

ESE535: Electronic Design Automation

Day 2: January 26, 2015
Covering

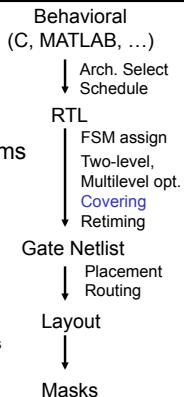
Work preclass exercise



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Today: Covering Problem

- Implement a “gate-level” netlist in terms of some library of primitives
- General Formulation
 - Make it easy to change technology
 - Make it easy to experiment with library requirements
 - Evaluate benefits of new cells...
 - Evaluate architecture with different primitives



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Input

- netlist (logical circuit)
 - library
- represent both in normal form:
 - nand gate
 - inverters

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Elements of a library - 1

Element/Area Cost Tree Representation (normal form)

INVERTER	2	A single input and output terminal with a triangle pointing to the output.	A tree diagram where the root node is an inverter, with one child node being the input and one child node being the output.
NAND2	3	A two-input AND gate with one output.	A tree diagram where the root node is a NAND2 gate, with two child nodes being the inputs and one child node being the output.
NAND3	4	A three-input AND gate with one output.	A tree diagram where the root node is a NAND3 gate, with three child nodes being the inputs and one child node being the output.
NAND4	5	A four-input AND gate with one output.	A tree diagram where the root node is a NAND4 gate, with four child nodes being the inputs and one child node being the output.
		A complex logic circuit composed of multiple inverters and NAND gates.	A tree diagram representing the logic circuit shown above, showing the hierarchical structure of the primitives used.

Example: Keutzer

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Elements of a library - 2

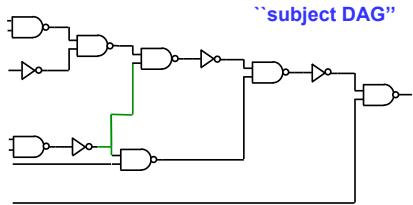
Element/Area Cost Tree Representation (normal form)

AOI21	4	An AND-Or-Invert gate with two inputs and one output.	A tree diagram where the root node is an AOI21 gate, with two child nodes being the inputs, one child node being the AND gate, and one child node being the inverter.
AOI22	5	An AND-Or-Invert gate with three inputs and one output.	A tree diagram where the root node is an AOI22 gate, with three child nodes being the inputs, one child node being the AND gate, and one child node being the inverter.

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Input Circuit Netlist

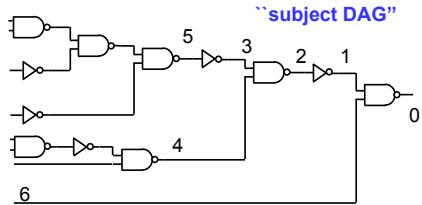


- Each wire is a network (net).
- Each net has a single source (the gate that drives it).
- In general, net may have multiple sinks (gates that take as input)

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Input Circuit Netlist



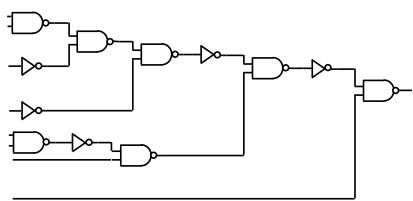
- A list of the nets (netlist) fully describes the circuit
- 0 nand 1 6
- 1 inv 2
- 2 nand 3 4

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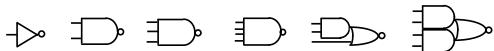
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Problem Statement

Find an "optimal" (in area, delay, power) mapping of this circuit (DAG)



into this library



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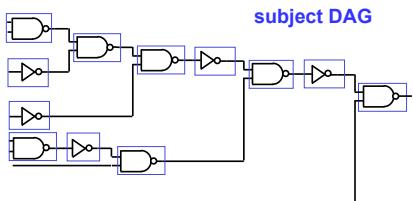
Why covering now?

- Nice/simple cost model
- Problem can be solved well
 - somewhat clever solution
- General/powerful technique
- Show off special cases
 - harder/easier cases
- Show off things that make hard
- Show off bounding

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What's the Problem? Trivial Covering



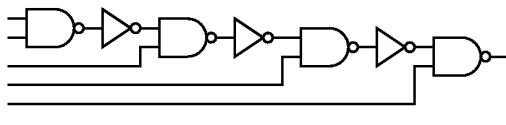
7 NAND2 (3) = 21
5 INV (2) = 10
Area cost 31

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Preclass 1

- Direct covering cost?



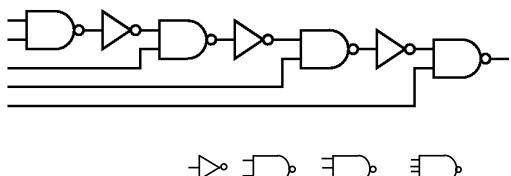
-> <--
2 3

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Preclass 3 & 4

- Least Area Cover? (associated area?)
– How did you get?



