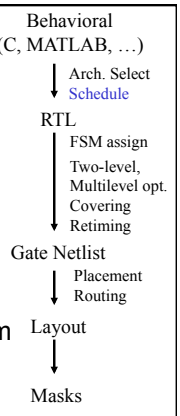
ESE535: Electronic Design Automation

Day 5: January 28, 2013
Scheduling
Variants and Approaches

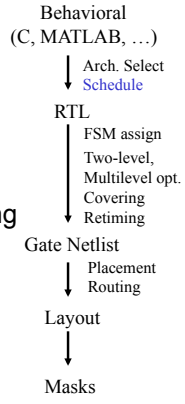
Previously

- Resources aren't free
- Share to reduce costs
- Schedule operations on resources
  - Fixed resources
- Greedy approximation algorithm
- List Scheduling for resource-constrained scheduling

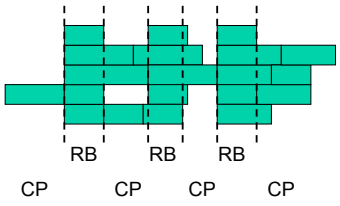


Today

- Tighter bounds on List Scheduling
- Time-Constrained Scheduling
  - Force Directed
- Few words on project architecture
- Resource-Constrained
  - Branch-and-Bound



Precedence



Precedence Constrained

- Optimal Length > All busy times
  - Optimal Length ≥ Resource Bound
  - Resource Bound ≥ All busy
- Optimal Length>This Path
  - Optimal Length ≥ Critical Path
  - Critical Path ≥ This Path
- List Schedule = This path + All busy times
- List Schedule ≤ 2 *(Optimal Length)

Conclude

- Scheduling of identical parallel machines with precedence constraints has a 2-approximation.

Tightening

- How could we do better?
- What is particularly pessimistic about the previous cases?
  - List Schedule = This path + All busy times
  - List Schedule ≤ 2 *(Optimal Length)

Tighten

- LS schedule ≤ Critical Path+Resource Bound
- LS schedule ≤ Min(CP,RB)+Max(CP,RB)
- Optimal schedule ≥ Max(CP,RB)
- LS/Opt ≤ 1+Min(CP,RB)/Max(CP,RB)
- The more one constraint dominates
  - ➔the closer the approximate solution to optimal
  - ↳(EEs think about 3dB point in frequency response)

Tightening

- Example of
  - More information about problem
  - More internal variables
  - …allow us to state a tighter result
- 2-approx for any graph
  - Since CP may = RB
- Tighter approx as CP and RB diverge

Multiple Resource

- Previous result for homogeneous functional units
- For heterogeneous resources:
  - also a 2-approximation
    - Lenstra+Shmoys+Tardos, Math. Programming v46p259
    - (not online, no precedence constraints)

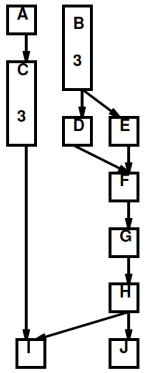
Bounds

- Precedence case, Identical machines
  - no polynomial approximation algorithm can achieve better than 4/3 bound
    - (unless P=NP)
- Heterogeneous machines (no precedence)
  - no polynomial approximation algorithm can achieve better than 3/2 bound

Preclass

Preclass

- Critical Path LB?
- Resources to keep RB < CP ?
- Resources to achieve CP?
  - Take poll: 4, 3, 2, 1
- What was trick to achieving?
- Why might List Schedule have a problem with this?



Force Directed

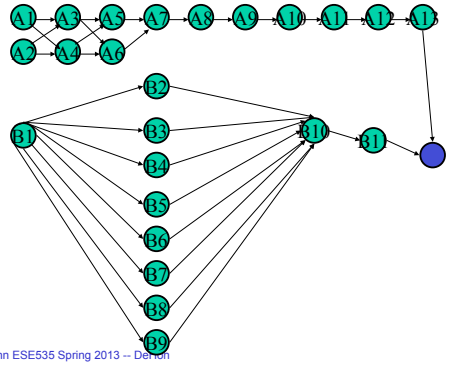
Force-Directed

- **Problem**: how exploit schedule freedom (slack) to minimize instantaneous resources
  - Directly solve time-constrained scheduling
    - (previously only solved indirectly)
  - Minimize resources with timing target

Force-Directed

- Given a node, can schedule anywhere between ASAP and ALAP schedule time
  - Between latest schedule predecessor and ALAP
  - Between ASAP and already scheduled successors
  - Between latest schedule predecessor and earliest schedule successor
- *That is:* Scheduling node will limit freedom of nodes in path

Single Resource Challenge



Force-Directed

- If everything where scheduled, **except** for the target node, what would we do?:
  - examine resource usage in all timeslots allowed by precedence
  - place in timeslot that has least increase maximum resources
    - Least energy
    - Where the forces are pulling it

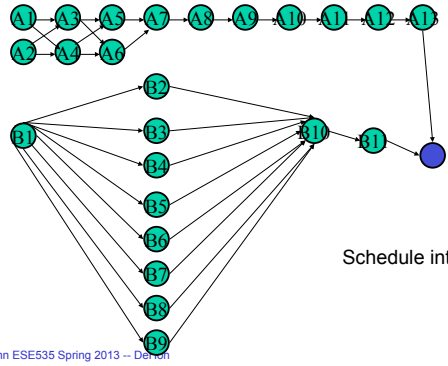
Force-Directed

- **Problem:** don't know resource utilization during scheduling
- **Strategy:** estimate resource utilization

