

# ESE535: Electronic Design Automation

Day 25: April 17, 2013  
Covering and Retiming



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## Previously

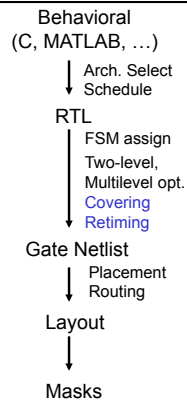
- Cover (map) LUTs for minimum delay
  - solve optimally for delay → flowmap
- Retiming for minimum clock period
  - solve optimally

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2

## Today

- Solving cover/retime separately **not** optimal
- Cover+retime



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3

## Preclass 1

	Circuit	3-LUTs?	critical path
A			
B			
C			

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4

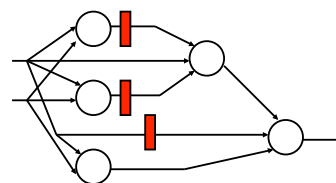
## Preclass 2

	Circuit	3-LUTs?	critical path
A			
B			

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5

## Example

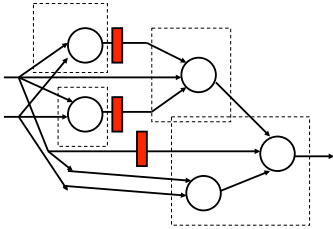


Cover with 4-LUTs

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6

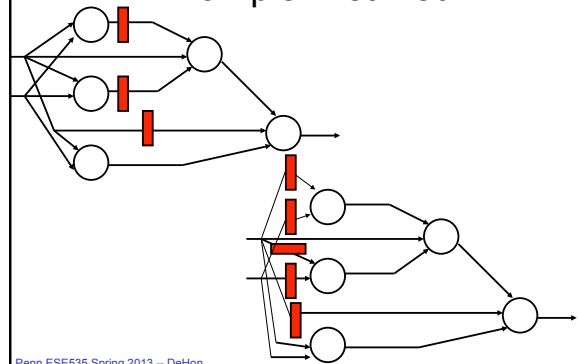
## Example



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7

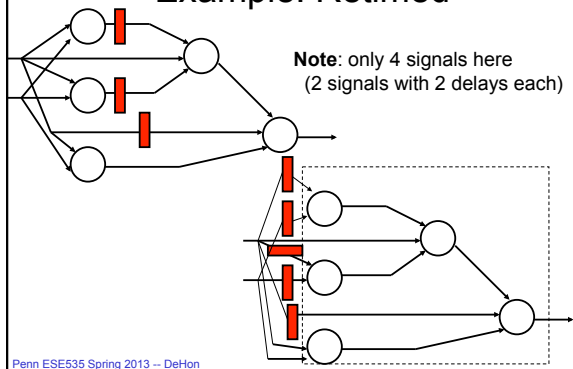
## Example: Retimed



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8

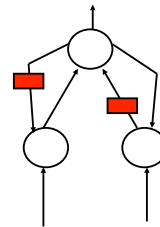
## Example: Retimed



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9

## Example 2

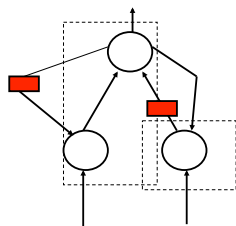


Cover with 4-LUTs

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10

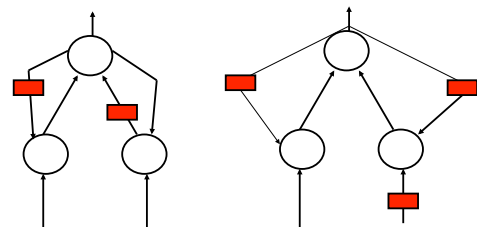
## Example 2



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11

## Example 2: retimed

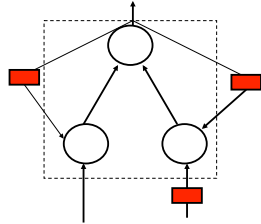


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12

## Example 2: retimed

Cycle Bound: 1

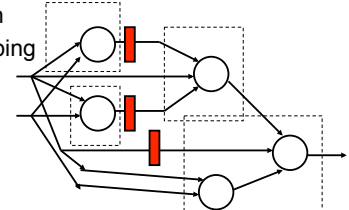


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13

## Basic Observation

- Registers break up circuit, limiting coverage
  - fragmentation
  - prevent grouping



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14

## Phase Ordering Problem

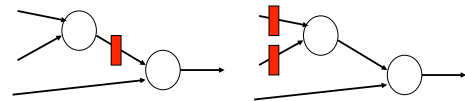
- General problem
  - don't know effect of other mapping step
  - Have seen this many places
- Here
  - don't know delay if retime first
    - don't know what can be packed into LUT
  - If we do not retime first
    - fragmentation: forced breaks at bad places

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15

## Observation #1

- Retiming flops to input of (fanout free) subgraph is trivial (and always doable)



- Does not change I/O into subgraph

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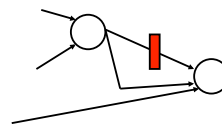
## Observation #1: Consequence

- Can cover *ignoring* flop placement
- Then retime flops to input of gates

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17

## Fanout Problem?

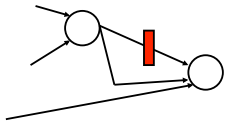


Can we use the same trick here?

