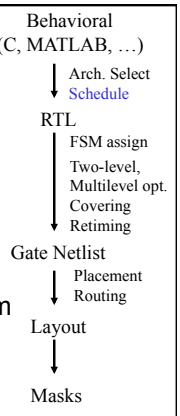
ESE535: Electronic Design Automation

Day 9: February 16, 2015
Scheduling
Variants and Approaches

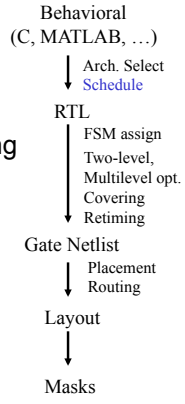
Previously

- Resources aren't free
- Share to reduce costs
- Schedule operations on resources
  - Fixed resources
- Greedy approximation algorithm
- List Scheduling for resource-constrained scheduling
  - Bounds on solution quality



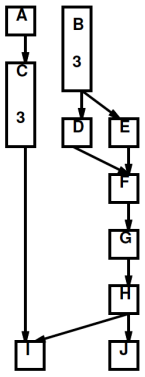
Today

- Time-Constrained Scheduling
  - Force Directed
- Resource-Constrained
  - Branch-and-Bound



Preclass

- Critical Path LB?
- Resources to keep RB < CP ?
- Resources to achieve CP?
  - Take poll: 4, 3, 2, 1
- What was trick to achieving?
- Why might List Schedule have a problem with this?



Force Directed

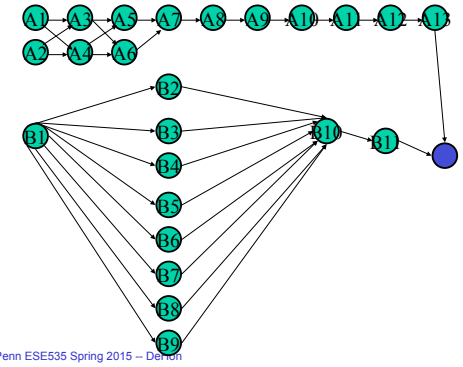
Force-Directed

- **Problem**: how exploit schedule freedom (slack) to minimize instantaneous resources
  - Directly solve time-constrained scheduling
    - (previously only solved indirectly)
  - Minimize resources with timing target

Force-Directed

- Given a node, can schedule anywhere between ASAP and ALAP schedule time
  - Between latest schedule predecessor and ALAP
  - Between ASAP and already scheduled successors
  - Between latest schedule predecessor and earliest schedule successor
- *That is:* Scheduling node will limit freedom of nodes in path

Single Resource Challenge



Force-Directed

- If everything where scheduled, **except** for the target node, what would we do?:
  - examine resource usage in all timeslots allowed by precedence
  - place in timeslot that has least increase maximum resources
    - Least energy
    - Where the forces are pulling it

Force-Directed

- **Problem:** don't know resource utilization during scheduling
- **Strategy:** estimate resource utilization

Force-Directed Estimate

- Assume a node is uniformly distributed within slack region
  - between earliest and latest possible schedule time
  - In all timesteps between ASAP and ALAP

$$W(node) = \frac{1}{(slack+1)}$$

Force-Directed Estimate

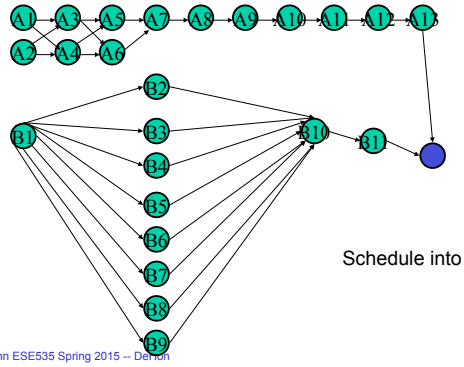
- Assume a node is uniformly distributed within slack region

$$W(node) = \frac{1}{(slack+1)}$$

- Use this estimate to identify most used timeslots

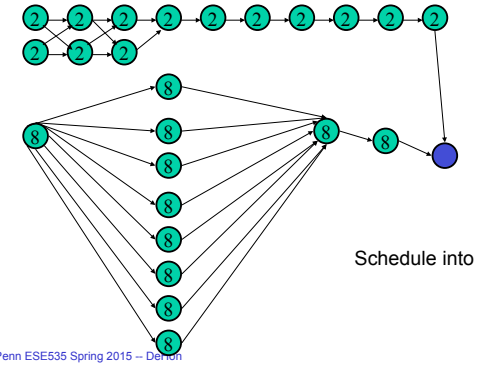
$$Cost(TS) = \sum_{node \in TS} W(node)$$

Single Resource Challenge



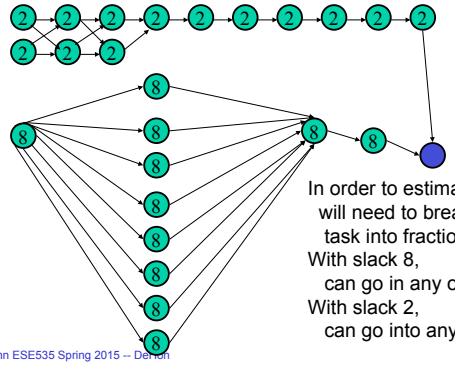
Schedule into 12 cycles

Slacks on all nodes



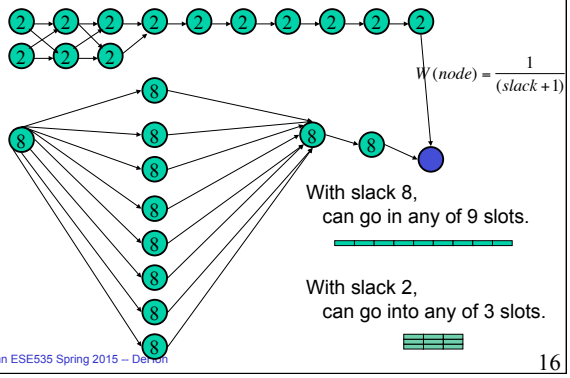
Schedule into 12 cycles

Slacks on all nodes



In order to estimate,
will need to break each
task into fractions.
With slack 8,
can go in any of 9 slots.
With slack 2,
can go into any of 3 slots.