Force-Directed Estimate

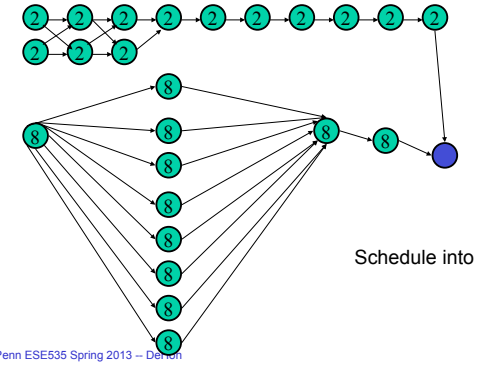
- Assume a node is uniformly distributed within slack region
  - between earliest and latest possible schedule time
- Use this estimate to identify most used timeslots

Single Resource Challenge



Schedule into 12 cycles

Slacks on all nodes



Schedule into 12 cycles

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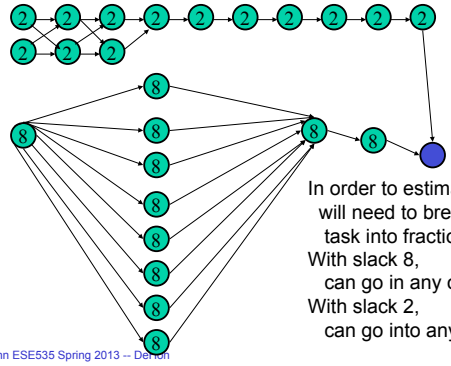
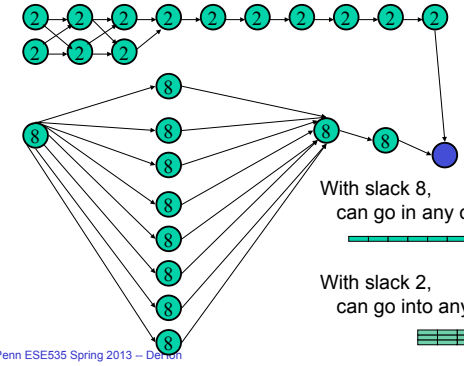


In order to estimate,
will need to break each
task into fractions.
With slack 8,
can go in any of 9 slots.
With slack 2,
can go into any of 3 slots.

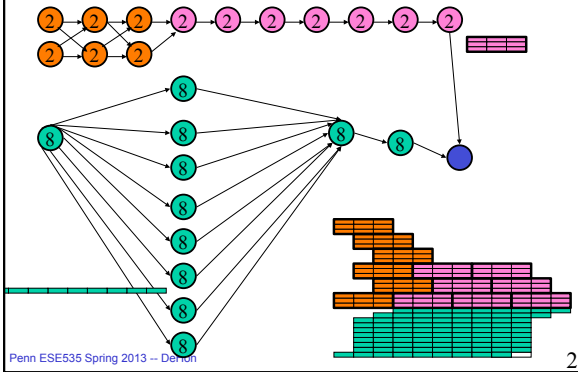
Slacks on all nodes



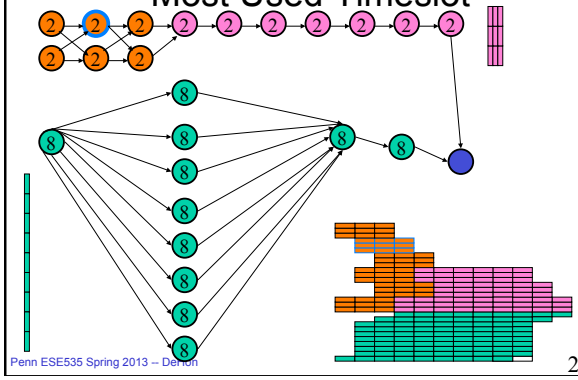
With slack 8,
can go in any of 9 slots.

With slack 2,
can go into any of 3 slots.

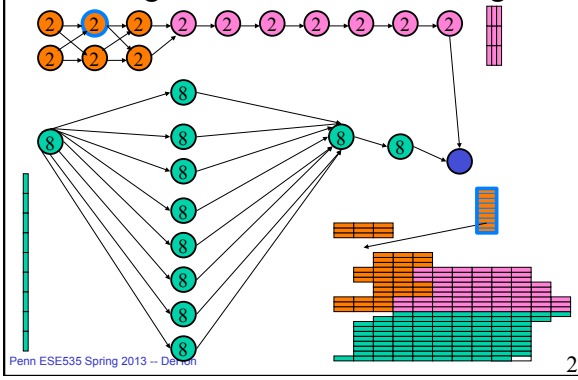
Uniform Distribution of Slack



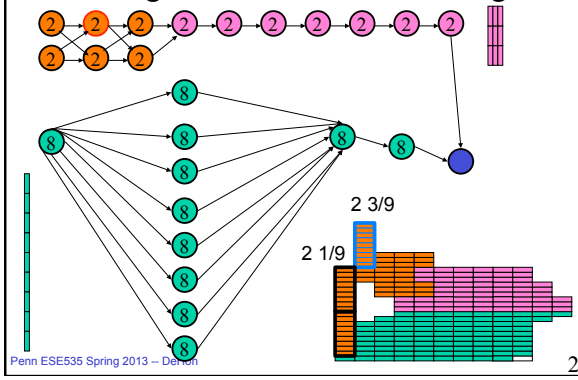
Most Constrained Node, Most Used Timeslot



