

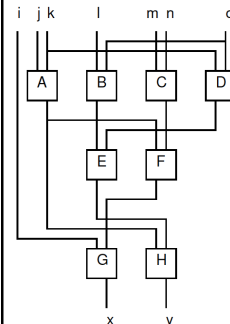
# ESE535: Electronic Design Automation

Day 6: January 30, 2013  
Partitioning  
(Intro, KLFM)



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## Preclass Warmup



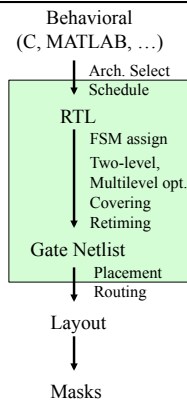
- What cut size were you able to achieve?

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## Today

- Partitioning
  - why important
    - Can be used as tool at many levels
  - practical attack
  - variations and issues



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## Motivation (1)

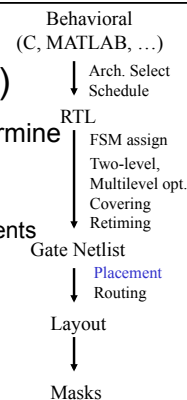
- Divide-and-conquer
  - trivial case: decomposition
  - smaller problems easier to solve
    - net win, if super linear
    - $Part(n) + 2 \times T(n/2) < T(n)$
  - problems with sparse connections or interactions
  - Exploit structure
    - limited cutsizes is a common structural property
    - random graphs would **not** have as small cuts

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## Motivation (2)

- Cut size (bandwidth) can determine
  - Area, energy
- Minimizing cuts
  - minimize interconnect requirements
  - increases signal locality
- Chip (board) partitioning
  - minimize IO
- Direct basis for placement

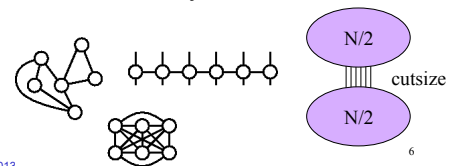


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## Bisection Width

- Partition design into two equal size halves
  - Minimize wires (nets) with ends in both halves
- Number of wires crossing is **bisection width**
- lower bw = more locality

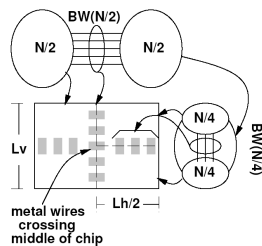


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## Interconnect Area

- Bisection width is lower-bound on IC width
  - When wire dominated, may be tight bound
- (recursively)



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## Classic Partitioning Problem

- **Given:** netlist of interconnect cells
- Partition into two (roughly) equal halves (A,B)
- minimize the number of nets shared by halves
- “Roughly Equal”
  - balance condition:  $(0.5-\delta)N \leq |A| \leq (0.5+\delta)N$

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## Balanced Partitioning

- NP-complete for general graphs
  - [ND17: Minimum Cut into Bounded Sets, Garey and Johnson]
  - Reduce SIMPLE MAX CUT
  - Reduce MAXIMUM 2-SAT to SMC
  - Unbalanced partitioning poly time
- Many heuristics/attacks

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## KL FM Partitioning Heuristic

- Greedy, iterative
  - pick cell that decreases cut and move it
  - repeat
- small amount of non-greediness:
  - look past moves that make locally worse
  - randomization

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## Fiduccia-Mattheyses (Kernighan-Lin refinement)

- Start with two halves (random split?)
- Repeat until no updates
  - Start with all cells free
  - Repeat until no cells free
    - Move cell with largest gain (balance allows)
    - Update costs of neighbors
    - Lock cell in place (record current cost)
  - Pick least cost point in previous sequence and use as next starting position
- Repeat for different random starting points

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## Efficiency

Tricks to make efficient:

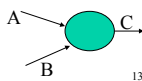
- Expend *little* work picking move candidate
  - Constant work  $\equiv O(1)$
  - Means amount of work not dependent on problem size
- Update costs on move cheaply  $[O(1)]$
- Efficient data structure
  - update costs cheap
  - cheap to find next move

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## Ordering and Cheap Update

- Keep track of Net gain on node == delta net crossings to move a node
  - cut cost after move = cost - gain
- Calculate node gain as  $\Sigma$  net gains for all nets at that node
  - Each node involved in several nets
- Sort nodes by gain
  - Avoid full resort every move

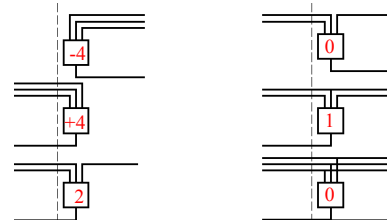


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## FM Cell Gains

Gain = Delta in number of nets crossing between partitions  
= Sum of net deltas for nets on the node



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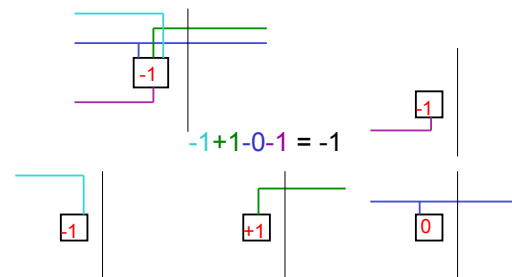
## After move node?

- Update cost
  - Newcost=cost-gain
- Also need to update gains
  - on all nets attached to moved node
  - but moves are nodes, so push to
    - all nodes affected by those nets

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## Composability of Net Gains

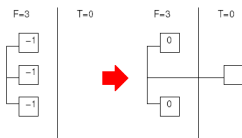


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## FM Recompute Cell Gain

- For each net, keep track of number of cells in each partition [F(net), T(net)]
- Move update:(for each net on moved cell)
  - if  $T(\text{net})=0$ , increment gain on F side of net
    - (think  $-1 \Rightarrow 0$ )

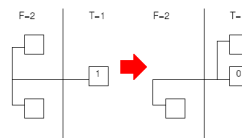


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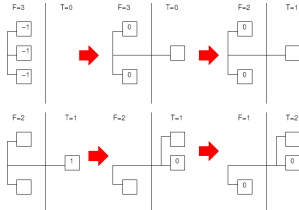


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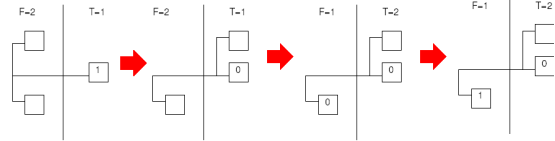


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  - if  $F(\text{net})=1$ , increment gain on F cell

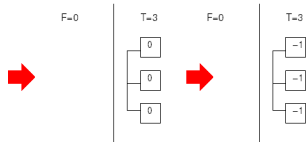


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## FM Recompute Cell Gain

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  - if  $F(\text{net})=1$ , increment gain on F cell
  - if  $F(\text{net})=0$ , decrement gain on all cells (T)



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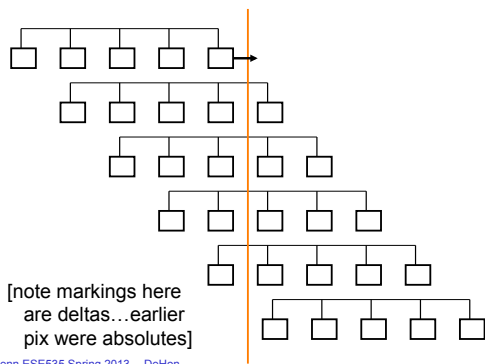
## FM Recompute Cell Gain

