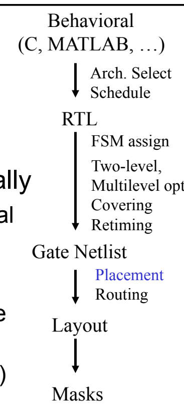
ESE535: Electronic Design Automation

Day 6: February 4, 2014
Partitioning 2
(spectral, network flow)

Today

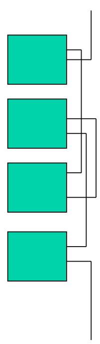
- Alternate views of partitioning
- Two things we can solve optimally
  - (but don't exactly solve our original problem)
- Techniques
  - Linear Placement w/ squared wire lengths
  - Network flow MinCut (time permit)



Optimization Target

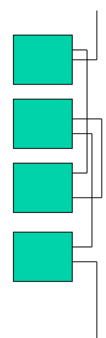
- Place cells
- In linear arrangement
- Wire length between connected cells:
  - distance=X_i - X_j
  - cost is sum of distance squared

Pick X_i's to minimize cost



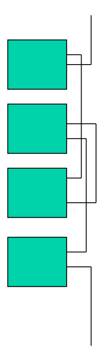
Why this Target?

- Minimize sum of squared wire distances
- Prefer:
  - **Area:** minimize channel width
  - **Delay:** minimize critical path length



Why this Target?

- Our preferred targets are discontinuous and discrete
- Cannot formulate analytically
- Not clear how to drive toward solution
  - Does reducing the channel width at a non-bottleneck help or not?
  - Does reducing a non-critical path help or not?



Preclass: Initial Placement

- Metrics:
  - Wirelength
  - Squared wirelength
  - Channel width
  - Critical path length

Spectral Ordering

Minimize Squared Wire length -- 1D layout

- Start with connection array C ($c_{i,j}$)
- "Placement" Vector X for x_i placement
- **Problem:**
  - Minimize cost = $0.5 \times \sum_i \sum_j c_{i,j} \left(x_i - x_j\right)^2$
  - cost sum is $X^T BX$
    - B = D-C
    - D=diagonal matrix, $d_{i,i} = \Sigma(\text{over } j)\ c_{i,j}$

Preclass Netlist

- Squared wire lengths:

$$(X_A \quad -X_G)^2$$
$$+(X_B \quad -X_G)^2$$
$$+(X_B \quad -X_H)^2$$
$$+(X_C \quad -X_H)^2$$
$$+(X_G \quad -X_O)^2$$
$$+(X_H - \quad X_O)^2$$

C Matrix

	A	B	C	G	H	O
A				1		
B				1	1	
C					1	
G	1	1				1
H		1	1			1
O				1	1	

D Matrix

	A	B	C	G	H	O
A	1			1		
B		2		1	1	
C			1		1	
G	1	1		3		1
H		1	1		3	1
O				1	1	2

B=D-C Matrix

	A	B	C	G	H	O
A	1			-1		
B		2		-1	-1	
C			1		-1	
G	-1	-1		3		-1
H		-1	-1		3	-1
O				-1	-1	2

BX

	A	B	C	G	H	O	X		BX
A	1			-1			X_A		$X_A - X_G$
B		2		-1	-1		X_B		$2X_B - X_G - X_H$
C			1		-1		X_C	=	$X_C - X_H$
G	-1	-1		3		-1	X_G		$3X_G - X_A - X_B - X_O$
H		-1	-1		3	-1	X_H		$3X_H - X_B - X_C - X_O$
O				-1	-1	2	X_O		$2X_O - X_G - X_H$