Single Resource Challenge



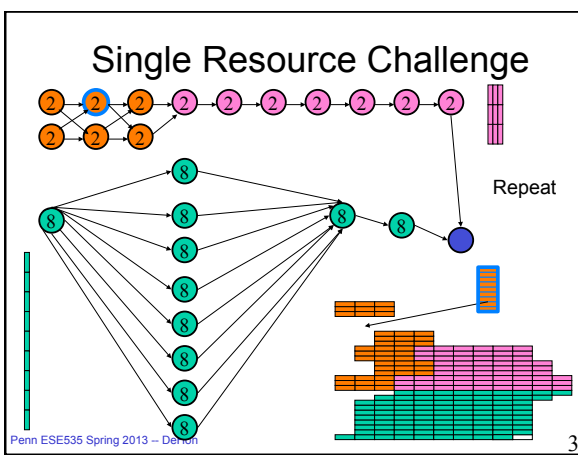
Single Resource Challenge

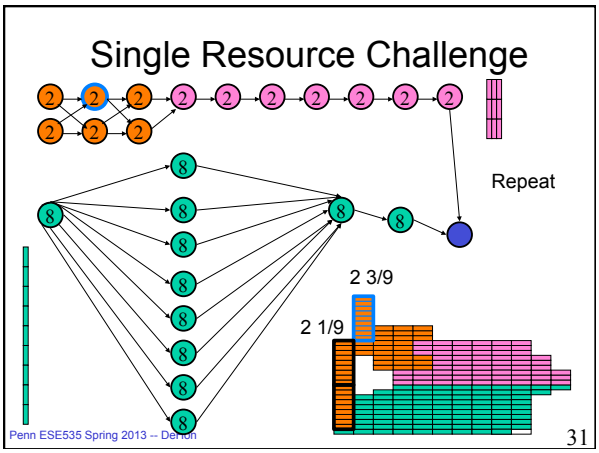


Force-Directed

- Scheduling a node will shift distribution
  - all of scheduled node's cost goes into one timeslot
  - predecessor/successors may have freedom limited so shift their contributions
- **Goal:** shift distribution to minimize maximum resource utilization (estimate)

Single Resource Challenge




Single Resource Challenge
Repeat
2 3/9
2 1/9
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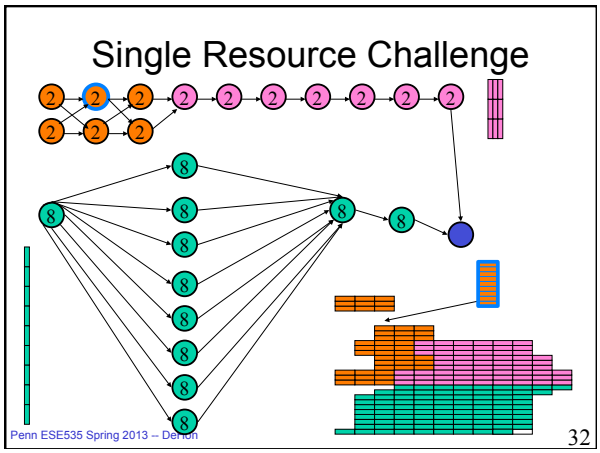
31


Single Resource Challenge
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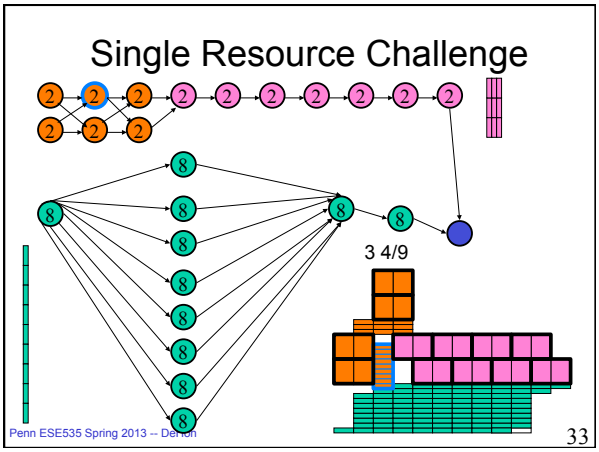
Single Resource Challenge
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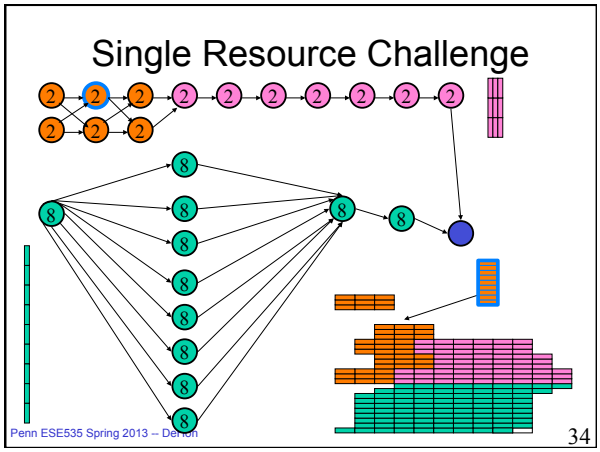
33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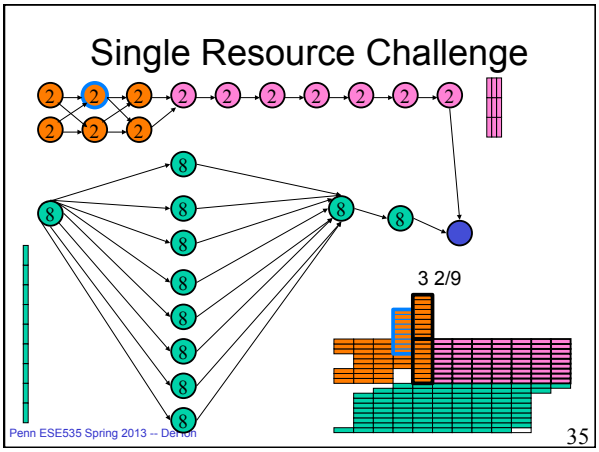
Single Resource Challenge
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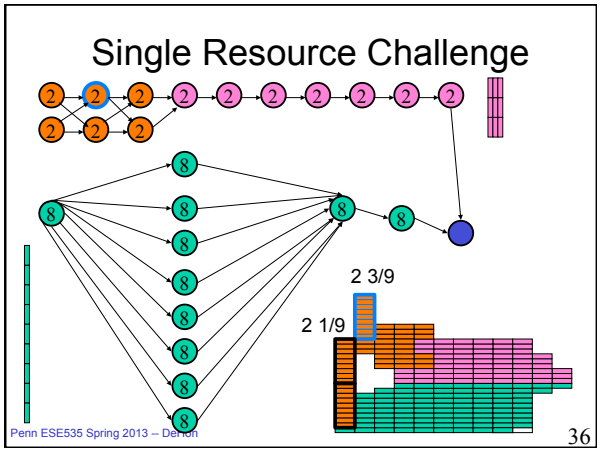
35


