 11	-01 11
010 00	01- 00
011 00	
100 00	100 00
101 11	
110 10	11- 10
111 10	

Only need 3 product terms (versus 4 w/ no sharing)

Prime Implicants

- Implicant -- cube in on-set
  - (not entirely in don't-care set)
- Prime Implicant -- implicant, not contained in any other cube
  - for y=a*b+a*c
    - a*b is a prime implicant
    - a*b*c is not a prime implicant (contained in ab, ac)
  - *I.e.* largest cube still in on-set (on+dc-sets)

Prime Implicants

- Minimum cover will be made up of primes
  - fewer products if cover more
  - fewer literals in prime than contained cubes
- Necessary but not sufficient that minimum cover contain only primes
  - y=ab+ac+b/c
  - y=ac+b/c
- Number of PI's can be exponential in input size
  - more than minterms, even!
  - Not all PI's will be in optimum cover

Restate Goal

- Goal in terms of PIs
  - Find minimum size set of PIs that cover the on-set.

Essential Prime Implicants

- Prime Implicant which contains a minterm not covered by any other PI
  - Essential PI **must** occur in any cover
  - y=ab+ac+b/c
  - ab 11- 110 111
  - ac 1-1 101 111 * essential (only 101)
  - b/c -10 110 010 * essential (only 010)

Computing Primes

- Start with minterms
  - for on-set and dc-set
- merge pairs (distance one apart)
- for each pair merged,
  - mark source cubes as covered
- repeat merging for resulting cube set
  - until no more merging possible
- retain all unmarked cubes which aren't entirely in dc-set

Compute Prime Example

0	0000
5	0101
7	0111
8	1000
9	1001
10	1010
11	1011
14	1110
15	1111

Note this is preclass 3.

(in-class assignments, back of preclass sheet; record solutions on board.)

Compute Prime Example

0 0000	0, 8 -000
5 0101	5, 7 01-1
7 0111	7,15 -111
8 1000	8, 9 100-
9 1001	8,10 10-0
10 1010	9,11 10-1
11 1011	10,11 101-
14 1110	10,14 1-10
15 1111	11,15 1-11
	14,15 111-

Compute Prime Example

0 0000	0, 8 -000	0, 8 -000		/b/c/d
5 0101	5, 7 01-1	5, 7 01-1		/abd
7 0111	7,15 -111	7,15 -111		bcd
8 1000	8, 9 100-	~~8, 9 100-~~		a/b
9 1001	8,10 10-0	8,10 10-0	8, 9,10,11 10--	ac
10 1010	9,11 10-1	9,11 10-1		
11 1011	10,11 101-	10,11 101-		
14 1110	10,14 1-10	~~10,14 1-10~~	10,11,14,15 1-1-	
15 1111	11,15 1-11	~~11,15 1-11~~		
	14,15 111-	14,15 111-		

Covering Matrix

- Minterms × Prime Implicants

	/b/c/d	/abd	bcd	a/b	ac
0000	X				
0101		X			
0111		X	X		
1000	X			X	
1001				X	
1010				X	X
1011				X	X
1110					X
1111			X		X

Goal: minimum cover

Essential Reduction

- Must pick essential PI
  - pick and eliminate row and column

Which essential?

	/b/c/d	/abd	bcd	a/b	ac
0000	X				
0101		X			
0111		X	X		
1000	X			X	
1001				X	
1010				X	X
1011				X	X
1110					X
1111			X		X

Essential Reduction

- This case:
  - Cover determined by essentials
  - Preclass 3: ac+a/b+/abd+/b/c/d
- General case:
  - Reduces size of problem

Dominators: Column

- If a column (PI) covers the same or strictly more than another column
  - can remove **dominated** column

	B	C	D	E	F	G	H
0101	X	X					
0111		X	X				
1000						X	X
1010					X	X	
1110				X	X		
1111			X	X			

C dominates B

Any others?

G dominates H

Dominators: Column

- If a column (PI) covers the same or strictly more than another column
  - can remove **dominated** column

	B	C	D	E	F	G	H
0101	X	X					
0111		X	X				
1000						X	X
1010					X	X	
1110				X	X		
1111			X	X			

	C	D	E	F	G
0101	X				
0111	X	X			
1000					X
1010				X	X
1110			X	X	
1111		X	X		

New Essentials

- Dominance reduction may yield new Essential PIs

What's essential?

	C	D	E	F	G
0101	X				
0111	X	X			
1000					X
1010				X	X
1110			X	X	
1111		X	X		

C,G now essential

	C	D	E	F	G
1110			X	X	
1111		X	X		

What now?