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Cost Models

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Cost Model: Area

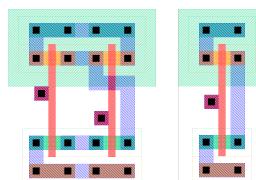
- **Assume:** Area in gates
- or, at least, can pick an area/gate
– so proportional to gates
- **e.g.**
 - Standard Cell design
 - Standard Cell/route over cell
 - Gate array

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Standard Cells

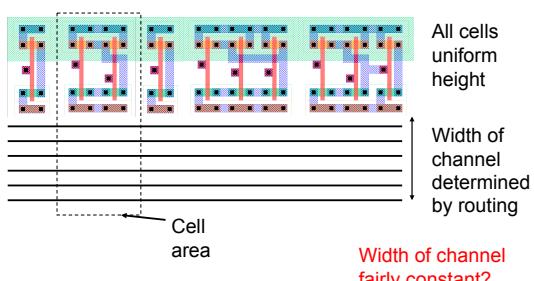
- Lay out gates so that heights match
 - Rows of adjacent cells
 - Standardized sizes
- Motivation: ease place and route



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Standard Cell Area



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Cost Model: Delay

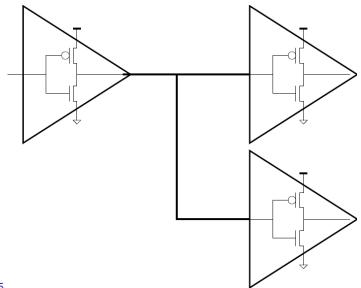
- Delay in gates
 - at least assignable to gates
 - $T_{wire} \ll T_{gate}$
 - $T_{wire} \approx \text{constant}$
 - delay exclusively/predominantly in gates
 - Gates have C_{out}, C_{in}
 - lump capacitance for output drive
 - delay $\sim T_{gate} + \text{fanout} \times C_{in}$
 - $C_{wire} \ll C_{in}$
 - or C_{wire} can lump with C_{out}/T_{gate}

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Logic Delay

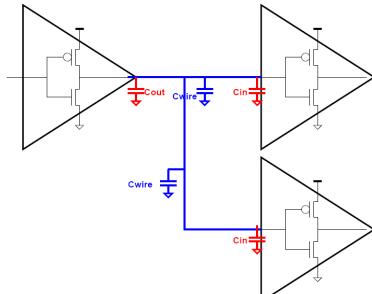
- How would we calculate delay?



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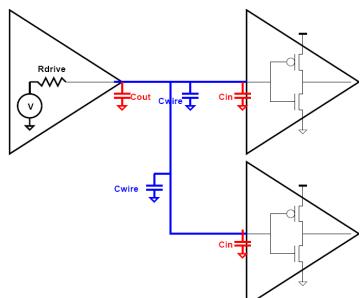
Parasitic Capacitances



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Delay of Net



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Cost Model: Delay

- Delay in gates
 - at least assignable to gates
 - $T_{\text{wire}} \ll T_{\text{gate}}$
 - $T_{\text{wire}} \approx \text{constant}$
 - delay exclusively/predominantly in gates
 - Gates have $C_{\text{out}}, C_{\text{in}}$
 - lump capacitance for output drive
 - delay $\sim T_{\text{gate}} + \text{fanout} \times C_{\text{in}}$
 - $C_{\text{wire}} \ll C_{\text{in}}$
 - or C_{wire} can lump with $C_{\text{out}}/T_{\text{gate}}$

$$\begin{aligned} F &= 22 \text{nm CMOS} \\ T_{\text{gate}}(\text{inv drive 4 inv}) &\sim 1 \text{ps} \\ T_{\text{wire}}(300 \mu\text{m}) &\sim 1 \text{ps} \\ W_{\text{gate}} &\sim 0.3 \mu\text{m} \end{aligned}$$

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Cost Models

- Why do I show you models?
 - not clear there's one "right" model
 - changes over time
 - you're going to encounter many different kinds of problems
 - want you to see formulations so can critique and develop own
 - simple cost models make problems tractable
 - are surprisingly adequate
 - simple, at least, help bound solutions
 - may be **wrong** today...need to rethink

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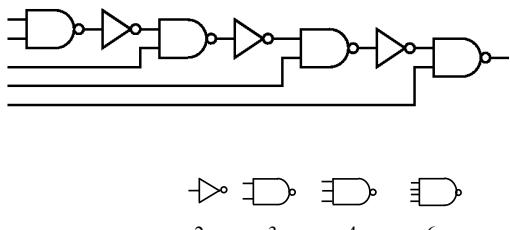
Approaches

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Greedy work?