Single Resource Challenge
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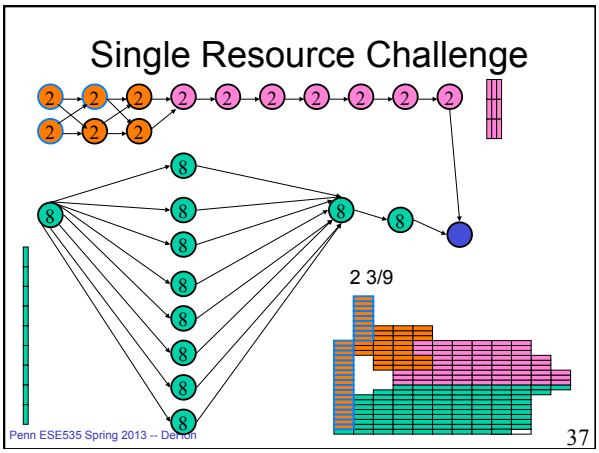
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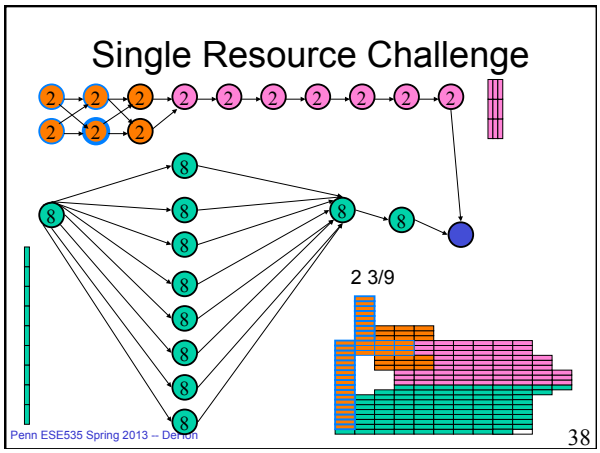
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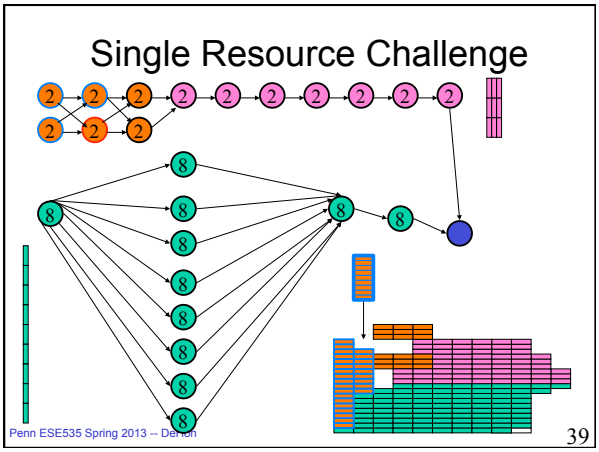
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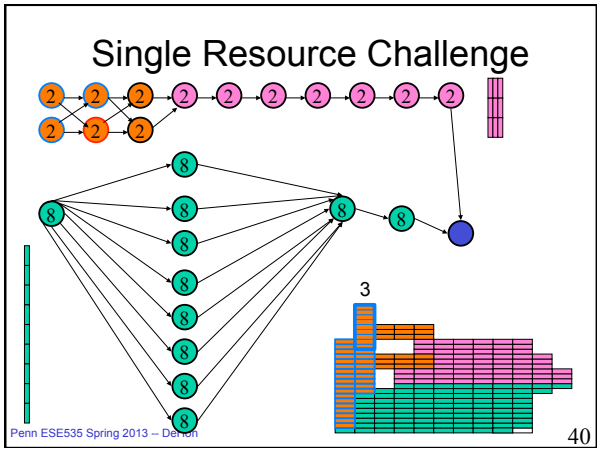
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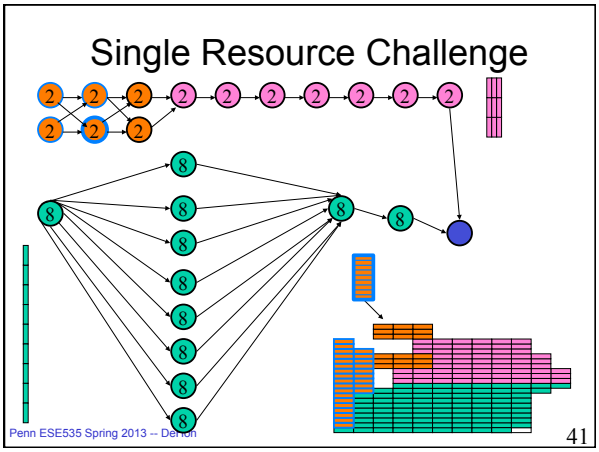
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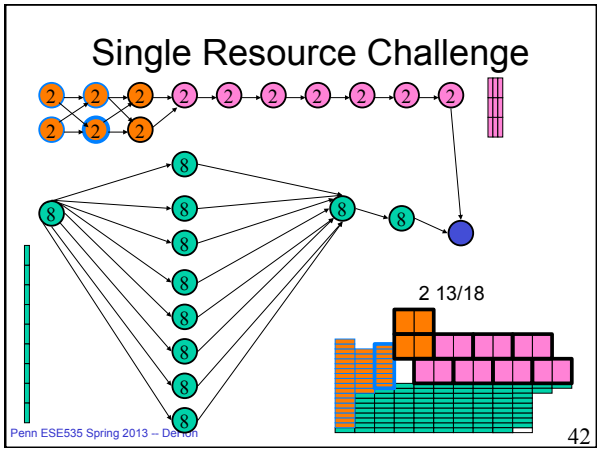
Single Resource Challenge