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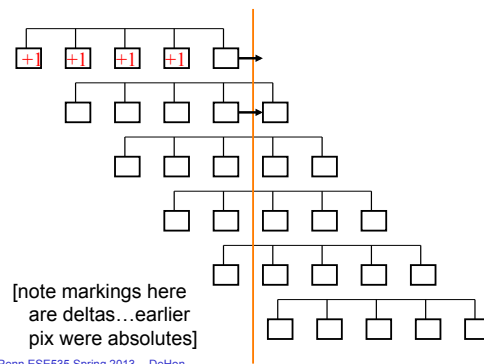
## FM Recompute (example)



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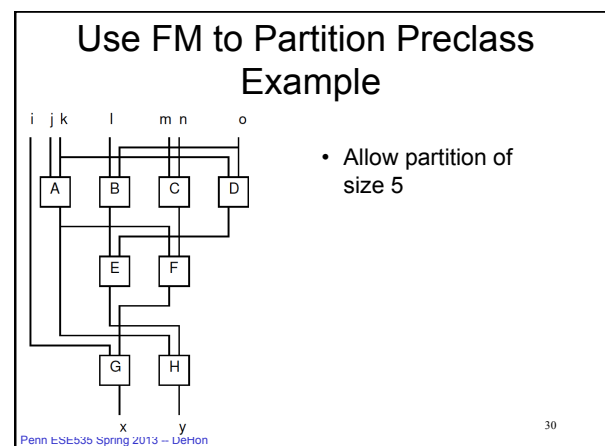
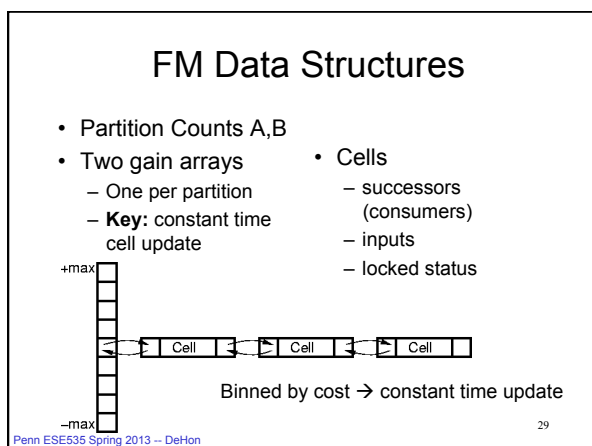
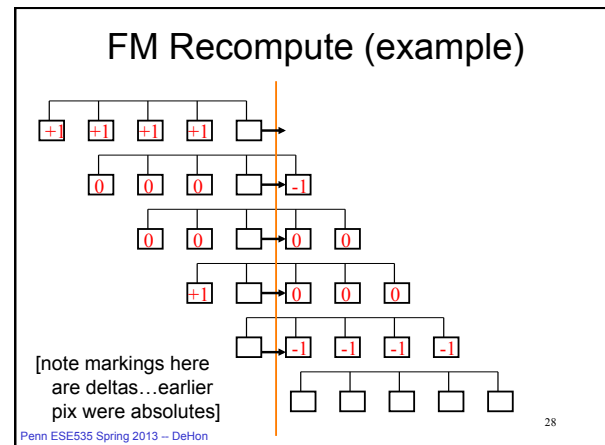
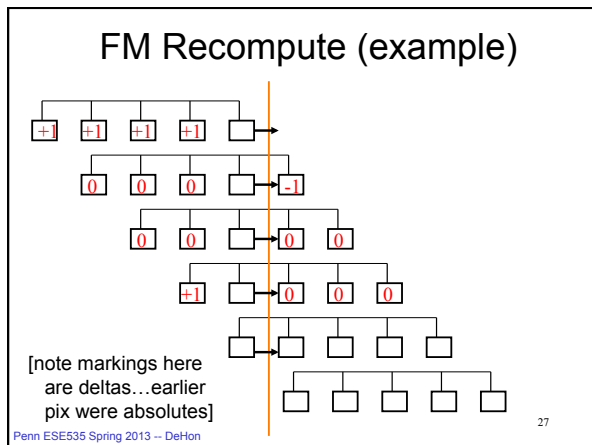
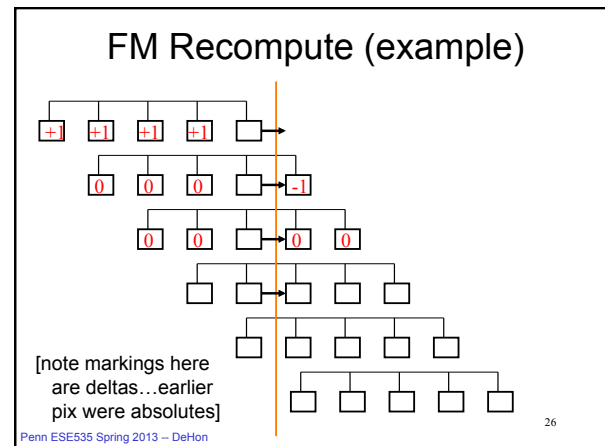
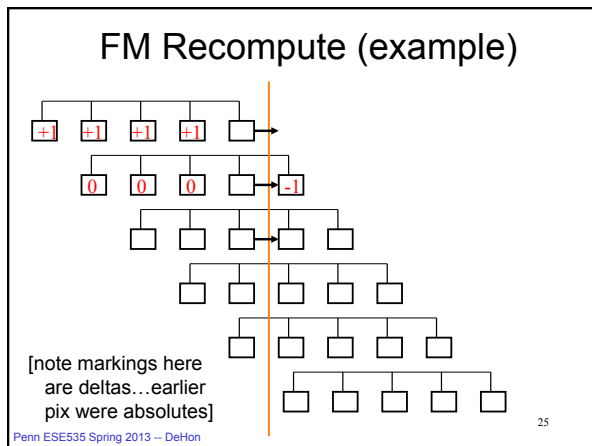
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## FM Recompute (example)