$X^T(BX)$

X_A	X_B	X_C	X_G	X_H	X_O

$X_A - X_G$
$2X_B - X_G - X_H$
$X_C - X_H$
$3X_G - X_A - X_B - X_O$
$3X_H - X_B - X_C - X_O$
$2X_O - X_G - X_H$

$X^T(BX)$

$X_A^2 - X_A X_G$

$+2X_B^2 - X_B X_G - X_B X_H$

$+X_C^2 - X_C X_H$

$+3X_G^2 - X_A X_G - X_B X_G - X_G X_O$

$+3X_H^2 - X_B X_H - X_C X_H - X_H X_O$

$+2X_O^2 - X_G X_O - X_H X_O$

$X^T(BX)$

$X_A^2 - X_A X_G$

$+2X_B^2 - X_B X_G - X_B X_H$

$+X_C^2 - X_C X_H$

$+3X_G^2 - X_A X_G - X_B X_G - X_G X_O$

$+3X_H^2 - X_B X_H - X_C X_H - X_H X_O$

$+2X_O^2 - X_G X_O - X_H X_O$

$(X_A - X_G)^2$

$+2X_B^2 - X_B X_G - X_B X_H$

$+X_C^2 - X_C X_H$

$+2X_G^2 - X_B X_G - X_G X_O$

$+3X_H^2 - X_B X_H - X_C X_H - X_H X_O$

$+2X_O^2 - X_G X_O - X_H X_O$

$X^T(BX)$

$(X_A - X_G)^2$

$+2X_B^2 - X_B X_G - X_B X_H$

$+X_C^2 - X_C X_H$

$+2X_G^2 - X_B X_G - X_G X_O$

$+3X_H^2 - X_B X_H - X_C X_H - X_H X_O$

$+2X_O^2 - X_G X_O - X_H X_O$

$(X_A - X_G)^2 + (X_B - X_G)^2$

$+X_B^2 - X_B X_H$

$+X_C^2 - X_C X_H$

$+X_G^2 - X_G X_O$

$+3X_H^2 - X_B X_H - X_C X_H - X_H X_O$

$+2X_O^2 - X_G X_O - X_H X_O$

Can See Will Converage To..

- Squared wire lengths:

$(X_A - X_G)^2$
$+(X_B - X_G)^2$
$+(X_B - X_H)^2$
$+(X_C - X_H)^2$
$+(X_G - X_O)^2$
$+(X_H - X_O)^2$

$(X_A - X_G)^2 + (X_B - X_G)^2$

$+X_B^2 - X_B X_H$

$+X_C^2 - X_C X_H$

$+X_G^2 - X_G X_O$

$+3X_H^2 - X_B X_H - X_C X_H - X_H X_O$

$+2X_O^2 - X_G X_O - X_H X_O$

Trying to Minimize

- Squared wire lengths:

$(X_A - X_G)^2$
$+(X_B - X_G)^2$
$+(X_B - X_H)^2$
$+(X_C - X_H)^2$
$+(X_G - X_O)^2$
$+(X_H - X_O)^2$

- Which we know is also X^TBX
- Make all X_i's same?
- …but, we probably need to be in unique positions.

Spectral Ordering

- Add constraint: $X^TX=1$
  - prevent trivial solution all x_i's =0
- Minimize cost=X^TBX w/ constraint
  - minimize $L=X^TBX-\lambda(X^TX-1)$
  - $\partial L/\partial X=2BX-2\lambda X=0$
  - $(B-\lambda I)X=0$
  - What does this tell us about X, λ ?
  - X → Eigenvector of B
  - cost is Eigenvalue λ

Spectral Solution

- Smallest eigenvalue is zero
  - Corresponds to case where all x_i's are the same → uninteresting
- **Second smallest** eigenvalue (eigenvector) is the solution we want

Eigenvector for B

For this **B Matrix**

	A	B	C	G	H	O
A	1			-1		
B		2		-1	-1	
C			1		-1	
G	-1	-1		3		-1
H		-1	-1		3	-1
O				-1	-1	2

Eigenvector is:

X_A		0.6533
X_B		1.116E-14
X_C	=	-0.6533
X_G		0.2706
X_H		-0.2706
X_O		1.934E-14

Spectral Ordering

- X (x_i's) continuous
- use to order nodes
  - We need at discrete locations
  - this is one case where can solve ILP from LP
    - Solve LP giving continuous x_i's
    - then move back to closest discrete point

Eigenvector is:

X_A		0.6533
X_B		1.116E-14
X_C	=	-0.6533
X_G		0.2706
X_H		-0.2706
X_O		1.934E-14

Eigenvector for B

Order?

