On Geographic Routing Without Location Information

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***Abstract*— One of the most interesting problems in wireless adhoc networks is finding scalable routing algorithms for message passing from source nodes to destination nodes in. Several routing algorithms exist in the literature. In particular, the ones that use geographic location information as the address seem to be more scalable to large networks. These methods however, require that all nodes in the network know the geographic location of all the others. While this is the case in certain sensor net applications, there are plenty of scenarios where geographic location information is not obtainable for all nodes. Recently, a new routing algorithm was proposed by Rao *et al.* which does not require geographic information for *all* of the nodes in the network, but assumes that peripheral nodes located at the boundary of the region of interest have location information. The algorithm is based on the use of a set of *virtual* coordinates which are calculated by averaging the $x-y$ coordinates of each node in the network with its nearest neighbors, while keeping the coordinates of the peripheral nodes on the boundary fixed. Simulations indicate that the virtual coordinates converge to fixed values when the graph is connected. It has been shown that such a routing scheme has a surprisingly high success rate, close to that of location aware routing algorithms, and performs even better when there are obstacles in the network.In this paper, we will explicitly calculate a closed form solution for the virtual coordinates, explore the connections to distributed coordination algorithms and provide a possible explanation for the high success rate of the algorithm. We further explore connections to several other areas such as distributed coordination algorithms as well as distance geometry, as well as the notion of resistance distance in graphs.**

I. Introduction

One of the most important problems in ad hoc networks is finding best routes for transmitting data between source-destination pairs in a multi-hop fashion. Several algorithms have been proposed for routing. These include on-demand routing [9] with some variations [10], distance vector routing [16] and geographic routing (cf. [17] and the references therein). In geographic routing each node's location is used as its address, and packets are forwarded in a greedy fashion towards the destination, i.e., packets are passed to the neighboring node which is closest in distance to the destination node. Of course one can envision a situation in which there are dead-ends, meaning that a node has no neighbor which is closer to destination than itself. Some variations of geographic routing such as GOAFR+ [12] have been able to successfully address this issue.

While geographic routing has many advantages, its main disadvantage is the assumption that georgraphic location of all the nodes are known. In certain settings such as sensor nets with GPS this assumption is natural, but more often than not, exact information is not available for all nodes. It is however, reasonable to assume that *some* nodes in the network know their exact location information while others do not [4]. In a recent paper [17], Rao *et al.* provide a simple but compelling algorithm for geographic routing when only the nodes on the boundary of the region have exact location information. (In [17], the authors also consider the more general setting where peripheral nodes do not have location information and use a certain triangularization algorithm to locate each other). First, the interior nodes of the network are labelled with some random initial coordinates. These random initial coordinates are then updated in a distributed iterative scheme, in which the x and y coordinates of each interior node is replaced by the average of the coordinates of its nearest neighbors (which could contain peripheral nodes). The converged value of these coordinates are then used for geographic routing, meaning that messages are passed to the node that is closest in distance (calculated using virtual coordinates) to the destination node. The above algorithm has shown to be surprisingly effective, in terms of the success rate of routing. More interestingly, simulations indicate that in the presence of random obstacles this type of routing tends to perform even better than geographic routing with location information [17]. Note that in this process the coordinates of the peripheral nodes are fixed and they do not change, however their value will influence the virtual coordinates of their neighbors (and the neighbors of neighbors, etc.)

The purpose of this paper is to find a close form solution for the value of the virtual coordinates in terms of the fixed coordinates of the peripheral nodes and provide a justification for the surprising performance of routing with virtual coordinates. It will be shown that if the network of nodes (including the peripheral ones) corresponds to a connected graph, the virtual coordinates always converge, and they converge to a point in the convex hull of the boundary nodes. Moreover, it will be shown that the coefficients of the convex combination are determined by the inverse of a diagonally perturbed Laplacian matrix of the graph of the interior (non-boundary) nodes of the network, or the

inverse of an $n-m$ dimensional principal submatrix of the Laplacian of the graph over n nodes, where n is the total number of nodes and m is the number of peripheral nodes. We note that numerous variations on the theme of "inverting" the Laplacian matrix of the graph have appeared in the graph theory literature. (See for example [14], [15], [3], [5]. We use these results to show that the value of the virtual coordinates conveys some information about a certain notion of distance on the graph, known as the "resistance distance". As a result, even though exact Euclidean distance information is not available, the nodes will have some information about distances. Further connections to a leader following distributed coordination algorithm [8] which appeared recently in the literature are discussed and some future research directions are presented.