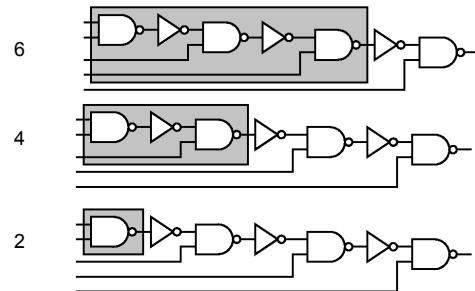
- Greedy = pick next locally “best” choice



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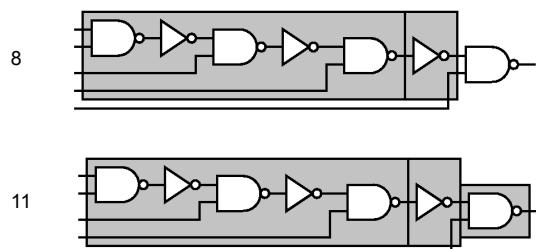
Greedy In→Out



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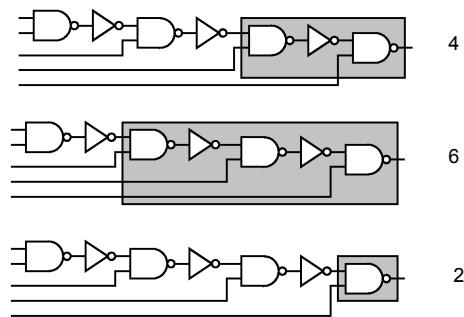
Greedy In→Out



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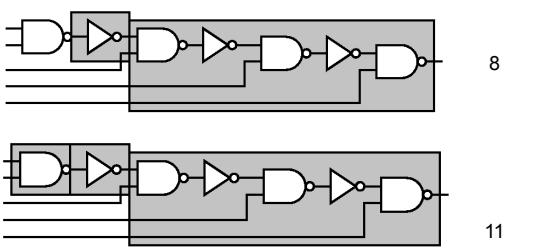
Greedy Out→In



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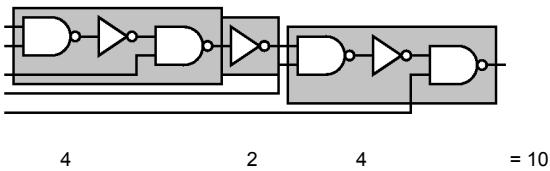
Greedy Out→In



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But...



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Greedy Problem

- What happens in the future (elsewhere in circuit) will determine what should be done at this point in the circuit.
- Can't just pick best thing for now and be done.

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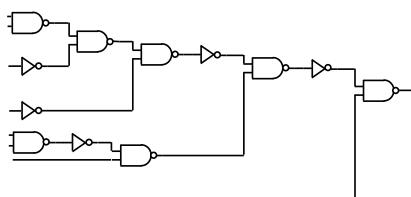
Brute force?

- Pick a node (output)
- Consider
 - all possible gates which may cover that node
 - branch on all inputs after cover
 - pick least cost node

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Pick a Node



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Brute force?

- Pick a node (output)
- Consider
 - all possible gates which may cover that node
 - recurse on all inputs after cover
 - pick least cost node
- Explore all possible covers
 - can find optimum

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Analyze brute force?

- Time?

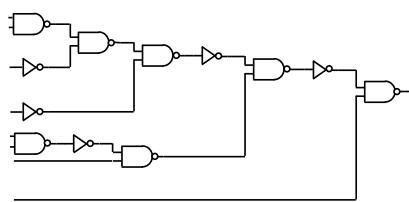
$$T_{brute}(node) = \sum_{i=0}^{\max \text{ pattern}} \left(T_{match}(P_i) + \sum_{j=0}^{\max \text{ in}} (T_{brute}(\text{in } j)) \right)$$
- Say P patterns, constant time to match each
 - (if patterns long could be $> O(1)$)
- P-way branch at each node...
 - How big is tree?
- ...exponential
 - $O((P)^{\text{depth}})$

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Structure inherent in problem to exploit?

- What structure exists?

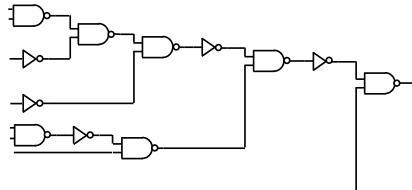


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Structure inherent in problem to exploit?

- There are only N unique nodes to cover!