E dominates D and F

Cover = {C,E,G}

Dominators: Row

- If a row has the same (or strictly more) PIs than another row, the larger row dominantes
  - we can remove the **dominating** row
    - (NOTE OPPOSITE OF COLUMN CASE)

	C	D	E	F	G
0101	X				
~~0111~~	~~X~~	~~X~~			
1000					X
~~1010~~				~~X~~	~~X~~
1110			X	X	
1111		X	X		

0111 dominates 0101
remove 0111

Others?

1010 dominates 1000
remove 1010

Cyclic Core

- After applying reductions
  - essential
  - column dominators
  - row dominators
- May still have a non-trivial covering matrix

- How do we move forward from here?

Example

	A	B	C	D	E	F	G	H
0000	X							X
0001	X	X						
0101		X	X					
0111			X	X				
1000							X	X
1010						X	X	
1110					X	X		
1111				X	X			

Cyclic Core

- Cannot select (e.g. essential) or exclude (e.g. dominated) a PI definitively.
- Make a guess
  - A in cover
  - A not in cover
- Proceed from there

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Example

	A	B	C	D	E	F	G	H
0000	X							X
0001	X	X						
0101		X	X					
0111			X	X				
1000							X	X
1010						X	X	
1110					X	X		
1111				X	X			

A in Cover:

What now?

	B	C	D	E	F	G	H
0101	X	X					
0111		X	X				
1000						X	X
1010					X	X	
1110				X	X		
1111			X	X			

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Example

	A	B	C	D	E	F	G	H
0000	X							X
0001	X	X						
0101		X	X					
0111			X	X				
1000							X	X
1010						X	X	
1110					X	X		
1111				X	X			

A in Cover:

	B	C	D	E	F	G	H
0101	X	X					
0111		X	X				
1000						X	X
1010					X	X	
1110				X	X		
1111			X	X			

C dominates B
G dominates H

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Example

	A	B	C	D	E	F	G	H
0000	X							X
0001	X	X						
0101		X	X					
0111			X	X				
1000							X	X
1010						X	X	
1110					X	X		
1111				X	X			

What now?

A not in Cover:

	B	C	D	E	F	G	H
0000							X
0001	X						
0101	X	X					
0111		X	X				
1000						X	X
1010					X	X	
1110				X	X		
1111			X	X			

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Basic Two-Level Minimization (espresso-exact)

- Generate Prime Implicants
- Reduce (essential, dominators)
- If not done,
  - pick a cube
  - branch (back to reduce) on selected/not
    - *i.e.* search tree … branch and bound
- Save smallest

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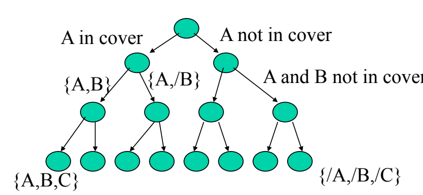
Branching Search



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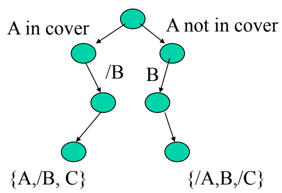
Branching Search w/ Implications



Implications Prune Tree
(like BCP in SAT)

Only exponential in decision where must branch

Covering Technique?

- Possibly useful for dataflow subgraph selection? (Day 15)
  – Treat application components as rows (minterms)
  – Treat patterns as columns (PIs)
- But, more general (complicated) cost model

Optimization

- Summarize Minterms (signature cubes)
  – rows represent collection of minterms with same primes
- Avoid generating full set of PIs
  – pre-combining dominators during generation
- Branch-and-bound pruning
  – get lower bound on remaining cost of a cover by computing independent set of primes
    - (not necessarily maximal, that would be NP-hard)

Heuristic Variant

- Don't backtrack when select prime for inclusion/exclusion
  – pick cover large set of minterms/signatures
  – weight to select "hard" to cover signatures
- Generate reduced set of PIs
- Iterative improvement

Canonical Form

- Can start with *any* form of logical expression
- Get unique truth-table/minterms
- Problem not sensitive to input statement
  – compare covering (decomposition)
  – compare sequential programming languages
- **Cost:** potentially exponential explosion in minterms/PIs

Summary

- Formulate as covering problem
- Solution space restricted to PIs
- Essentials must be in solution
- Use dominators to further reduce space
- Then branching/pruning to explore rest of PIs
- Ways to reduce work
  – group minterms/PIs together early
  – mostly fall into this general scheme

Big Ideas

- Canonical Form
  - eliminate bias of input specification
- Technique:
  - branch-and-bound
  - pruning search – exploit structure
  - Dominators

Admin

- Assign5b due today
- Reading for Wednesday online (web)
- Withdraw date is Friday
  - Probably won't get anything further graded
- Assign 6 & 7 out today
- No office hours on Tuesday
  - Makeup office hours Wed. 5pm
  - …or send email, make appointment