II. Preliminaries

A. Graph Theory Preliminaries

In this section we introduce some standard graph theoretic notation and terminology. For more information, the interested reader is referred to [6].

An (undirected) graph $\mathcal{G}$ consists of a vertex set, $\mathcal{V}$, and an edge set $\mathcal{E}$, where an edge is an unordered pair of distinct vertices in $\mathcal{G}$. If $x, y \in \mathcal{V}$, and $(x, y) \in \mathcal{E}$, then x and y are said to be adjacent, or neighbors and we denote this by writing $x \sim y$. The number of neighbors of each vertex is its valence. A path of length r from vertex x to vertex y is a sequence of $r+1$ distinct vertices starting with x and ending with y such that consecutive vertices are adjacent. If there is a path between any two vertices of a graph $\mathcal{G}$, then $\mathcal{G}$ is said to be connected. If there is such a path on a directed graph ignoring the direction of the edges, then the graph is weakly connected.

The adjacency matrix $A(\mathcal{G}) = [a_{ij}]$ of an (undirected) graph $\mathcal{G}$ is a symmetric matrix with rows and columns indexed by the vertices of $\mathcal{G}$, such that $a_{ij} = 1$ if vertex i and vertex j are neighbors and $a_{ij} = 0$, otherwise. The valence matrix $D(\mathcal{G})$ of a graph $\mathcal{G}$ is a diagonal matrix with rows and columns indexed by $\mathcal{V}$, in which the (i,i)-entry is the valence of vertex i. The (un)directed graph of a (symmetric) matrix is a graph whose adjacency matrix is constructed by replacing all nonzero entries of the matrix with 1.

The symmetric singular matrix defined as:

$$L(\mathcal{G}) = D(\mathcal{G}) - A(\mathcal{G})$$

is called the Laplacian of $\mathcal{G}$. The Laplacian matrix captures many topological properties of the graph. The Laplacian L is always a positive semidefinite M-matrix (defined below) and the algebraic multiplicity of its zero eigenvalue is equal to the number of connected components in the graph. The n-dimensional eigenvector associated with the zero eigenvalue is the vector of ones, $\mathbf{1}_n$.

B. Nonnegative matrices

We review some standard results from the theory of nonnegative matrices. Interested reader is referred to [7] and [2] for further details. A matrix $M = [m_{ij}]$ is called nonnegative (positive) if $m_{ij} \geq 0 (> 0)$ for all i and j. A nonnegative matrix is called (row) stochastic, if all the row sums add up to 1. This implies that 1 is an eigenvalue, corresponding to an eigenvector which is the vector of all ones, denoted by $\mathbf{1}$. An $n \times n$ nonnegative matrix M is called irreducible if $(I+M)^{n-1} > 0$. It is well known that a symmetric nonnegative matrix is irreducible if the *graph* of the matrix is connected. The celebrated Perron Frobenius theorem implies that a nonnegative irreducible matrix always has its spectral radius $\rho(M) := \max_i |\lambda_i(M)|$ as an eigenvalue, with the corresponding left and right eigenvectors being positive. It turns out that parts of this theorem generalize to arbitrary nonnegative matrices: If M is an arbitrary nonnegative matrix, then $\rho(M)$ is an eigenvalue of M and there is a nonnegative vector $x \geq 0$, $x \neq 0$ such that $Mx = \rho(M)x$.