Eigenvector is:

X_A		0.6533
X_B		1.116E-14
X_C	=	-0.6533
X_G		0.2706
X_H		-0.2706
X_O		1.934E-14

Order from Eigenvector

A
G
O
B
H
C

Quality of this solution?
all metrics

Anyone get a solution with a better metric?

Eigenvector is:

X_A		0.6533
X_B		1.116E-14
X_C	=	-0.6533
X_G		0.2706
X_H		-0.2706
X_O		1.934E-14

Spectral Ordering Option

- Can encourage "closeness"
  - Making some $c_{i,j}$ larger
  - Must allow some to be not close
- Could use $c_{i,j}$ for power opt
  - $c_{i,j}=P_{switch}$

	A	B	C	G	H	O
A	1			-1		
B		2		-1	-1	
C			1		-1	
G	-1	-1		3		-1
H		-1	-1		3	-1
O				-1	-1	2

Spectral Ordering Option

- With iteration, can reweigh connections to change cost model being optimized
  - linear
  - $(\text{distance})^{1.X}$

$$C_{i,j} = \frac{1}{\sqrt{|X_i - X_j|}}$$

$$C_{i,j}(X_i - X_j)^2 = \frac{(X_i - X_j)^2}{\sqrt{|X_i - X_j|}} = (X_i - X_j)^{1.5}$$

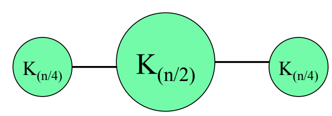
	A	B	C	G	H	O
A	1			-1		
B		2		-1	-1	
C			1		-1	
G	-1	-1		3		-1
H		-1	-1		3	-1
O				-1	-1	2

Spectral Partitioning

- Can form a basis for partitioning
- Attempts to cluster together connected components
- Create partition from ordering
  - *E.g.* Left half of ordering is one half, right half is the other

Spectral Ordering

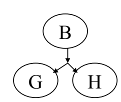
- Midpoint bisect isn't necessarily best place to cut, consider:



Spectral Partitioning Options

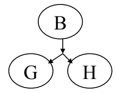
- Can bisect by choosing midpoint
  - (not strictly optimizing for minimum bisect)
- Can relax cut critera
  - min cut w/in some δ of balance
- Ratio Cut
  - Minimize (cut/|A||B|)
    - idea tradeoff imbalance for smaller cut
      - more imbalance →smaller |A||B|
      - so cut must be much smaller to accept
  - Easy to explore once have spectral ordering
    - Compute at each cut point in O(N) time

Fanout

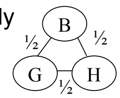


- How do we treat fanout?
- As described assumes point-to-point nets
- For partitioning, pay price when cut something once
  - *I.e.* the accounting did last time for KLFM
- Also a discrete optimization problem
  - Hard to model analytically

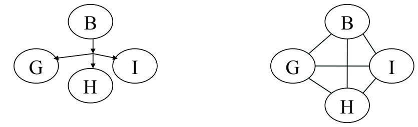
Spectral Fanout



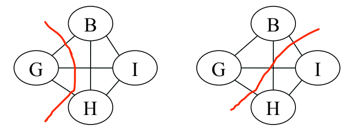
- Typically:
  - Treat all nodes on a single net as fully connected
  - Model links between all of them
  - Weight connections so cutting in half counts as cutting the wire – e.g. 1/(nodes-1)
  - Threshold out high fanout nodes
    - If connect too many things give no information



Spectral Fanout Cut Approximation



Weight edges: 1/(4-1)=1/3



Spectral vs. FM

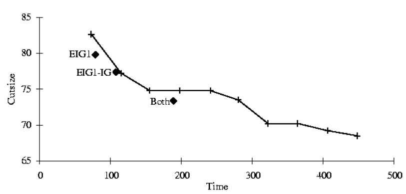
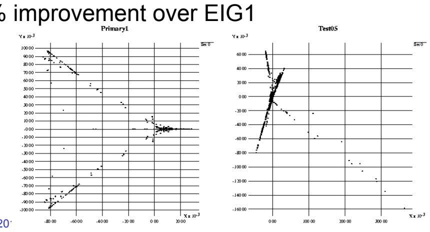


Figure 5. Graphs of cutsizes for different numbers of runs of our optimized version of KLFM versus the spectral initialization approaches. Values shown are the geometric means of the results for the 9 test circuits (all but industry3).