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Structure

- If subtree solutions do not depend on what happens outside of its subtree
 - separate tree
 - farther up tree
- Should only have to look at N nodes.
- Time(N) = $N \cdot P \cdot T(\text{match})$
 - w/ P fixed/bounded \rightarrow linear in N
 - w/ cleverness work isn't $P \cdot T(\text{match})$ at every node

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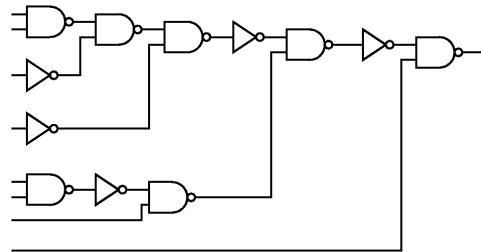
Idea Re-iterated

- Work from inputs
- Optimal solution to subproblem is contained in optimal, global solution
- Find optimal cover for each node
- Optimal cover:
 - examine all gates at this node
 - look at cost of gate and its inputs
 - pick least

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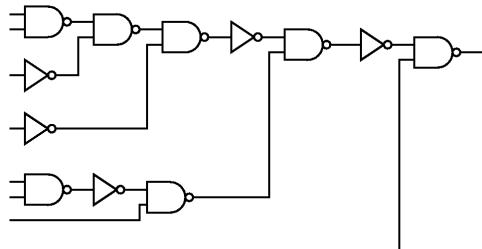
Work front-to-back



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Work Example (area)



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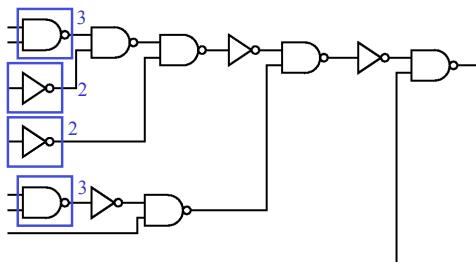
5

4

5

41

Work Example (area)



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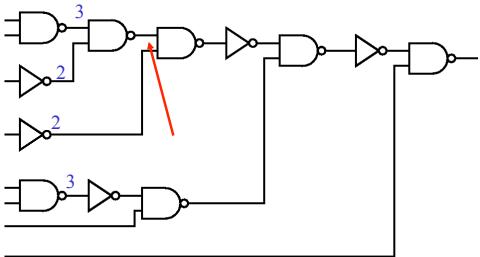
5

4

5

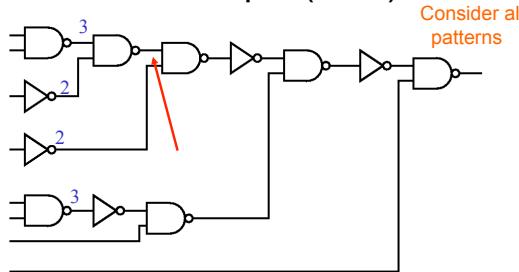
42

Work Example (area)



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Work Example (area)



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Elements of a library - 1

Element/Area Cost Tree Representation (normal form)

INVERTER	2		
NAND2	3		
NAND3	4		
NAND4	5		

Example: Keutzer

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Elements of a library - 2

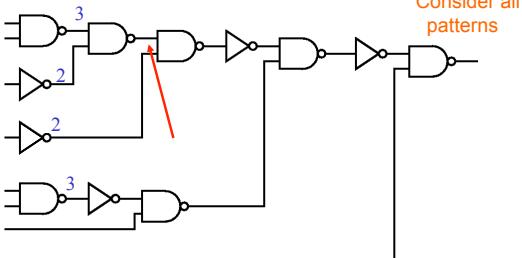
Element/Area Cost Tree Representation (normal form)

AOI21	4		
AOI22	5		

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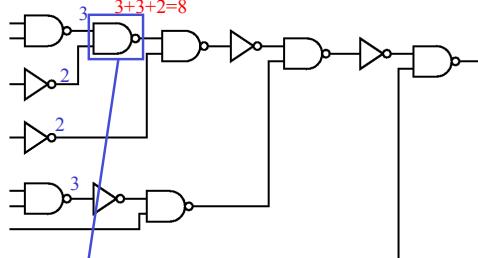
46

Work Example (area)



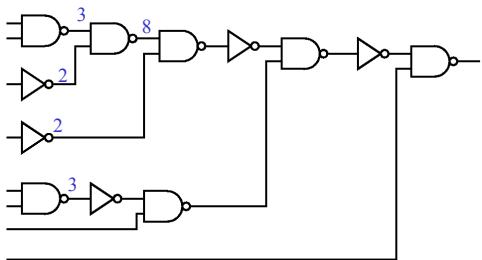
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Work Example (area)



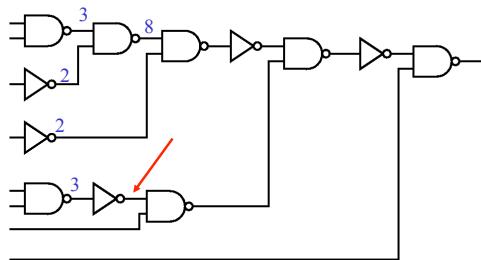
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Work Example (area)