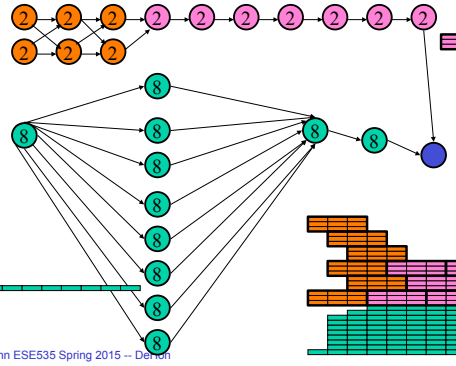
Slacks on all nodes



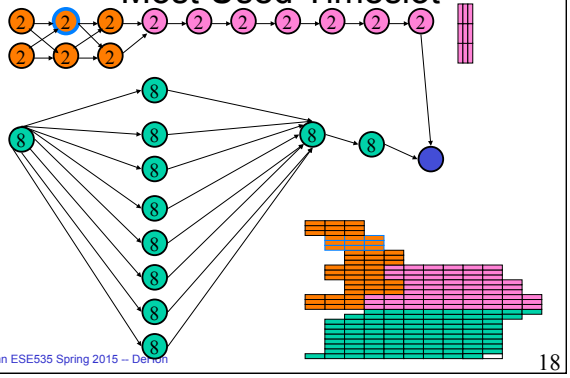
With slack 8,
can go in any of 9 slots.

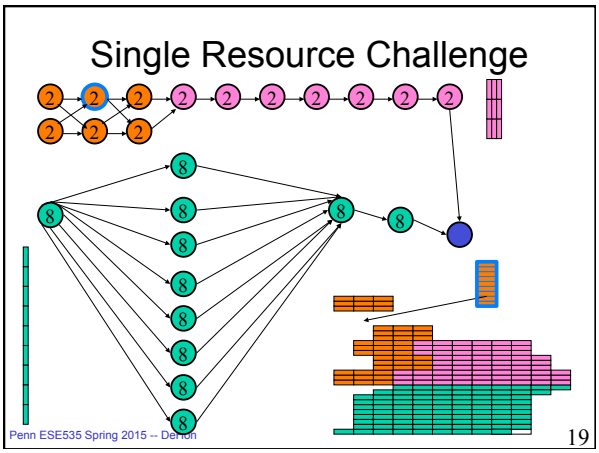
With slack 2,
can go into any of 3 slots.