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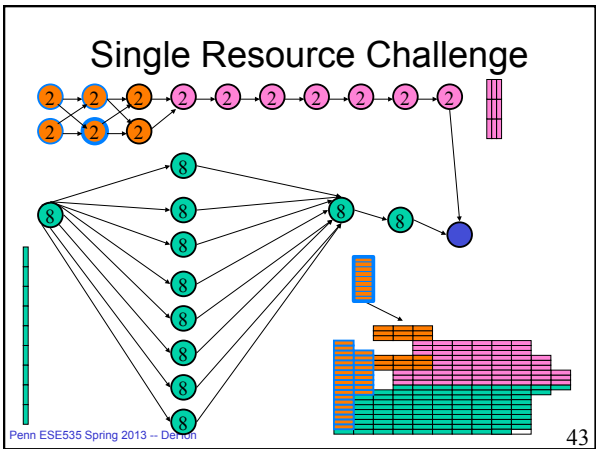
42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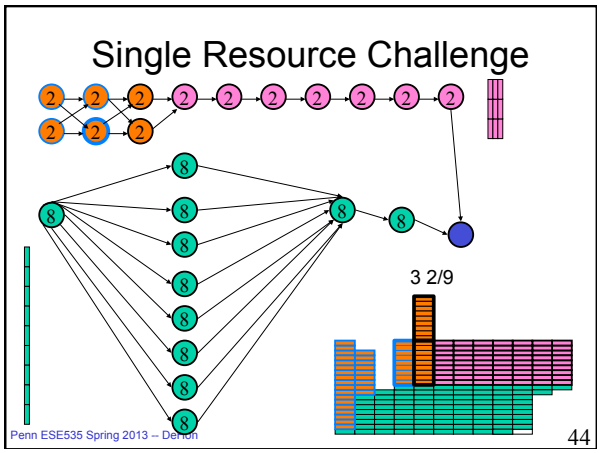
Single Resource Challenge
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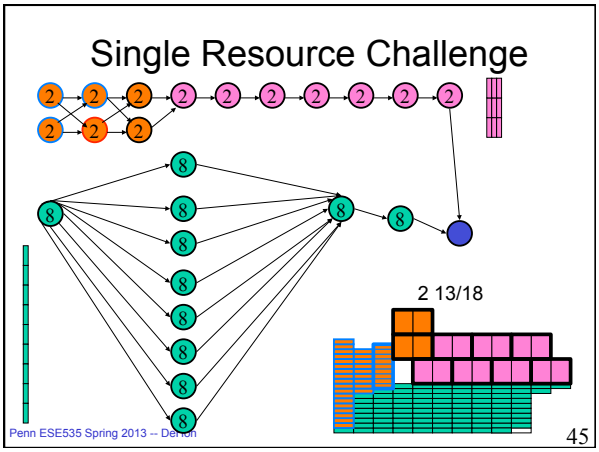
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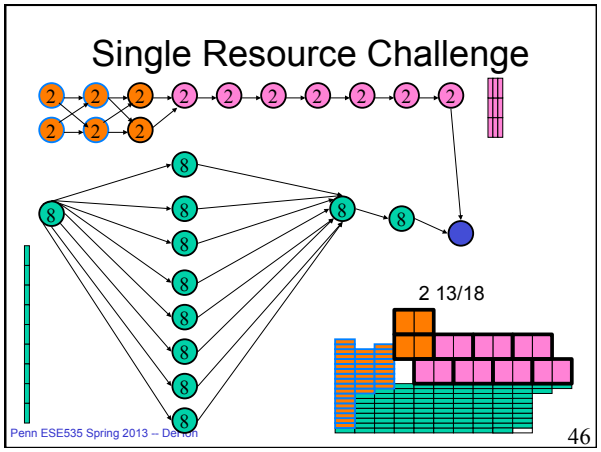
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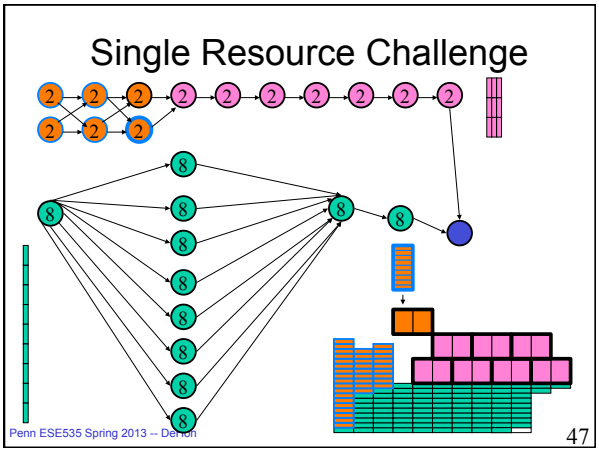
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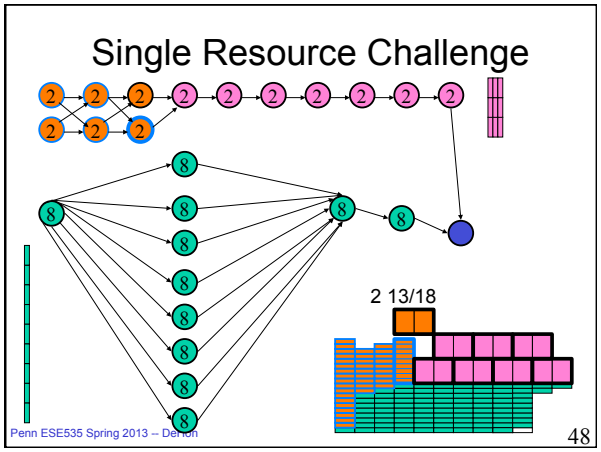
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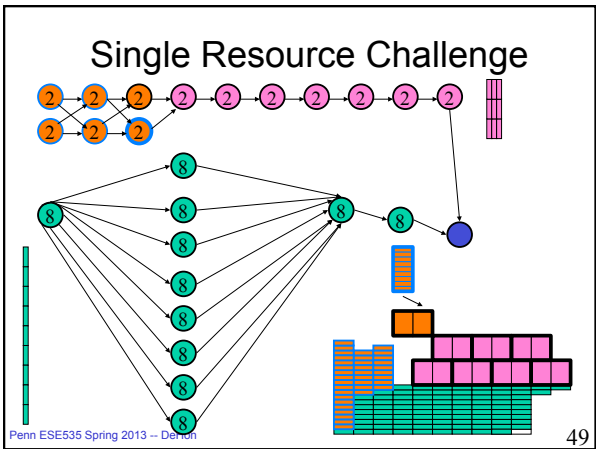
