Stability of Graph Neural Networks

- Let $S$ and $B$ be two GSOs such that $d(B, S) \leq \varepsilon / 2$
- Relative error $E$ with structural constraint $\|E - I\| \leq \varepsilon$
- Let $\Phi(S)$ be the output of a GST
- Let $\|\Phi(S) - \Phi(B)\| \leq C$ and $\|\Phi(S)\| \leq B$

Stability of Graph Scattering Transforms

- It holds that $\|\Phi(B(S, x)) - \Phi(S, x)\| \leq C \|B - I\| \|x\|$

The result is linear in the size of the perturbation $\varepsilon$
- The result is linear in the integral Lipschitz constant $C$
- More layers $\Rightarrow$ Less stability
- Consider a perturbation of the form $S - \alpha I$ (edge removal): $\varepsilon$-Eigenvalues change proportionally to their value $\lambda_i \Rightarrow (1 - \alpha)\lambda_i$
- Larger eigenvalues change more than smaller eigenvalues

Numerical Experiments

- Rich representation $\Rightarrow$ Good accuracy in classification problems
- Examples: authorship attribution, source localization
- Performance is better with other non-trainable representations (GFT)
- Can be identified with stable linear filters
- Performance is worse than trainable GNN $\Rightarrow$ Can not be applied to other architectures

Conclusions

- Studied the stability properties of graph scattering transforms (GSTs)
- Graph wavelets need to be linearizable
- Introduced relative perturbation model
- Proved stability of GST to changes in the underlying graph support
- Proportional to perturbation size, independent of graph
- Showed resulting GST representation is rich enough