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## FM Optimization Sequence (ex)

+3	+2	-1
+3	+1	-1
+2	0	-1
+2	-1	0
+1	0	+1
0	-1	-1
+1	-1	-1
0	-2	-2
0	-2	-2
-1	-1	-2
+1	0	-1
-1	-1	-2
-2	-2	-1
-2	-3	-3
-3	-3	-3
-3	-3	-3
+12	+3	0

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## FM Running Time?

- Randomly partition into two halves
- Repeat until no updates
  - Start with all cells free
  - Repeat until no cells free
    - Move cell with largest gain
    - Update costs of neighbors
    - Lock cell in place (record current cost)
  - Pick least cost point in previous sequence and use as next starting position
- Repeat for different random starting points

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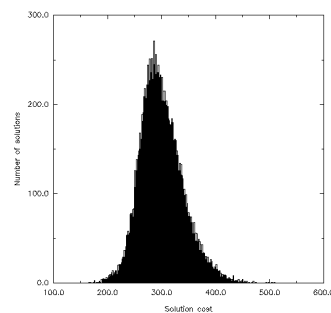
## FM Running Time

- **Assume:**
  - constant number of passes to converge
  - constant number of random starts
- N cell updates each round (swap)
- Updates K + fanout work (avg. fanout K)
  - assume at most K inputs to each node
  - For every net attached (K+1)
    - For every node attached to those nets (O(K))
- Maintain ordered list O(1) per move
  - every io move up/down by 1
- Running time: O(K<sup>2</sup>N)
  - Algorithm significant for its speed
    - (more than quality)

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## FM Starts?



So, FM gives a **not bad** solution quickly

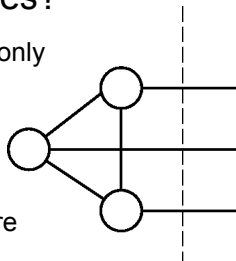
21K random starts, 3K network -- Alpert/Kahng

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## Weaknesses?