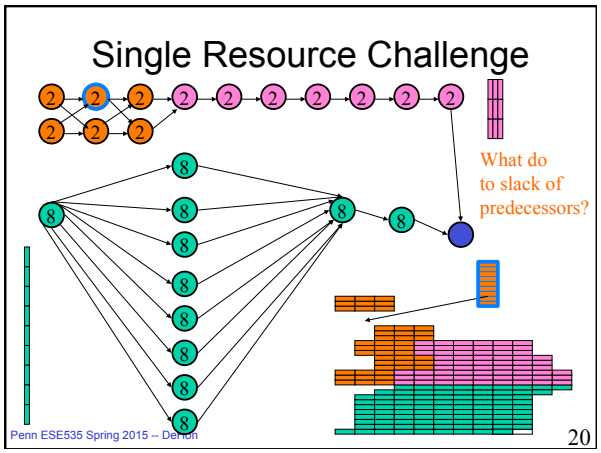
Uniform Distribution of Slack

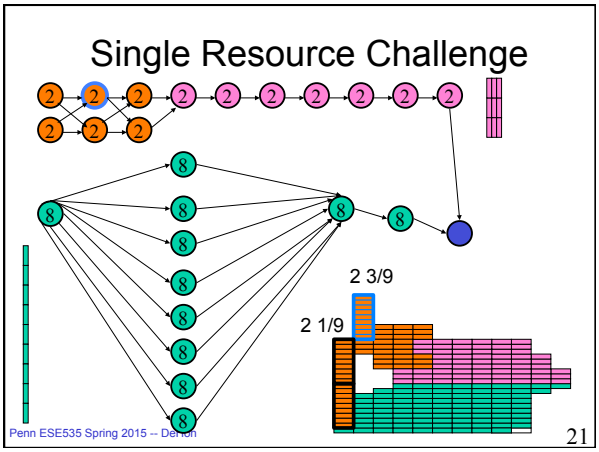


Most Constrained Node, Most Used Timeslot



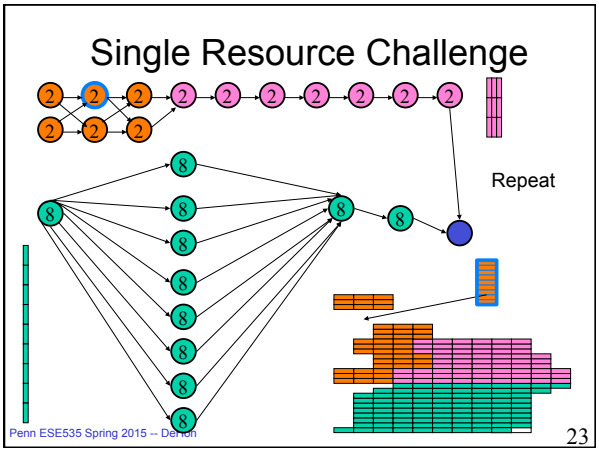


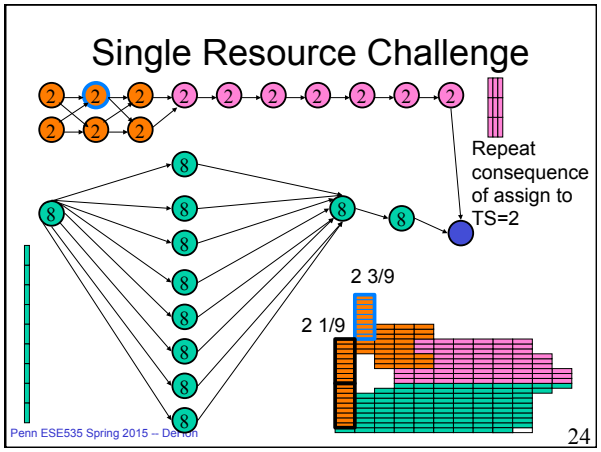


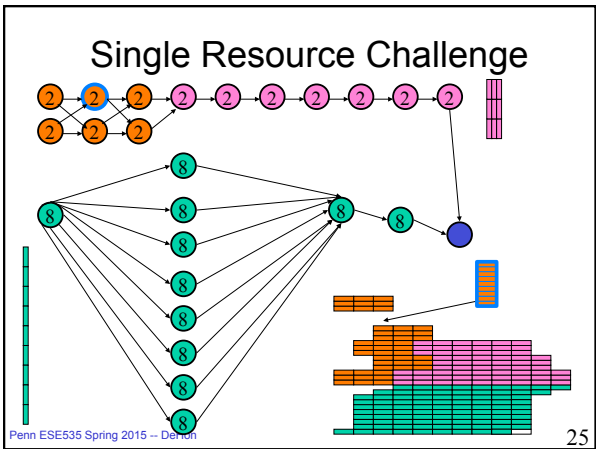


Force-Directed

- Scheduling a node will shift distribution
  - all of scheduled node's cost goes into one timeslot
  - predecessor/successors may have freedom limited so shift their contributions
- **Goal:** shift distribution to minimize maximum resource utilization (estimate)