A closely related class of matrices are Metzler matrices or M-matrices. An M-matrix is a matrix with non-positive off diagonal entries and positive diagonal entries. It can be shown that any M-matrix L can be written as $L = sI - B$ where $s > 0$ and $B \geq 0$ with $s > \rho(B)$. Also, any matrix in this form is an M-matrix. M-matrices appear frequently in a variety of disciplines such as biology, economics, graph theory and theory of Markov chains. An important example is the Laplacian matrix of a graph, which is a singular M-matrix. In [2] (pp 134-138), more than 50 equivalent conditions are stated and proven to be equivalent to a matrix being a nonsingular M-matrix. An M-matrix L is called irreducible, if and only if the nonnegative matrix B in $L = sI - B$ is irreducible. As an example, Laplacian matrices of connected graphs are all irreducible M-matrices. A nonsingular M-matrix is always monotone, meaning that $x \geq 0$ implies $Lx \geq 0$. Moreover the inverse of L, if exists, is a nonnegative matrix. The inverse is positive if and only if the M-matrix is irreducible.

III. Problem Description

We now describe the problem setting as in [17]. Consider n nodes in a plane forming a multihop radio network. We assume that the nodes are fixed. Each node is capable of communicating in a bidirectional way, with any other node within a disk of a certain radius r. The radius r is assumed to be fixed and is determined by the amount of power available to each node. The neighborhood relationship between nodes can be represented by a simple undirected graph in which each node of the network is a node of the graph and there is an edge between two nodes if they are within the communication range of each other. The nodes are assumed to be distributed over a bounded region of the plane. It is assumed that there are exactly m nodes on the boundary of the region with $m < n$ (and usually $m << n$), denoted as peripheral or boundary nodes. The algorithm in [17]

which provides a scalable geographic routing scheme in the absence of location information for the interior nodes is based on a distributed iteration scheme in which each node i is assigned an initial guess as $x_i(0)$ and $y_i(0)$. The values of x_i and y_i at the subsequent time steps are determined by the average of the x and y coordinates of all of the nearest neighbors of node i as

$$\begin{aligned}
\begin{bmatrix} x_i(k+1) \\ y_i(k+1) \end{bmatrix} &= \begin{bmatrix} x_i(k) \\ y_i(k) \end{bmatrix} \qquad i = 1, \ldots, m \\
\begin{bmatrix} x_i(k+1) \\ y_i(k+1) \end{bmatrix} &= \begin{bmatrix} \frac{\sum_{j \in \mathcal{N}_i} x_j(k)}{d_i} \\ \frac{\sum_{j \in \mathcal{N}_i} y_j(k)}{d_i} \end{bmatrix} \\
i &= m+1, \ldots, n\,, \qquad (1)
\end{aligned}$$

where the first m nodes are at the boundary and have fixed known coordinates, and the remaining $n-m$ nodes are in the interior of the region whose coordinates change in accordance of the above iteration. The set $\mathcal{N}_i$ denotes the set of neighbors of the ith node and d_i denotes the cardinality of that set. Let $A = [a_{ij}]_{n-m \times n-m}$ be the adjacency matrix of the interior nodes of the graph, excluding the boundary nodes and the edges that are adjacent to them. Also, let D be the diagonal matrix of valencies of this graph. Furthermore, we define a set of m vectors denoted by $b_1, \cdots, b_m$ of dimension $n-m$ each, with the jth element of b_i being equal to 1 if and only if the jth internal node is a neighbor of ith peripheral node and 0 otherwise. We define the diagonal of the diagonal matrix $B_{n-m \times n-m}$ to be equal to the sum of b_is, i.e., $B = \text{diag}(\sum_{i=1}^{m} b_i)$. In essence, the ith diagonal entry of B will denote the number of peripheral nodes connected to the ith internal node. If an internal node is not a neighbor of any peripheral node, then the corresponding diagonal element of B is zero. We denote the $m \times 1$ vector of the x coordinates of the peripheral nodes as $x_p := [x_{1p} \ldots x_{mp}]^T$, and the y coordinates accordingly as y_p. Using the above definitions, we can write equation 1 in matrix form as