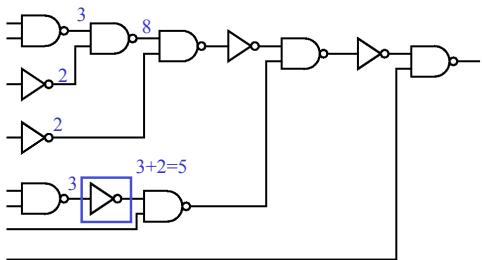
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Work Example (area)



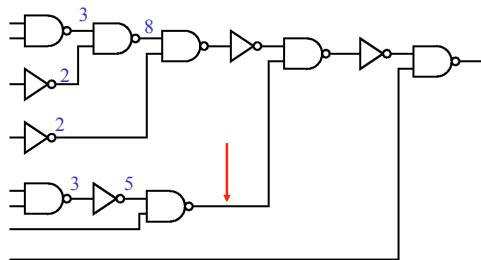
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Work Example (area)



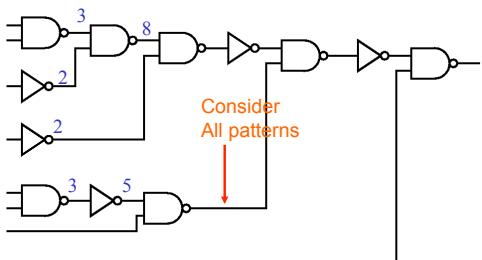
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Work Example (area)



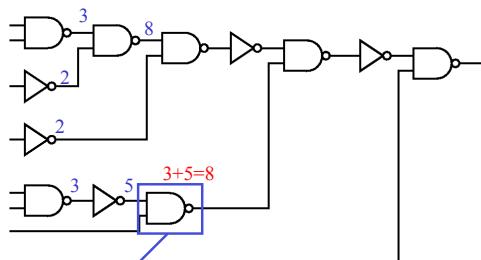
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Work Example (area)



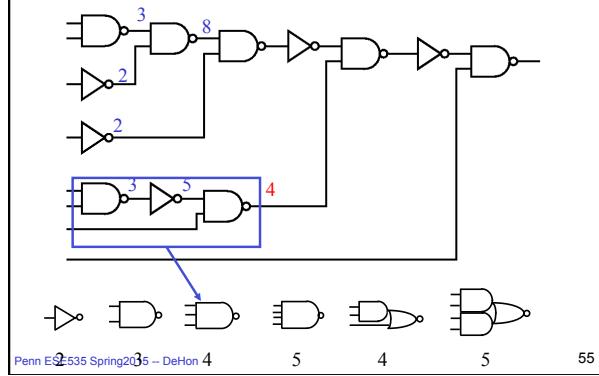
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Work Example (area)



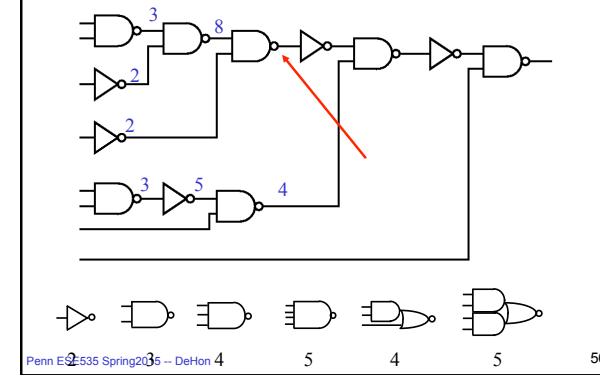
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Work Example (area)



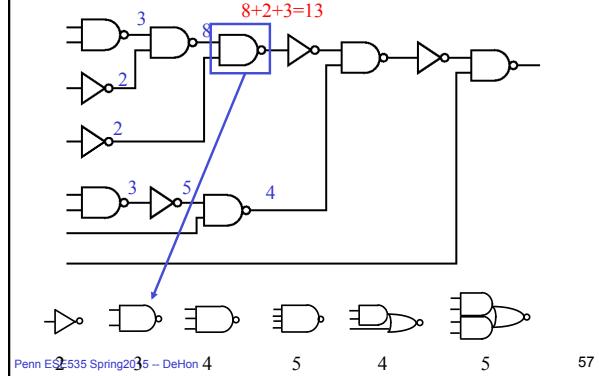
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Work Example (area)



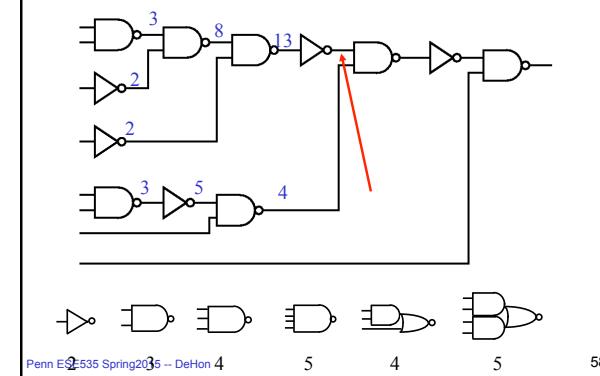
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Work Example (area)