Single Resource Challenge
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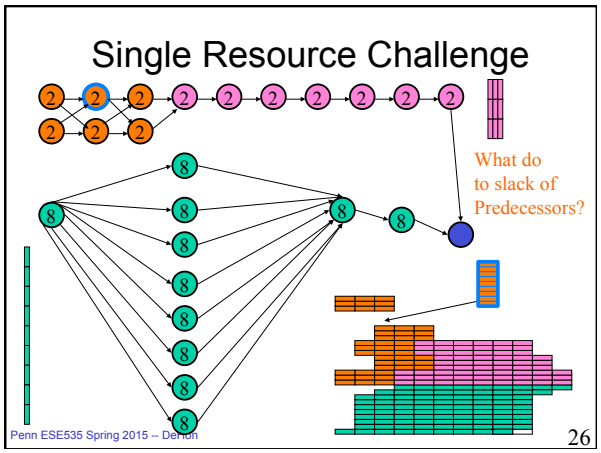
Single Resource Challenge
What do to slack of Predecessors?
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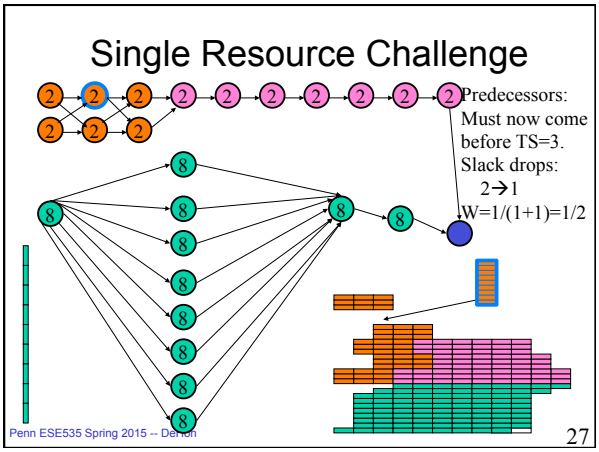
Single Resource Challenge
Predecessors: Must now come before TS=3. Slack drops: 2→1 W=1/(1+1)=1/2
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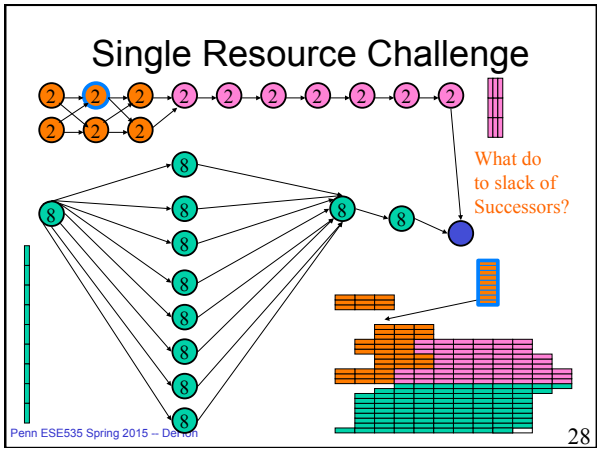
Single Resource Challenge
What do to slack of Successors?
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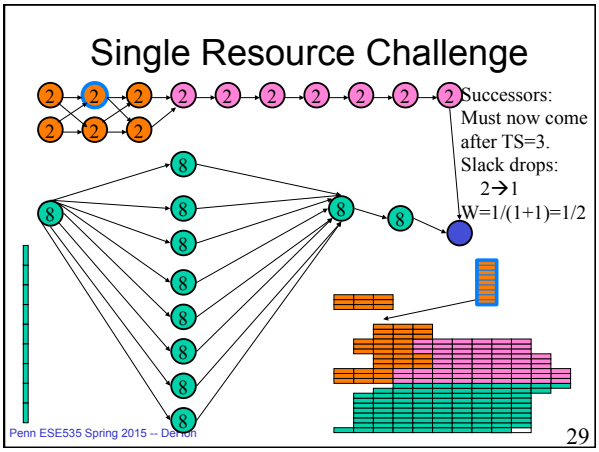
Single Resource Challenge
Successors: Must now come after TS=3. Slack drops: 2→1 W=1/(1+1)=1/2
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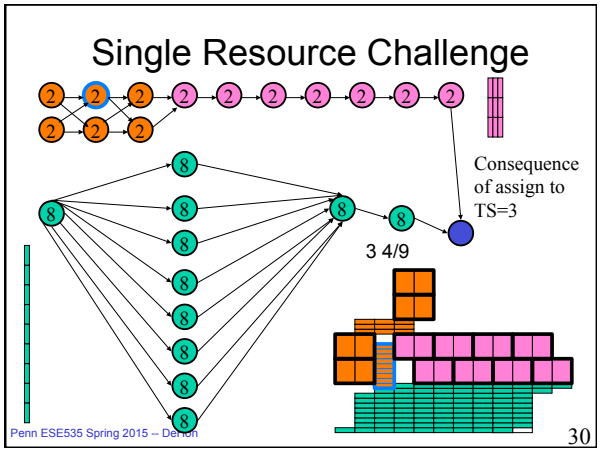
Single Resource Challenge
Consequence of assign to TS=3
3 4/9
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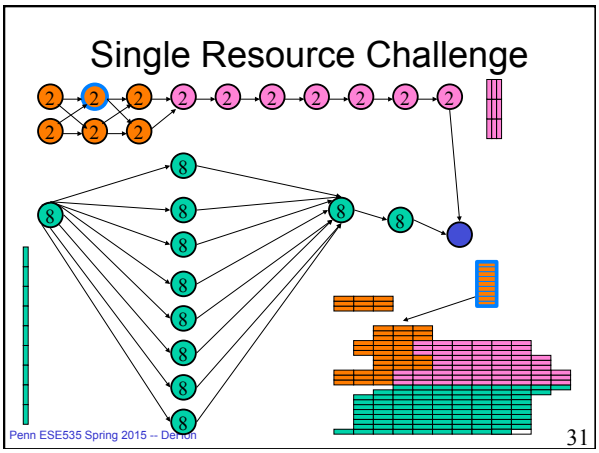
Single Resource Challenge
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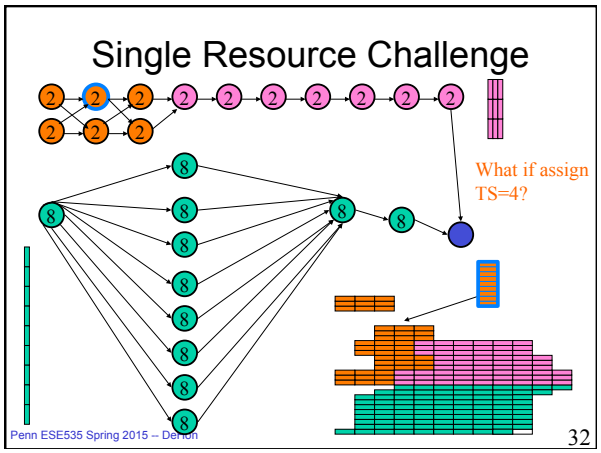
Single Resource Challenge
What if assign TS=4?
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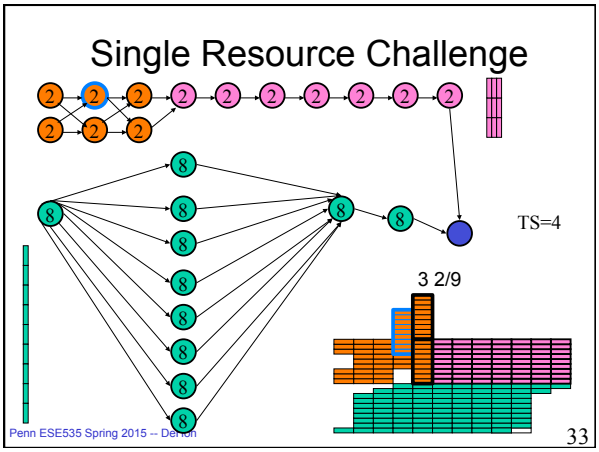
Single Resource Challenge
TS=4
3 2/9