From Hauck/Boriello '96

Improving Spectral

- More Eigenvalues
  - look at clusters in n-d space
    - **But:** 2 eigenvectors is not opt. solution to 2D placement
    - Partition cut is plane in this higher-dimensional space
  - 5--70% improvement over EIG1



Spectral Note

- Unlike KLFM, attacks **global** connectivity characteristics
- Good for finding "natural" clusters
  - hence use as clustering heuristic for multilevel algorithms
- After doing spectral
  - Can often improve incrementally using KLFM pass
  - Remember spectral optimizing squared wirelength, not directly cut width

If Time Permits

Max Flow

MinCut

MinCut Goal

- Find maximum flow (mincut) between a source and a sink
  - no balance guarantee

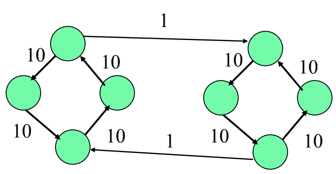
MaxFlow

- Set all edge flows to zero
  - F[u,v]=0
- While there is a path from s,t
  - (breadth-first-search)
  - for each edge in path f[u,v]=f[u,v]+1
  - f[v,u]=-f[u,v]
  - When c[v,u]=f[v,u] remove edge from search
- O(|E|*cutsize)
- [Our problem simpler than general case CLR]

Technical Details

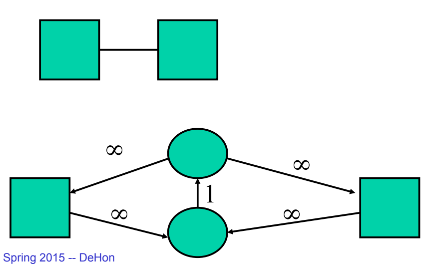
- For min-cut in graphs,
  - Don't really care about directionality of cut
  - Just want to minimize wire crossings
- Fanout
  - Want to charge discretely …cut or not cut
- Pick start and end nodes?

Directionality

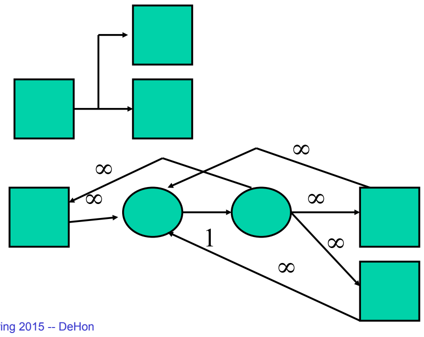


For logic net: cutting a net is the same regardless of which way the signal flows

Directionality Construct



Fanout Construct



Extend to Balanced Cut

- Pick a start node and a finish node
- Compute min-cut start to finish
- If halves sufficiently balanced, done
- else
  - collapse all nodes in smaller half into one node
  - pick a node adjacent to smaller half
  - collapse that node into smaller half
  - repeat from min-cut computation

FBB -- Yang/Wong ICCAD'94

Observation

- Can use residual flow from previous cut when computing next cuts
- Consequently, work of multiple network flows is only O(|E|*final_cut_cost)

Picking Nodes

- Optimal:
  - would look at all s,t pairs
    - Just for first cut is merely N-1 "others"
      - …N/2 to guarantee something in second half
    - Anything you pick **must** be in separate halves
    - Assuming there is a perfect/ideal bisection
      - If pick randomly, probability different halves: 50%
      - Few random selections likely to yield s,t in different halves
  - would also look at all nodes to collapse into smaller
  - could formulate as branching search

Picking Nodes

- Randomly pick
  - (maybe try several starting points)
- With small number of adjacent nodes,
  - could afford to branch on all

Big Ideas

- Divide-and-Conquer
- Techniques
  - flow based
  - numerical/linear-programming based
  - Transformation constructs
- Exploit problems we can solve optimally
  - Mincut
  - Linear ordering

Admin

- Assign 3 due on Thursday
- Reading for Monday online
- Assignment 4 exercise out
  - Should be small part
  - Most effort on partitioning project