$$x(k+1) = Tx(k) \qquad (2)$$

where

$$T := \begin{bmatrix} I_m & 0_{m \times (n-m)} \\ (D+B)^{-1}[b_1 \ldots b_m] & (D+B)^{-1}A \end{bmatrix},$$

x is an n-vector of horizontal coordinates of all nodes. As the first m nodes are fixed, we can rewrite the equations in terms of the $n-m$ internal nodes as states and the m peripheral nodes as inputs. to this end, we partition the n vector x as $x = [x_p \; x_{int}]^T$.

$$\begin{aligned}
x_{int}(k+1) &= (D+B)^{-1}A \; x_{int}(k) \\
&+ (D+B)^{-1}[b_1 \ldots b_m]x_p \\
y_{int}(k+1) &= (D+B)^{-1}A \; y_{int}(k) \\
&+ (D+B)^{-1}[b_1 \ldots b_m]y_p. \qquad (3)
\end{aligned}$$

Our goal in the next section is to find closed form solutions for the steady state values of the virtual coordinates x and y.

IV. Steady state values of virtual coordinates

It is clear from equation 3 that existence of steady state values for the virtual coordinates depend on the stability of the $n-m$ dimensional matrix $F := (D+B)^{-1}A$. An immediate consequence of the definition of A, B and D is that F is an (entry-wise) nonnegative matrix, with some row sums adding to 1, for the row corresponding to nodes that are not neighboring any boundary node, and strictly less than 1, for those neighboring boundary nodes.

Standing Assumption: The graph of all n nodes is weakly connected with edges corresponding to the peripheral nodes being unidirectional, while the edges corresponding to the internal nodes are bidirectional.

The above assumption states that the peripheral nodes are fixed and do not update their coordinates, but they transmit their location information to their one-hop neighbors. From any peripheral node, there is a path to all internal nodes. In the following proposition we will prove that all of the eigenvalues of the F matrix are real and have a magnitude strictly less than unity. In other words, the spectral radius of the matrix F, denoted by $\rho(F)$ is strictly less than 1.

Proposition IV.1 *Let D and A be the valence and adjacency matrix of the graph of the internal nodes. Also let $B = \sum_{i=1}^{m} b_i$, with $(b_i)_j l = 1$ if lth internal node is neighbor to ith peripheral node and 0 otherwise. Then*

$$\rho(F) = \rho((D+B)^{-1}A) < 1.$$

Proof: Consider the graph of all n nodes of the network ignoring the direction of the edges connecting the peripheral nodes to the internal ones. Suppose we devise an iteration in which *each* of the n nodes (including peripherals) averages its coordinates with its nearest neighbors, similar to the problem formulation in [8]. It is clear that under the standing assumption, the resulting undirected graph is connected, hence its adjacency matrix is irreducible. Let $x = [x_{per} \; x_{int}]^T$ with x_{per} corresponding to the m peripheral nodes and x_{int} corresponding to the $n-m$ internal nodes. The nearest neighbor averaging can be written as $x(k+1) = \bar{F}x(k)$ with $\bar{F}$ partitioned accordingly as $\begin{pmatrix} \bar{F}_{11} & \bar{F}_{12} \\ \bar{F}_{21} & \bar{F}_{22} \end{pmatrix}$. A simple comparison between the matrix $\bar{F}$ and the matrix T defined earlier reveals that they both have the same 22 block, meaning that their $n-m$ dimensional principal submatrices are the same, i.e., $\bar{F}_{22} = (D+B)^{-1}A$. In other words, whether the peripheral nodes update their states or not, will not change the update equation for the internal nodes. Since the graph over n vertices is connected, $\bar{F}$ is an irreducible matrix. It is also stochastic, since the positions are averaged with nearest neighbors. As a result, its spectral radius is equal to unity. That $\rho((D+B)^{-1}A) \leq 1$ should be clear: the infinity norm of the matrix (its maximum absolute row sum) is equal to 1, and since the spectral radius is always no greater than any induced matrix norm, the inequality holds. However,