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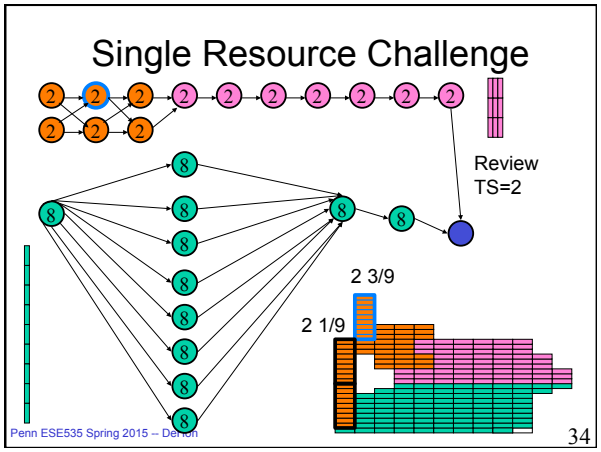
Single Resource Challenge
Review TS=2
2 3/9
2 1/9
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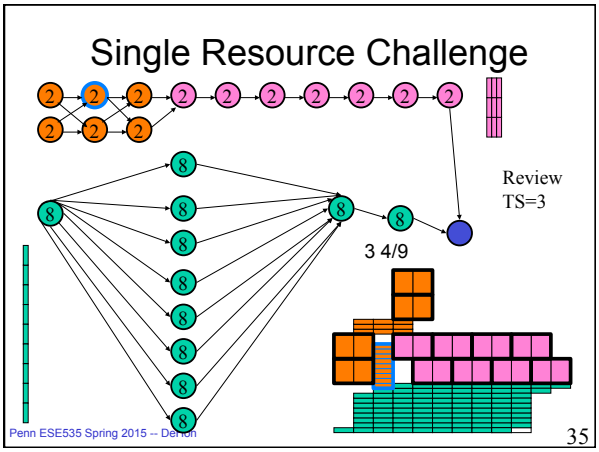
Single Resource Challenge
Review TS=3
3 4/9
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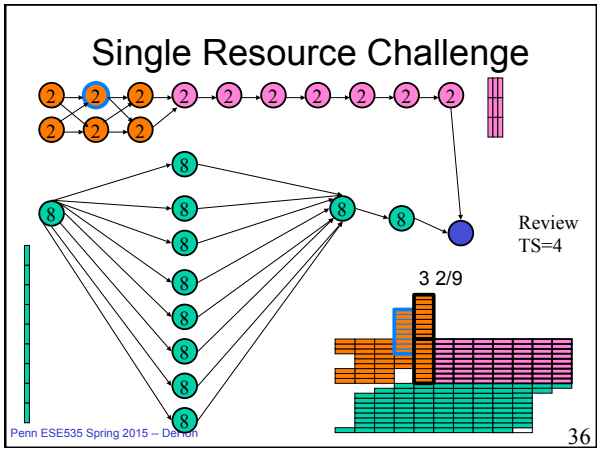
Single Resource Challenge
Review TS=4
3 2/9
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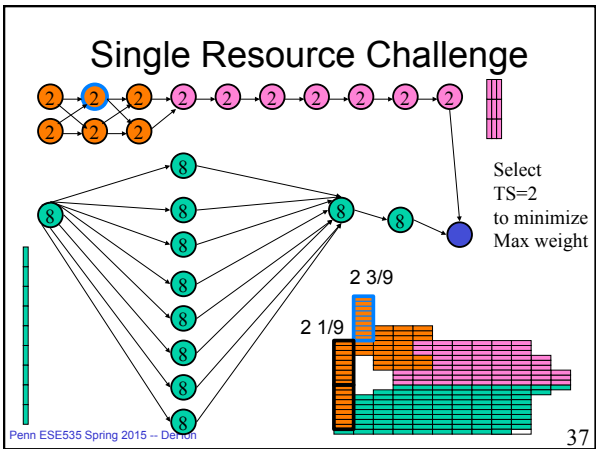
Single Resource Challenge
Select
TS=2
to minimize
Max weight
2 3/9
2 1/9
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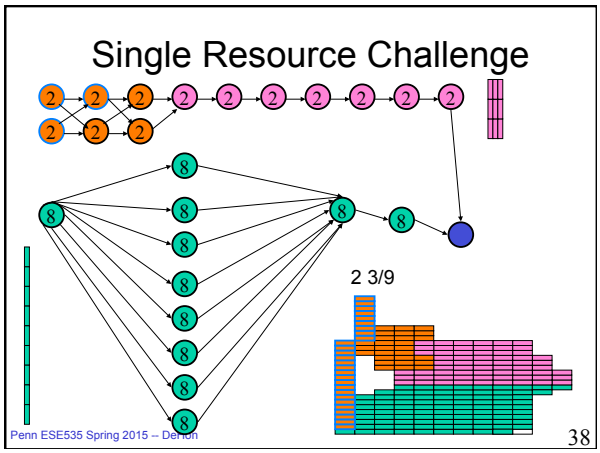
Single Resource Challenge
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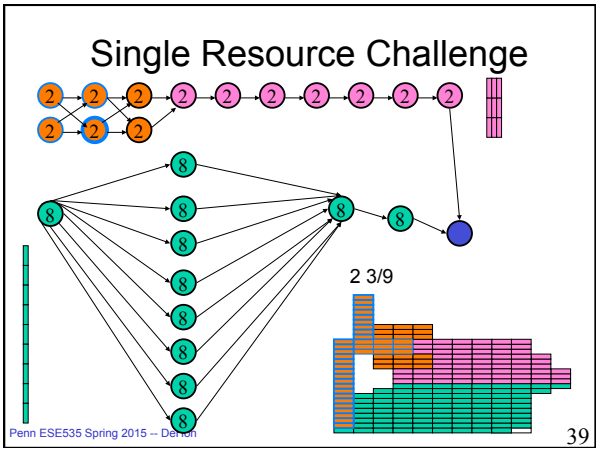
Single Resource Challenge
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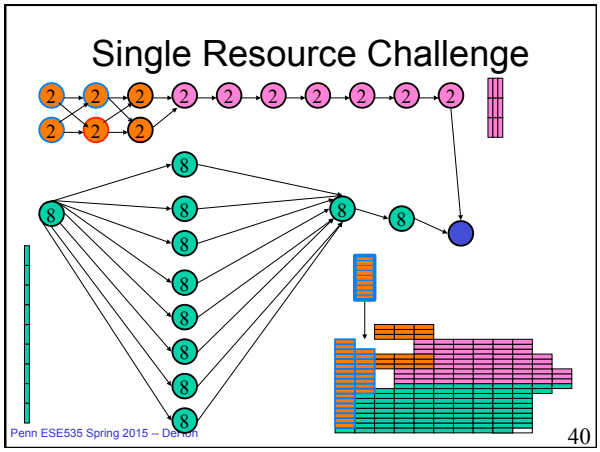
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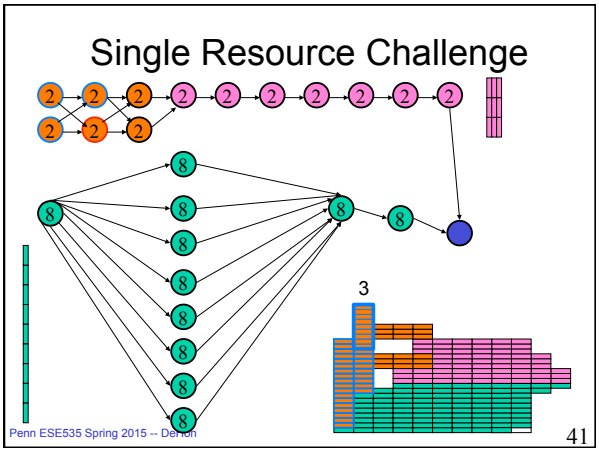
Single Resource Challenge
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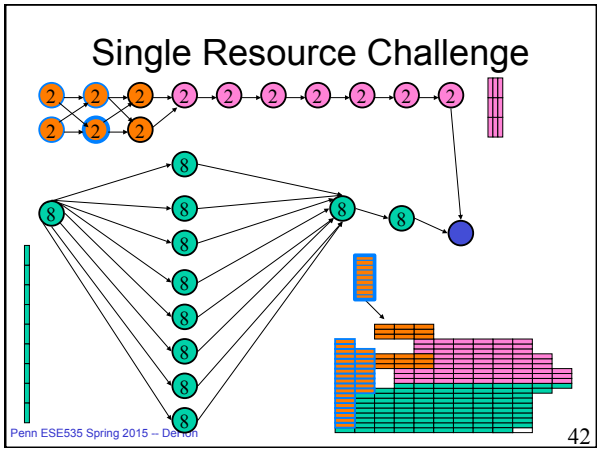
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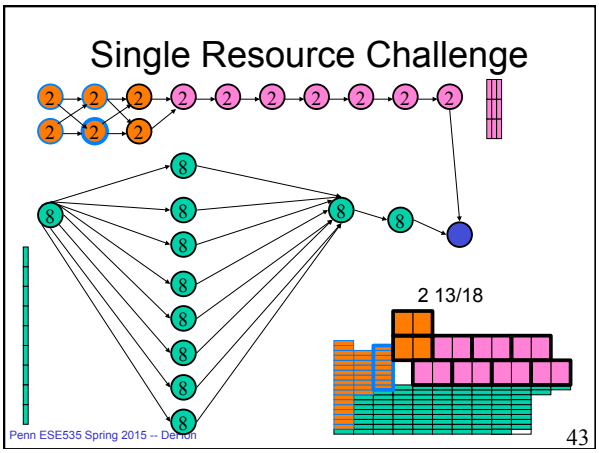
Single Resource Challenge