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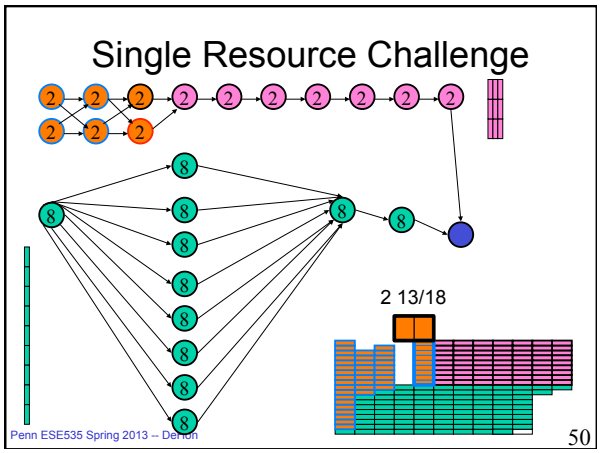
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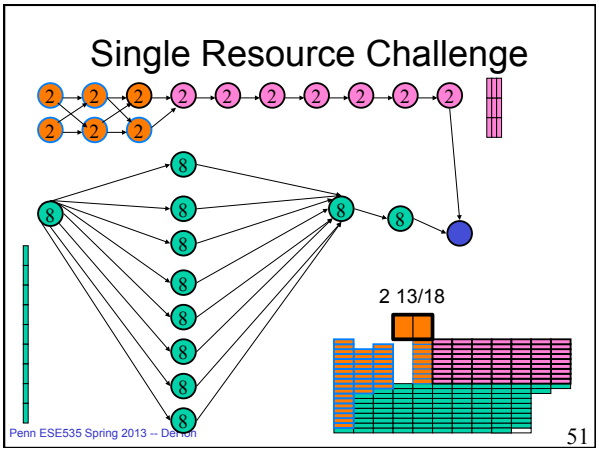
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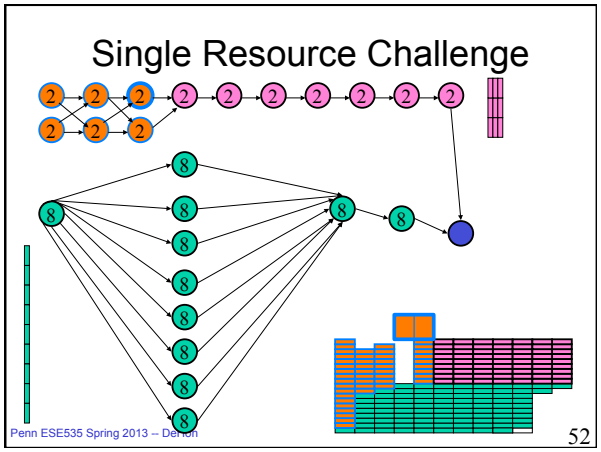
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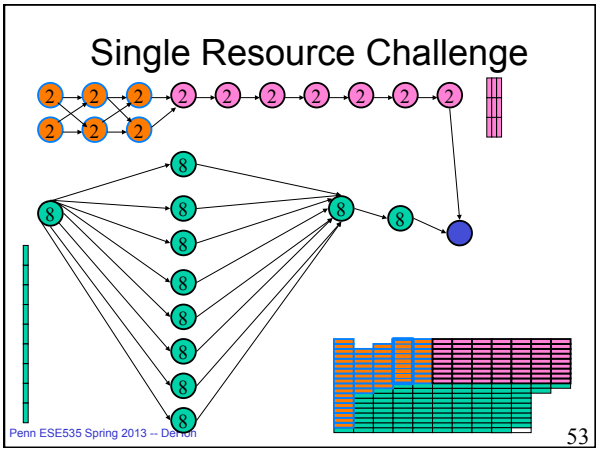
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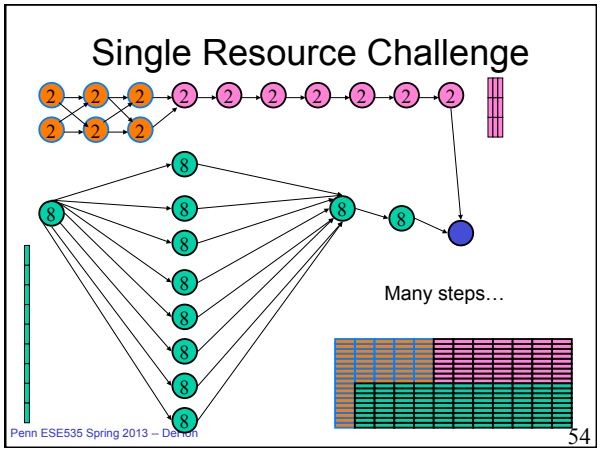
Single Resource Challenge
Many steps…
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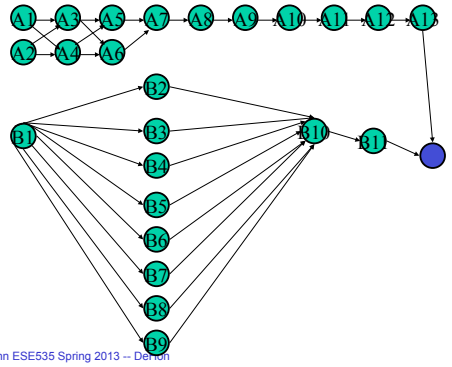
54