we need to prove that the inequality is strict. In order to do this, the next lemma we will prove that the spectral radius of a principal submatrix of any irreducible matrix has a strictly smaller spectral radius than that of the matrix itself. In other words, the above inequality is strict. The fact that the eigenvalues are real can be easily determined by noting that the matrix is similar to the symmetric matrix $(D+B)^{0.5}A(D+B)^{0.5}$. ■

We now prove the lemma that was used in proving the above proposition:

Lemma IV.2 *Let* $M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$ *be an irreducible nonnegative matrix with* M_{22} *as its* $n-m$ *dimensional principal submatrix. Then* $\rho(M_{22}) < \rho(M)$.

Proof: The fact that $\rho(M_{22}) \leq \rho(M)$ is well known, and can be found in [7] (Corollary 8.1.20, page 491). What remains to be proven is to show that the inequality is strict. In order to prove this, we use a simple contradiction argument. Suppose $\rho(M_{22}) = \rho(M) = \rho$. As the matrix M_{22} is also nonnegative, its spectral radius is an eigenvalue, furthermore, the corresponding eigenvector is always nonnegative. Therefore $M_{22}v = \rho v \geq 0$. We now augment the eigenvector v with $n-m$ zeros and form the vector q as

$$q = \begin{bmatrix} 0_{m\times 1} \\ v \end{bmatrix}.$$

The product of Mq is therefore equal to

$$Mq = \begin{bmatrix} M_{12}v \\ M_{22}v \end{bmatrix} = \begin{bmatrix} M_{12}v \\ \rho v \end{bmatrix} \geq \rho q,$$

where the last inequality is a direct consequence of nonnegativity of M_{12}, v, and their product. We therefore have shown that there exists a nonnegative vector $q \geq 0$ which satisfies

$$Mq \geq \rho q.$$

Since M is irreducible, it has a positive left and right eigenvector (Frobenius Theorem). Because of this and Theorem 8.3.4 of [7], the only way the inequality can hold is to have $Mq = \rho q$, implying that q is indeed the *Perron vector* of M, i.e., the positive eigenvector corresponding to the spectral radius ρ. But this results in a contradiction, since q is not element-wise positive and has zero elements by construction. Therefore our assumption that $\rho(M_{22}) = \rho(M)$ is false. As a result, $\rho(M_{22}) < \rho(M)$, and the proof is complete. ■

Now that we established the stability of the matrix $(D+B)^{-1}A$, we can explicitly calculate the value of virtual coordinates.

Theorem IV.3 Main result*: Consider the iteration scheme given in 3, where the virtual coordinates are averaged with coordinates of the nearest neighbors and the peripheral nodes are fixed. In steady state, the virtual coordinates converge to points in the convex hull of the boundary nodes. Moreover, the coefficients of the virtual coordinates can be explicitly calculated as*

$$\begin{aligned} \bar{x} &= \lim_{k\to\infty} x_{int}(k) = (D-A+B)^{-1}[b_1 \dots b_m]x_p \\ \bar{y} &= \lim_{k\to\infty} y_{int}(k) = (D-A+B)^{-1}[b_1 \dots b_m]y_p. \end{aligned} \quad (4)$$

Proof: From proposition 1, we know that the spectral radius of $(D+B)^{-1}A$ is strictly less than one, therefore, $\lim_{k\to\infty} x_{int}(k)$ exists. This can be seen by considering the iteration as a stable linear system driven by m constant inputs. Alternatively, we can view the equation in terms of the positions of all n nodes, and then view it as an autonomous system with the system matrix T, which is a nonnegative, triangular stochastic matrix. The matrix could also be viewed as the transition probability matrix of a Markov chain m ergodic classes. In either case, we can write

$$\begin{aligned} \bar{x} &= \lim_{k\to\infty} x_{int}(k) = \lim_{k\to\infty} [(D+B)^{-1}A]^k x_{int}(0) \\ &+ \lim_{k\to\infty} \sum_{i=0}^{k-1} [(D+B)^{-1}A]^k (D+B)^{-1}[b_1 \dots b_m]x_p. \end{aligned}$$