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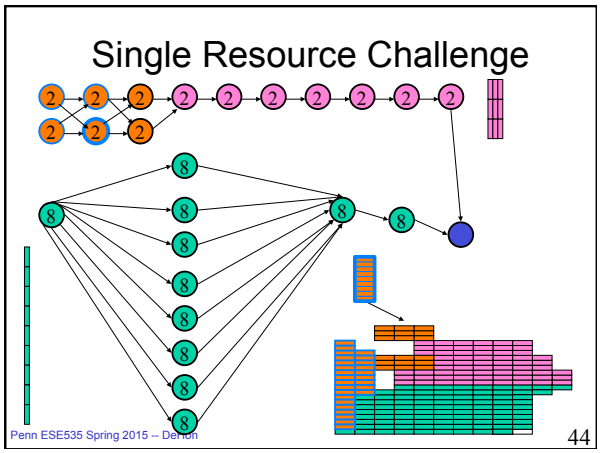
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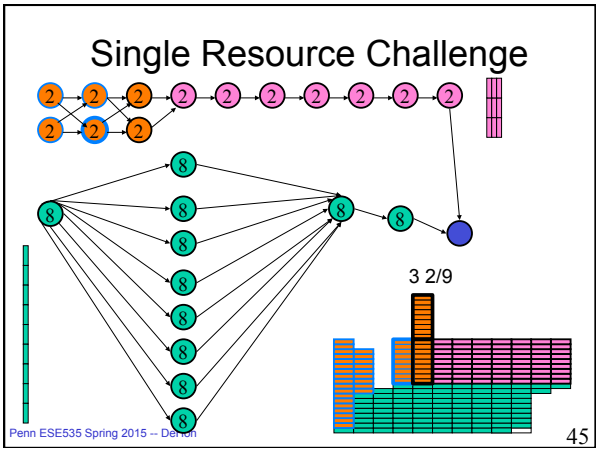
Single Resource Challenge
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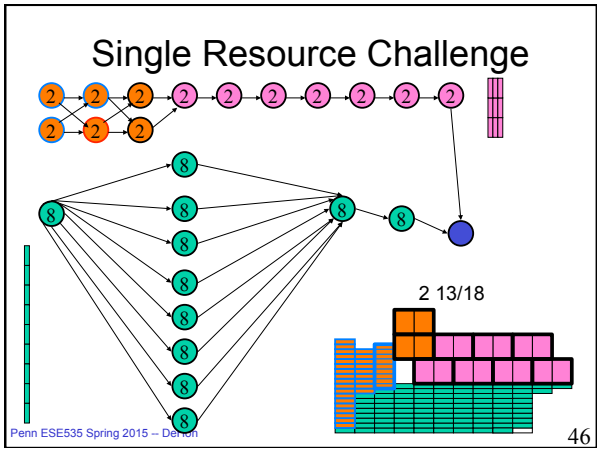
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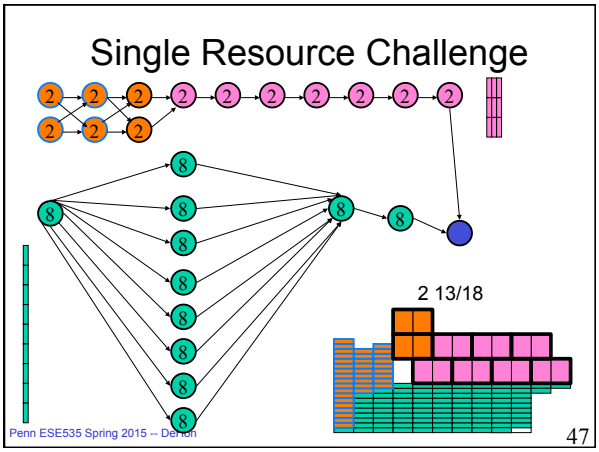
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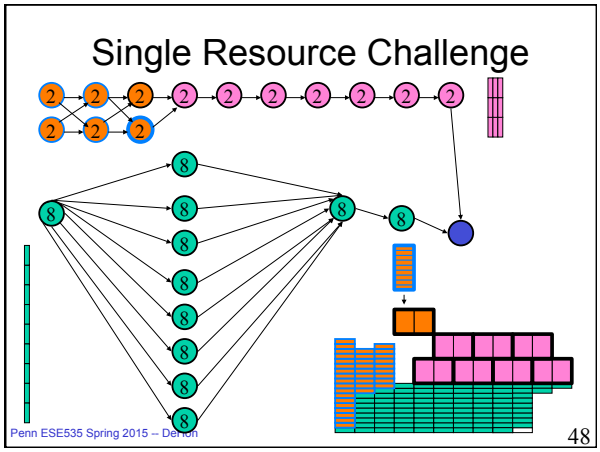
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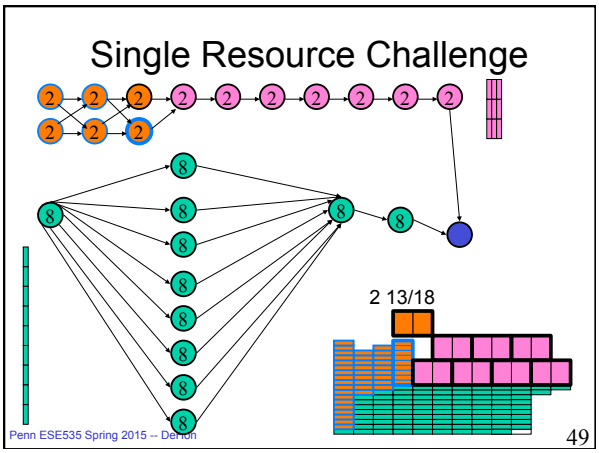
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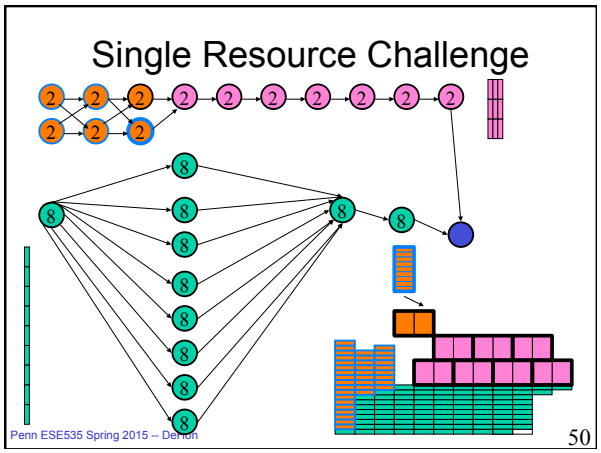
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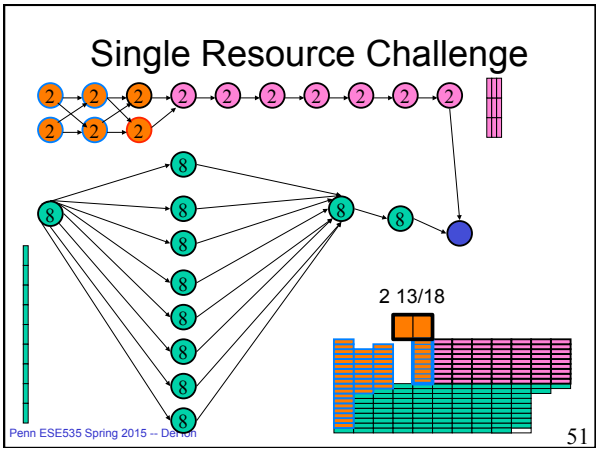
Single Resource Challenge
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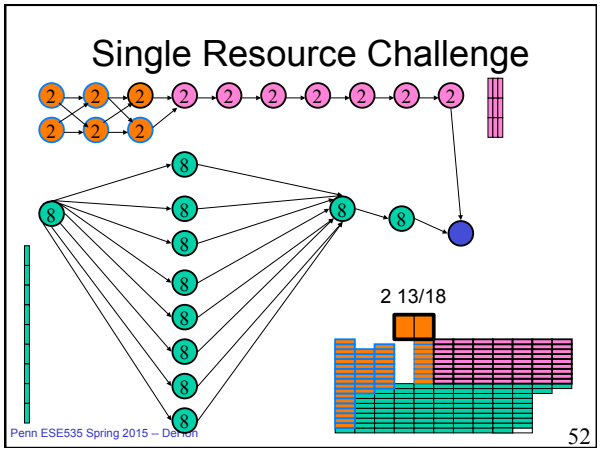
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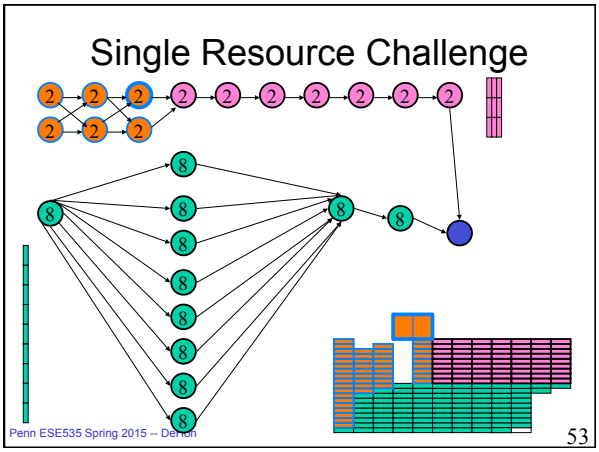
Single Resource Challenge
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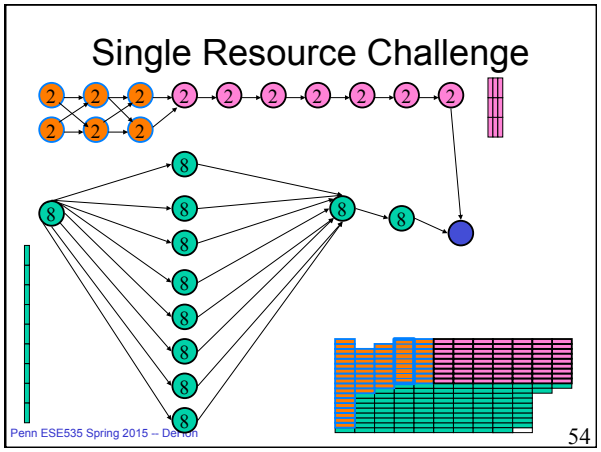
Single Resource Challenge
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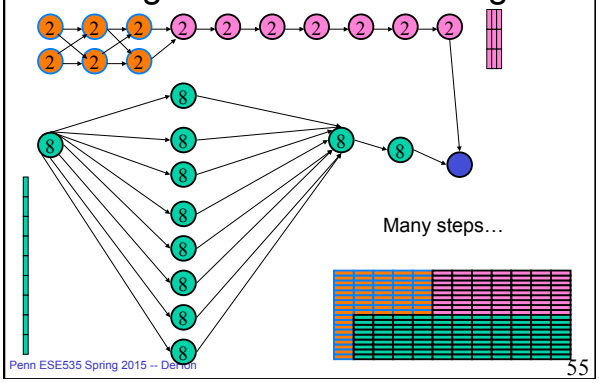
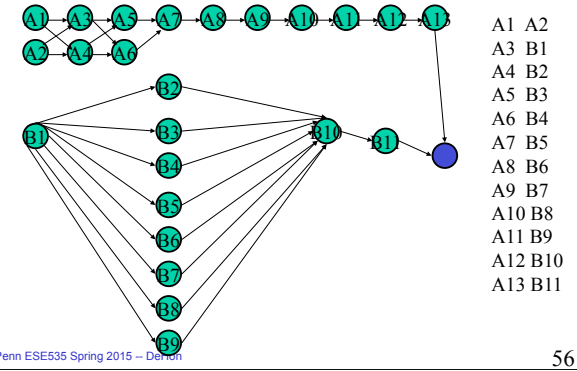
Single Resource Challenge