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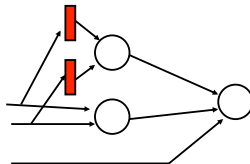
18

## Fanout Problem?



Cannot retime without replicating.

Replicating increases I/O (so cut size).



...but I/O is what defined feasible covers for LUTs

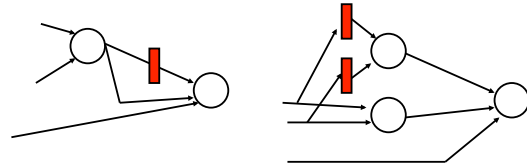
3 cut  $\rightarrow$  5 cut

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## Observation #2

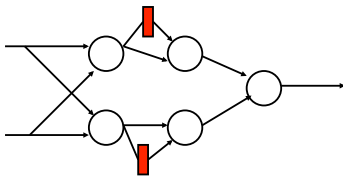
- Retiming flops to input of a subgraph with fanout may change the subgraph I/O



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20

## Different Replication Problem

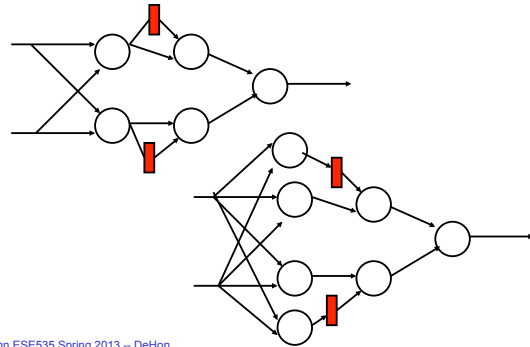


What does this do to I/O?

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21

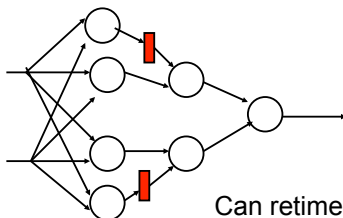
## Different Replication Problem



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22

## Different Replication Problem



Can retime and cover with single 4-LUT.

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23

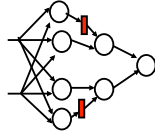
## Replication

- Once add registers
  - can't just grab max flow and get replication
    - (compare flowmap)
- Or, can't just ignore flop placement when have reconvergent fanout through flop

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24

## Replication



- Key idea:
  - represent timing paths in graph
  - differentiating based on number of registers in path
- **new graph**: all paths from node to output have same number of flip-flops
- label nodes  $u^d$  where  $d$  is flip-flops to output

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25

## Deal with Replication

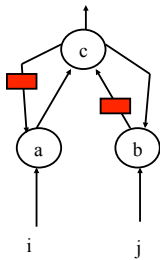
- **Expanded Graph**:
  - start with target output node
  - for each input  $u^d$  to current expanded graph
    - grab its input edge ( $x \rightarrow u$ ) with weight ( $w(e)$ )
    - add node  $x^{(d+w(e))}$  to graph (if necessary)
    - add edge  $x^{(d+w(e))} \rightarrow u^d$  with weight ( $w(e)$ )
  - continue breadth first until have enough
    - at most  $k \times n$  node depth required

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26

## Example

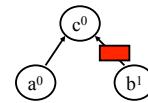
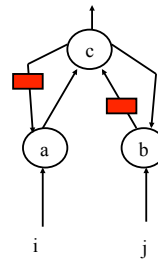
Build expanded graph



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27

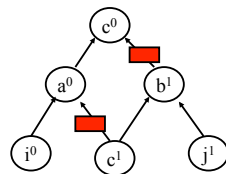
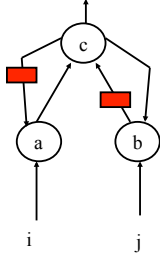
## Example



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28

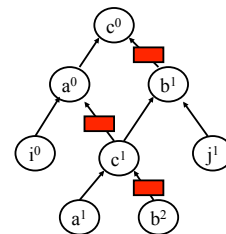
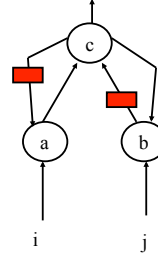
## Example