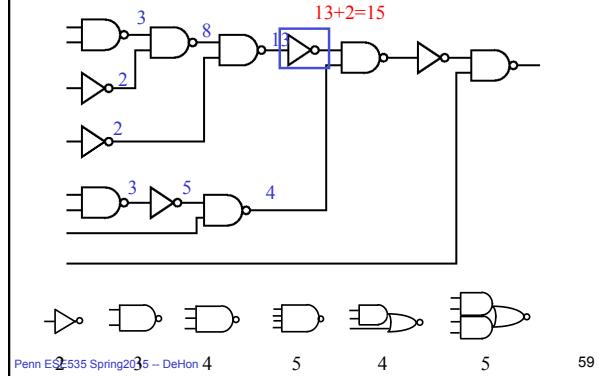
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Work Example (area)



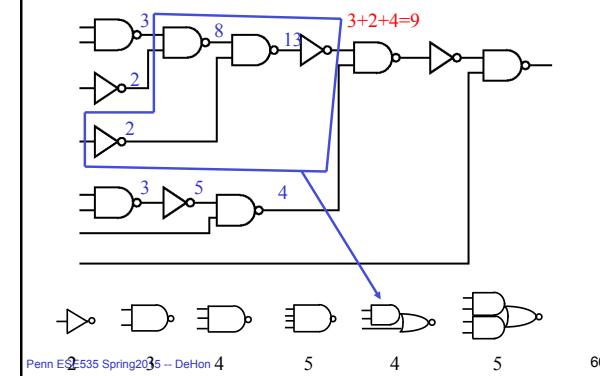
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Work Example (area)



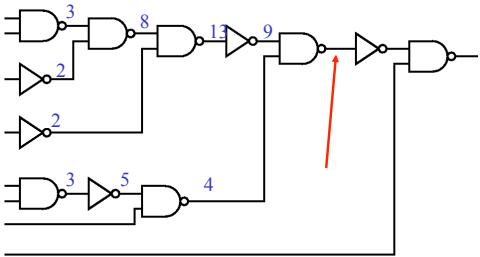
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Work Example (area)



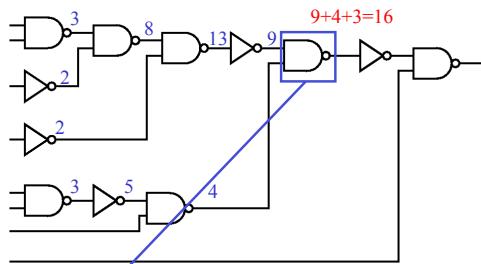
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Work Example (area)



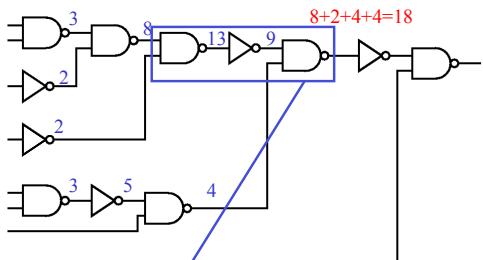
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Work Example (area)



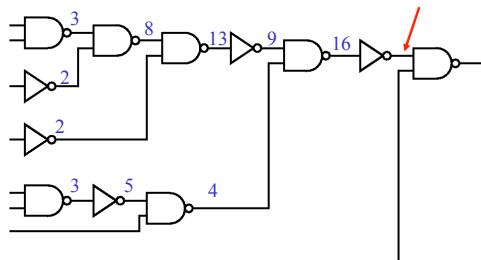
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Work Example (area)



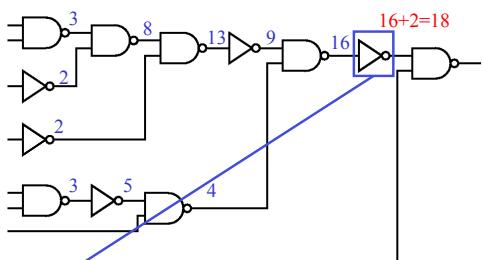
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Work Example (area)



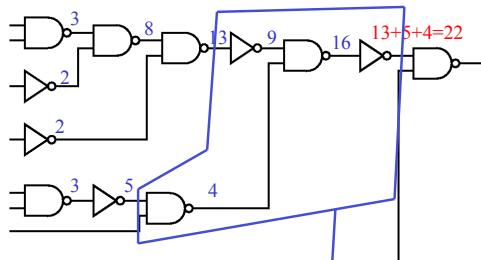
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Work Example (area)



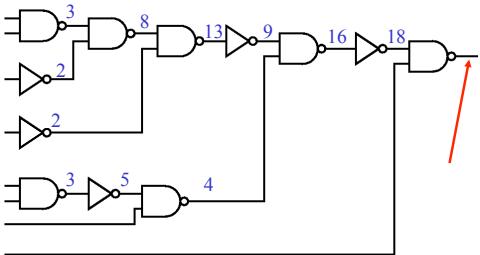
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Work Example (area)