Single Resource Hard (5)



Force-Directed Algorithm

1. ASAP/ALAP schedule to determine range of times for each node
2. Compute estimated resource usage
3. Pick most constrained node (in largest time slot…)
   - Evaluate effects of placing in feasible time slots (compute forces)
   - Place in minimum cost slot and update estimates
   - Repeat until done

Force-Directed Runtime

- Evaluate force of putting in timeslot O(N)
  - Potentially perturbing slack on net prefix/ postfix for this node → N
- Each node potentially in T slots: ×T
  - T = schedule target
- N nodes to place: ×N
- $O(N^2T)$
  - Loose bound--don't get both T slots and N perturbations

Force-Directed Algorithm (from reading)

1. ASAP/ALAP schedule to determine range of times for each node
2. Compute estimated resource usage
3. Select a move
   - For each unscheduled op
     - Evaluate effects of placing in feasible time slots (compute forces)
   - Select move results in minimum cost
   - Repeat until done

Force-Directed Runtime (from reading)

- Evaluate force of putting in timeslot O(N)
  - Potentially perturbing slack on net prefix/ postfix for this node → N
- Each selection N nodes to consider: ×N
- Each node potentially in T slots: ×T
  - T = schedule target
- N nodes to place: ×N
- $O(N^3T)$
  - Loose bound

How Greedy?