Single Resource Hard (5)



A1 A2
A3 B1
A4 B2
A5 B3
A6 B4
A7 B5
A8 B6
A9 B7
A10 B8
A11 B9
A12 B10
A13 B11

Force-Directed Algorithm

1. ASAP/ALAP schedule to determine range of times for each node
2. Compute estimated resource usage
3. Pick most constrained node (in largest time slot…)
   - Evaluate effects of placing in feasible time slots (compute forces)
   - Place in minimum cost slot and update estimates
   - Repeat until done

Force-Directed Runtime

- Evaluate force of putting in timeslot O(N)
  - Potentially perturbing slack on net prefix/ postfix for this node → N
- Each node potentially in T slots: ×T
  - T = schedule target
- N nodes to place: ×N
- $O(N^2T)$
  - Loose bound--don't get both T slots and N perturbations

Force-Directed Algorithm (from reading)

1. ASAP/ALAP schedule to determine range of times for each node
2. Compute estimated resource usage
3. Select a move
   - For each unscheduled op
     - Evaluate effects of placing in feasible time slots (compute forces)
   - Select move results in minimum cost
   - Repeat until done

Force-Directed Runtime (from reading)

- Evaluate force of putting in timeslot O(N)
  - Potentially perturbing slack on net prefix/ postfix for this node → N
- Each selection N nodes to consider: ×N
- Each node potentially in T slots: ×T
  - T = schedule target
- N nodes to place: ×N
- $O(N^3T)$
  - Loose bound

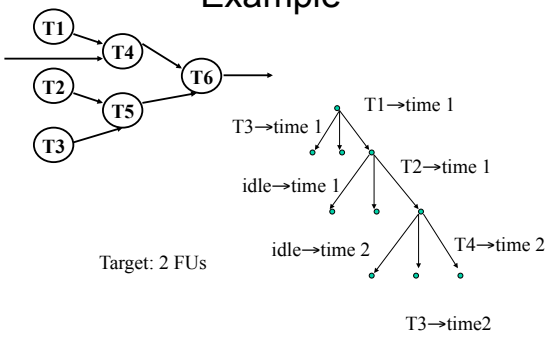
Branch-and-Bound

(for resource-constrained scheduling)

Brute-Force Scheduling (Exhaustive Search)

- Try all schedules
- Branching/Backtracking Search
- Start w/ nothing scheduled (ready queue)
- At each move (branch) pick:
  - available resource time slot
  - ready task (predecessors completed)
  - schedule task on resource
    - Update ready queue

Example



Branching Search

- Explores entire state space
  - finds optimum schedule
- Exponential work
  - O ($N^{(resources*time-slots)}$)
- Many schedules completely uninteresting

Reducing Work

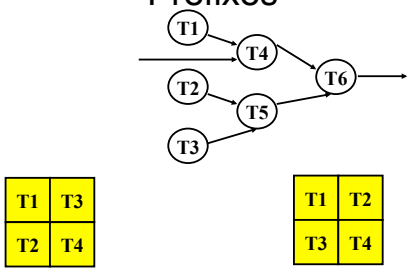
1. Canonicalize "**equivalent**" schedule configurations
2. Identify "**dominating**" schedule configurations
3. **Prune** partial configurations which will lead to worse (or unacceptable results)

"Equivalent" Schedules

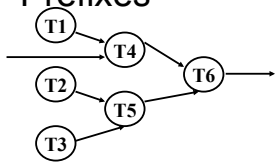
- If multiple resources of same type
  - assignment of task to particular resource at a particular timeslot is not distinguishing

Keep track of resource usage by capacity at time-slot.