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29

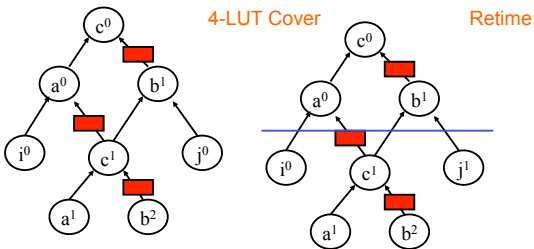
## Example



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30

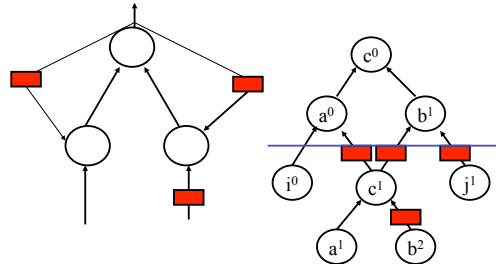
## Example



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31

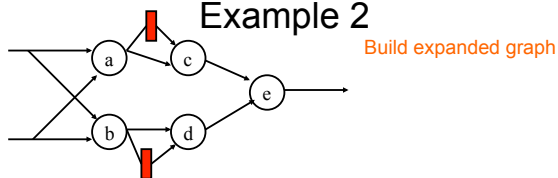
## Example



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32

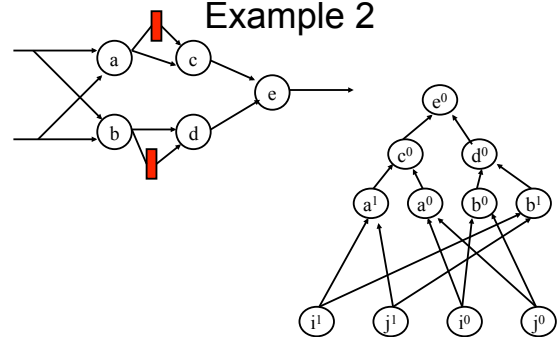
## Example 2



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33

## Example 2



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## Expanded Graph

- Expanded graph does not have fanout of different flip-flop depths from the *same* node.
  - Captures IO after register retiming
- Can now cover ignoring flip-flops and trivially retime.

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## Intuition on Solution

- Phase ordering problem arise form
  - need to capture I/O effects before covering
  - but also need to model delay for register movement
    - But don't know register movement until after covering
- So, break retime into two pieces
  - Expanded graph (capture I/O)
  - Actual retime (moves registers)
- Do expanded graph piece before cover and register movement after

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36

## Intuition on Solution

- Break retime into two pieces
  1. Expanded graph (capture I/O)
  2. Actual retime (moves registers)
- Do expanded graph piece before cover and register movement after
- Not quite that simple since how much of expanded graph need depends on covering
  - So really doing just-in-time expansion in the middle of covering...
    - Before each cover/cut computation

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37

## Labeling

- Key idea #1:
  - compute distances/delay like flowmap
    - Try collapse and compute flow cut
    - Dynamic programming to compute min delay covers
- Key idea #2:
  - count distance from register
    - like  $G-1/c$  graph

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## Labeling: Edge Weights

- To target clock period  $c$ 
  - use graph  $G-1/c$
  - paper:
    - assign weight  $-c \cdot w(e) + 1$
    - (same thing scaled by  $c$  and negated)

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39

## Labeling: Edge Weight Idea

- same idea:
  - will need register ever  $c$  LUT delays
  - credit with registers as encounter
  - charge a fraction  $(1/c)$  every LUT delay
  - know net distance at each point
  - if negative (delays  $> c \cdot \text{registers}$ )
    - cannot distribute to achieve  $c$
  - otherwise
    - labeling tells where to distribute

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## Labeling: Flow cut

- Label node as before (flowmap)
  - $L(v) = \min\{l(u) + d \mid \exists u \rightarrow v\}$
  - trivially can be  $L(v) - 1/c == \text{new LUT}$ 
    - Correspond to flowmap case:  $L(v) + 1$
    - note min vs. max and  $-1/c$  vs.  $+1$  due to rescaling to match retiming formulation and  $G-1/c$  graph
    - in this formulation, a combinational circuit of depth 4 would have  $L(v) = -4/c$
  - if can put this and all  $L(v)$ 's in one LUT
    - this can be  $L(v)$
    - construct and compute flow cut to test

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41

## LUT Map and Retime

- Start with outputs
- Cover with LUT based on cut
  - move flip-flops to inputs of LUT
    - don't have meaningful labels for covered nodes
    - Know can do this by expanded graph construction
- Recursively cover inputs
- Use label to retime
 
$$r(v) = \lceil l(v) \rceil - 1$$

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42

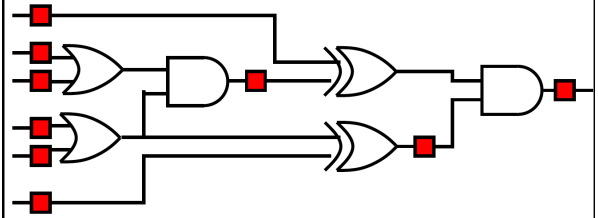
## Target Clock Period $c$

- As before (retiming)
  - binary search to find optimal  $c$

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43

## Example



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44

## Summary

- Can optimally solve
  - LUT map for delay
  - retiming for minimum clock period
- But, solving separately does not give optimal solution to problem
- Can solve problems together
  - Account for registers on paths
  - Label based on register placement and (flow) cover ignoring registers
  - Labeling gives delay, covering, retiming

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## Today's Big Ideas

- Exploit freedom
- Cost of decomposition
  - benefit of composite solution
- Technique:
  - dynamic programming
  - network flow

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## Admin

- Monday reading online
- HW7 final due Monday

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47