- Class FD:
  - Greedy selection of what to schedule next
  - Exhaustively consider where might go
- Reading FD:
  - Exhaustively consider what to schedule next **and** where might go
  - Greedy node assignment
    - Never revisit
- Exhaustive
  - Don't commit to an assignment
    - Consider all possible assignments

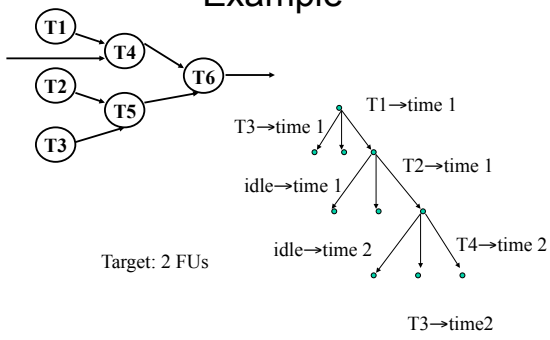
Branch-and-Bound

(for resource-constrained scheduling)

Brute-Force Scheduling (Exhaustive Search)

- Try all schedules
- Branching/Backtracking Search
- Start w/ nothing scheduled (ready queue)
- At each move (branch) pick:
  - available resource time slot
  - ready task (predecessors completed)
  - schedule task on resource
    - Update ready queue

Example



Branching Search

- Explores entire state space
  - finds optimum schedule
- Exponential work
  - O ($N^{(resources*time\text{-}slots)}$)
- Many schedules completely uninteresting

Reducing Work

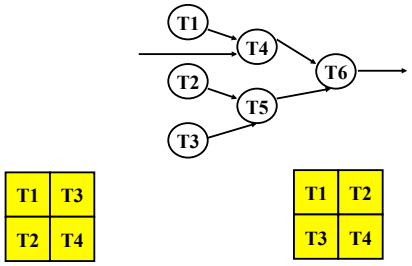
1. Canonicalize "**equivalent**" schedule configurations
2. Identify "**dominating**" schedule configurations
3. **Prune** partial configurations which will lead to worse (or unacceptable results)

"Equivalent" Schedules

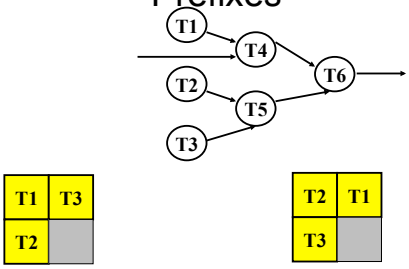
- If multiple resources of same type
  - assignment of task to particular resource at a particular timeslot is not distinguishing

Keep track of resource usage by capacity at time-slot.

"Equivalent" Schedule Prefixes



"Non-Equivalent" Schedule Prefixes

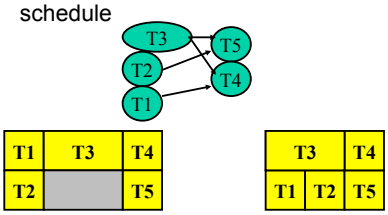


Pruning Prefixes

- Keep track of scheduled set
- Recognize when solving same sub-problem
  - Like dynamic programming finding same sub-problems
  - But no guarantee of small number of subproblems
    - set is power-set so 2^N
    - …but not all feasible,
      - so shape of graph may simplify

Dominant Schedules

- A strictly shorter schedule
  - scheduling the same or more tasks
  - will always be superior to the longer schedule



Pruning

- If can establish a particular schedule path will be worse than one we've already seen
  - we can discard it w/out further exploration
- In particular:
  - LB=current schedule time + lower_bound_estimate
  - if LB greater than known solution, prune

Pruning Techniques

Establish Lower Bound on schedule time

- Critical Path (ASAP schedule)
- Resource Bound

Alpha-Beta Search

- Generalization
  - keep both upper and lower bound estimates on partial schedule
    - Lower bounds from CP, RB
    - Upper bounds with List Scheduling
  - expand most promising paths
    - (least upper bound, least lower bound)
  - prune based on lower bounds exceeding known upper bound
  - (technique typically used in games/Chess)

Alpha-Beta

- Each scheduling decision will tighten
  - lower/upper bound estimates
- Can choose to expand
  - least current time (breadth first)
  - least lower bound remaining (depth first)
  - least lower bound estimate
  - least upper bound estimate
- Can control greediness
  - weighting lower/upper bound
  - selecting "most promising"

Note

- Aggressive pruning and ordering
  - can sometimes make polynomial time in practice
  - often cannot *prove* will be polynomial time
  - usually represents problem structure we still need to understand
- Coudert shows scheduling
  - Exact Coloring of Real-Life Graphs is Easy", in Proc. of *34th DAC, Anaheim, CA, June 1997.*

Multiple Resources

- Works for multiple resource case
- Computing lower-bounds per resource
  - resource constrained
- Sometimes deal with resource coupling
  - *e.g.* must have 1 A and 1 B simultaneously or in fixed time slot relation
    - *e.g.* bus and memory port

Summary

- Resource estimates and refinement
- Branch-and-bound search
  - "equivalent" states
  - dominators
  - estimates/pruning

Big Ideas:

- Estimate Resource Usage
- Use dominators to reduce work
- Techniques:
  - Force-Directed
  - Search
    - Branch-and-Bound
    - Alpha-Beta

Admin

- Assignment 5 due Thursday
- Reading for Wednesday online
- Office Hours Tuesday