- Local, incremental moves only
  - hard to move clusters
  - no lookahead
  - Stuck in local minima?
- Looks only at local structure



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## Improving FM

- Clustering
- Initial partitions
- Runs
- Partition size freedom

Following comparisons from Hauck and Boriello '96

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## Clustering

- Group together several leaf cells into cluster
- Run partition on clusters
- Uncluster (keep partitions)
  - iteratively
- Run partition again
  - using prior result as starting point
    - instead of random start

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## Clustering Benefits

- Catch local connectivity which FM might miss
  - moving one element at a time, hard to see move whole connected groups across partition
- Faster (smaller N)
  - METIS -- fastest research partitioner exploits heavily

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## How Cluster?

- Random
  - cheap, some benefits for speed
- Greedy “connectivity”
  - examine in random order
  - cluster to most highly connected
  - 30% better cut, 16% faster than random
- Spectral (next week)
  - look for clusters in placement
  - (ratio-cut like)
- Brute-force connectivity (can be  $O(N^2)$ )

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## Initial Partitions?

- Random
- Pick Random node for one side
  - start imbalanced
  - run FM from there
- Pick random node and Breadth-first search to fill one half
- Pick random node and Depth-first search to fill half
- Start with Spectral partition

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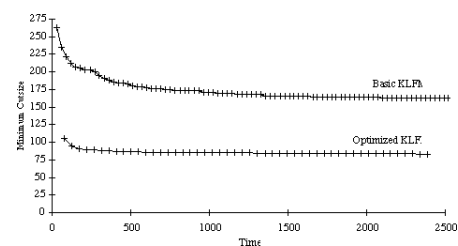
## Initial Partitions

- If run several times
  - pure random tends to win out
  - more freedom / variety of starts
  - more variation from run to run
  - others trapped in local minima

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## Number of Runs

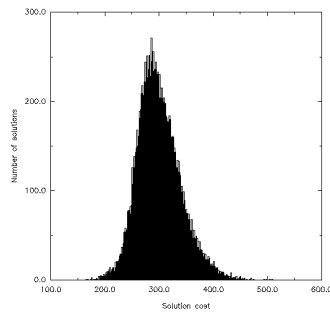


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## Number of Runs

- 2 - 10%
- 10 - 18%
- 20 < 20%
- 50 < 22%
- ...but?

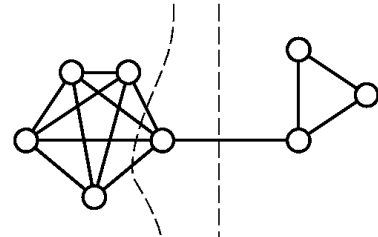


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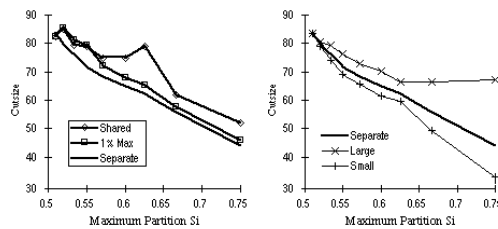
## Unbalanced Cuts

- Increasing slack in partitions  
– may allow lower cut size



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## Unbalanced Partitions



Small/large is benchmark size not small/large partition IO.

Following comparisons from Hauck and Boriello '96

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## Partitioning Summary

- Decompose problem
- Find locality
- NP-complete problem
- linear heuristic (KLFM)
- many ways to tweak  
– Hauck/Boriello, Karypis

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## Today's Big Ideas:

- Divide-and-Conquer
- Exploit Structure  
– Look for sparsity/locality of interaction
- Techniques:  
– greedy  
– incremental improvement  
– randomness avoid bad cases, local minima  
– incremental cost updates (time cost)  
– efficient data structures

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## Admin

- Reading for Monday online
- Assignment 2A due on Monday

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