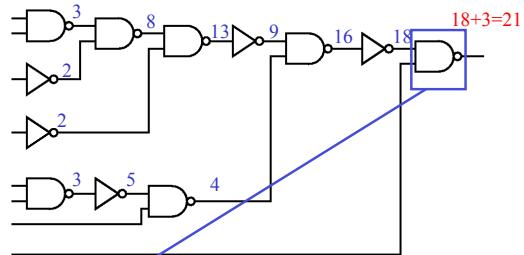
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Work Example (area)



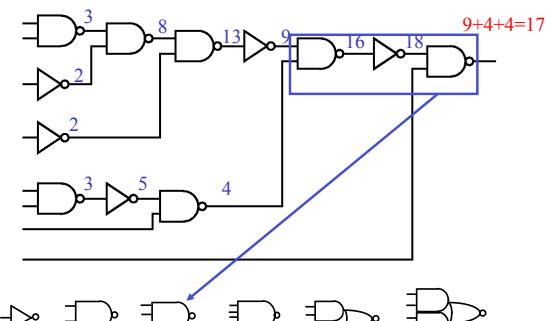
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Work Example (area)



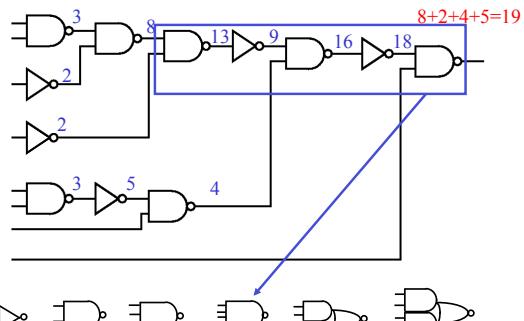
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Work Example (area)



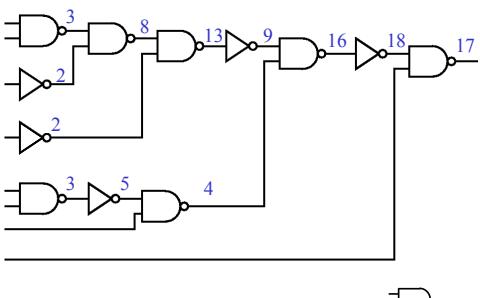
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Work Example (area)



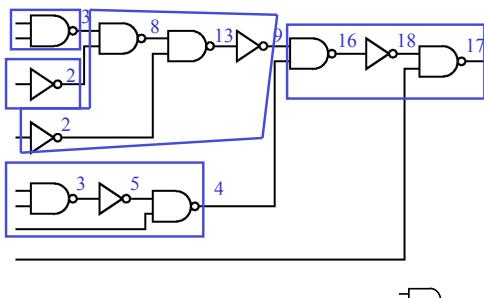
Penn E2535 Spring2035 - DeHon 4 5 4 5 70

Work Example (area)



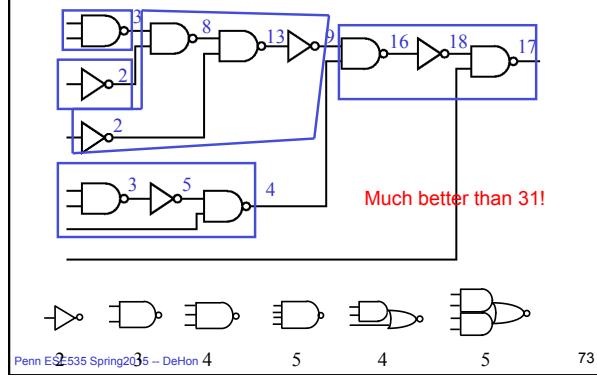
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Optimal Cover



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Optimal Cover



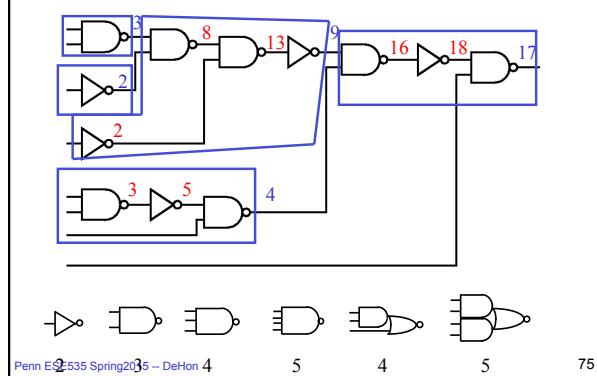
Note

- There are nodes we cover that will **not** appear in final solution.