"Equivalent" Schedule Prefixes



"Non-Equivalent" Schedule Prefixes

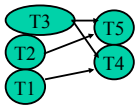


Pruning Prefixes

- Keep track of scheduled set
- Recognize when solving same sub-problem
  - Like dynamic programming finding same sub-problems
  - But no guarantee of small number of subproblems
    - set is power-set so 2^N
    - …but not all feasible,
      - so shape of graph may simplify

Dominant Schedules

- A strictly shorter schedule
  - scheduling the same or more tasks
  - will always be superior to the longer schedule



Pruning

- If can establish a particular schedule path will be worse than one we've already seen
  - we can discard it w/out further exploration
- In particular:
  - LB=current schedule time + lower_bound_estimate
  - if LB greater than known solution, prune

Pruning Techniques

Establish Lower Bound on schedule time

- Critical Path (ASAP schedule)
- Resource Bound

Alpha-Beta Search

- Generalization
  - keep both upper and lower bound estimates on partial schedule
    - Lower bounds from CP, RB
    - Upper bounds with List Scheduling
  - expand most promising paths
    - (least upper bound, least lower bound)
  - prune based on lower bounds exceeding known upper bound
  - (technique typically used in games/Chess)

Alpha-Beta

- Each scheduling decision will tighten
  - lower/upper bound estimates
- Can choose to expand
  - least current time (breadth first)
  - least lower bound remaining (depth first)
  - least lower bound estimate
  - least upper bound estimate
- Can control greediness
  - weighting lower/upper bound
  - selecting "most promising"

Note

- Aggressive pruning and ordering
  - can sometimes make polynomial time in practice
  - often cannot *prove* will be polynomial time
  - usually represents problem structure we still need to understand
- Coudert shows scheduling
  - Exact Coloring of Real-Life Graphs is Easy", in Proc. of *34th DAC, Anaheim, CA, June 1997.*

Multiple Resources

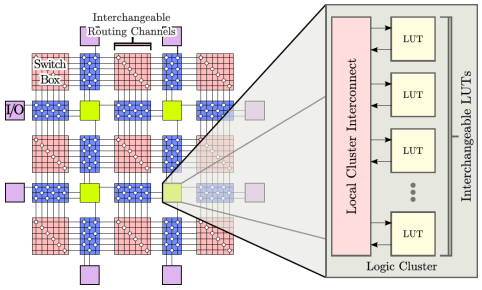
- Works for multiple resource case
- Computing lower-bounds per resource
  - resource constrained
- Sometimes deal with resource coupling
  - *e.g.* must have 1 A and 1 B simultaneously or in fixed time slot relation
    - *e.g.* bus and memory port

Summary

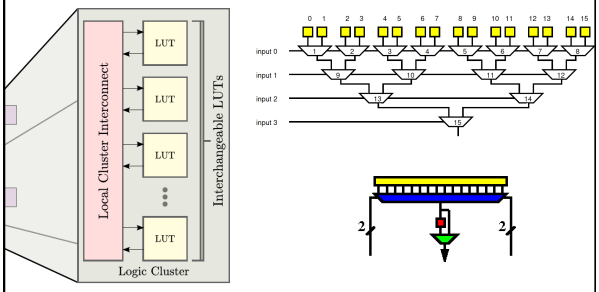
- Resource estimates and refinement
- Branch-and-bound search
  - "equivalent" states
  - dominators
  - estimates/pruning

Project Architecture

FPGA Architecture

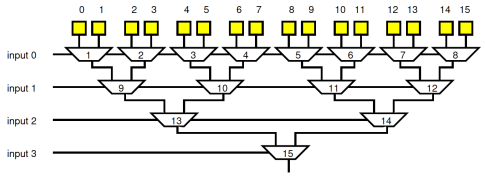


FPGA Lookup Tables (LUTs)



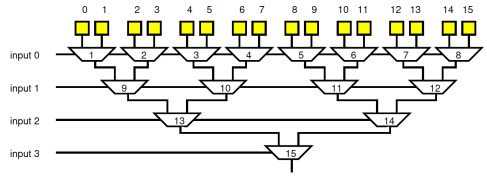
FPGA Lookup Tables (LUTs)

- Common mux failure: cannot switch 0, 1
  - Can hold a constant 0 or 1 output



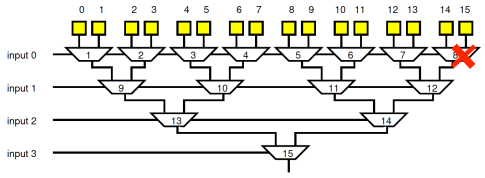
Fully Funtional LUT

- Compute A*B*C*D with:



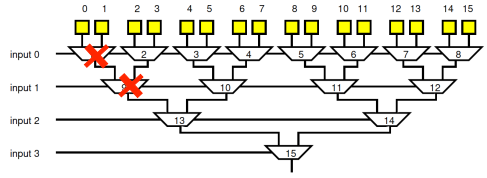
Using Partially Defective LUT

- Compute A*B*C*D with:



Using Partially Defective LUT

- Compute A*B*C*D with:



Big Ideas:

- Estimate Resource Usage
- Use dominators to reduce work
- Techniques:
  - Force-Directed
  - Search
    - Branch-and-Bound
    - Alpha-Beta

Admin

- Assignment 1 was due at class start
- Reading
  - Wednesday online
- Assignment 2 out
  - Part A due next Monday
  - Part B following