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"Unused" Nodes



Dynamic Programming Solution

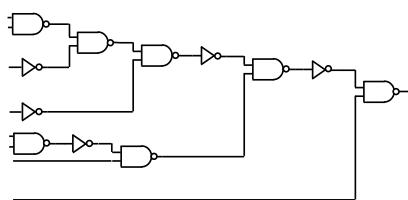
- Solution described is general instance of dynamic programming
- Require:
 - optimal solution to subproblems is optimal solution to whole problem
 - (all optimal solutions equally good)
 - divide-and-conquer gets same (finite/small) number of subproblems
- Same technique used for instruction selection in code generation for processors

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Delay

- Similar
 - $\text{Delay}(\text{node}) = \text{Delay}(\text{gate}) + \text{Max}(\text{Delay}(\text{input}))$

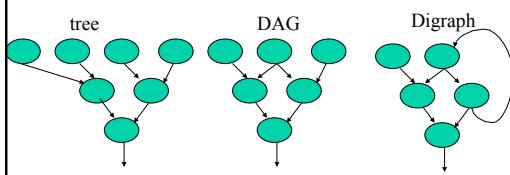


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DAG

- DAG = Directed Acyclic Graph
 - Distinguish from tree ($\text{tree} \subset \text{DAG}$)
 - Distinguish from cyclic Graph
 - $\text{DAG} \subset \text{Directed Graph (digraph)}$



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Trees vs. DAGs

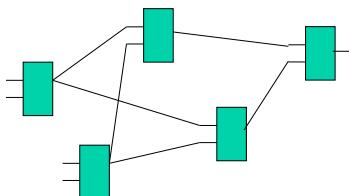
- Optimal for trees
 - why?
 - Delay
 - Area

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Not optimal for DAGs

- Why?

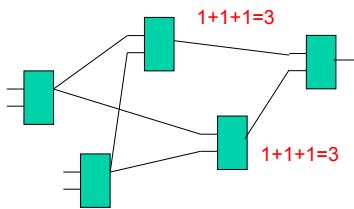


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Not optimal for DAGs

- Why?

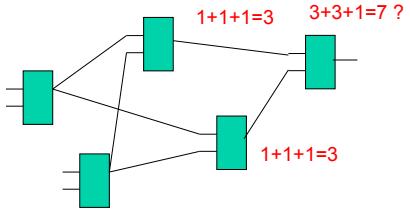


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Not optimal for DAGs

- Why?



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Not Optimal for DAGs (area)

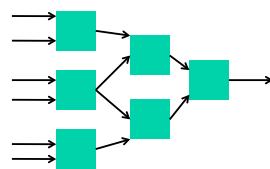
- $\text{Cost}(N) = \text{Cost}(\text{gate}) + \sum \text{Cost}(\text{input nodes})$
- think of sets
- cost is magnitude of set union
- Problem:** minimum cost (magnitude) solution isn't necessarily the best pick
 - get interaction between subproblems
 - subproblem optimum not global...

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DAG Example

- Cover with 3 input gates

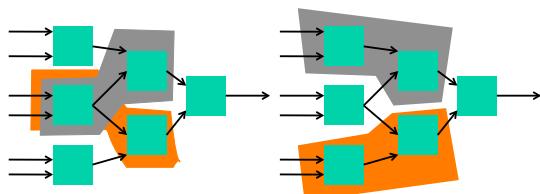


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DAG Example

- Cover with 3 input gates



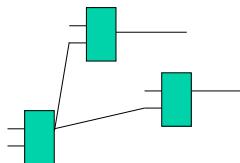
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Not Optimal for DAGs

- Delay:

- in fanout model, depends on problem you haven't already solved (delay of node depends on number of uses)



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What do people do?

- Cut DAGs at fanout nodes
- optimally solve resulting trees
- Area
 - guarantees covered once
 - get accurate costs in covering trees, made "premature" assignment of nodes to trees
- Delay
 - know where fanout is

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Bounding

- Tree solution give bounds (esp. for delay)
 - single path, optimal covering for delay
 - (also make tree by replicating nodes at fanout points)
- no fanout cost give lower bounds
 - know you can't do better
- delay lower bounds useful, too
 - know what you're giving up for area
 - when delay matters

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(Multiple Objectives?)

- Like to say, get delay, then area
 - won't get minimum area for that delay
 - algorithm only keep best delay
 - ...but best delay on off critical path piece not matter
 - ...could have accepted more delay there
 - don't know if on critical path while building subtree
 - (iterate, keep multiple solutions)

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Many more details...

- Implement well
- Combine criteria
- ...but now you know the main idea

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Big Ideas

- simple cost models
- problem formulation
- identifying structure in the problem
- special structure
- characteristics that make problems hard
- bounding solutions

Admin

- Reading for today: canvas
- Reading for Wednesday:
 - online/ACM DL
 - Highly relevant to assignment 3..6
- Office Hour: T4:30pm
 - Or make an appointment