Since $\rho((D+B)^{-1}A) < 1$, the first limit goes to zero, and as a result the initial values of the *virtual* coordinates for the internal nodes will be *forgotten*. Because the matrix $(D+B)^{-1}A$ has subunit spectral radius,

$$\sum_{i=0}^{\infty} [(D+B)^{-1}A]^i = [I-(D+B)^{-1}A]^{-1},$$

which therefore implies that

$$\bar{x} = [I-(D+B)^{-1}A]^{-1}(D+B)^{-1}[b_1 \dots b_m]x_p.$$

By factoring the $(D+B)^{-1}$ term inside of the brackets we have

$$\bar{x} = (D+B-A)^{-1}[b_1 \dots b_m]x_p.$$

Similarly we can write the exact same thing for the final value of the y coordinates:

$$\bar{y} = (D+B-A)^{-1}[b_1 \dots b_m]y_p.$$

Note that the same formula could be obtained by applying the Final Value Theorem of the z-transform to the discrete-time linear system 3.

The matrix $(D-A+B)$ is the Laplacian matrix of the graph of the internal nodes perturbed by the diagonal matrix B (equivalently, we can think of it as the principal submatrix of the Laplacian of the graph over n nodes). It is nonsingular, symmetric and positive definite. A symmetric nonsingular M-matrix is called a Stieltjes matrix. It is known that the inverse of an irreducible Stieltjes matrix is always positive (element-wise) and positive definite (cf. [2], page 171). As a result, $(D-A+B)^{-1}$ is a totally positive matrix. We will now show that the product of $(D+B-A)^{-1}[b_1 \dots b_m]$ is a positive, $n-m \times m$ row stochastic matrix. This can be seen immediately by looking at the matrix T and noting that the powers of this matrix are stochastic as well, or by the following argument.

Let $\mathbf{1_m}$ be the m dimensional vector of all ones. we will show that $(D+B-A)^{-1}[b_1 \dots b_m]\mathbf{1_m} = \mathbf{1_{n-m}}$. To see this multiply both sides by $(D+B-A)$, and note that $(D-A)\mathbf{1_{n-m}} = 0$ (since the Laplacian has zero row sums). Also, note that $B\mathbf{1_{n-m}} = b_1 + \dots + b_m$. The result then follows immediately. We have therefore shown that the final value of the virtual coordinates of each node is equal to some convex combination of the coordinates of the peripheral nodes, with coefficients of convex combinations being positive. ■

Remark IV.4 *The above theorem suggests that the location of the value of the coordinates is determined directly from the inverse of the diagonally perturbed Laplacian of the graph. While the Laplacian matrix $L := D - A$ is not invertible, addition of nonzero diagonal matrix (with at least one non-zero entry) will make the matrix weakly diagonally dominant and invertible.*

Remark IV.5 *Invertibility of $(D+B-A)$ implies that no two rows (or columns) in $(D+B-A)^{-1}$ are identical. However, it is still possible to have two different nodes (with different coordinates) with the exact same virtual coordinate.*

The matrix $(D-A+B)^{-1}$ contains information about the "resistance distance" between the nodes of the interior graph. Formally, the resistance distance, also known as the electric distance, is the effective resistance between any two nodes of the graph, if each edge of the graph is though of as a unit resistor. It can be shown that resistance distance is indeed a distance function [11], and it collapses to the usual graphic distance (i.e., minimum path lengths between nodes) if the graph is a tree. If there are multiple paths between two nodes in the graph, the resistance distance between these two nodes is less than the total number of hops in each path. The next theorem, whose proof can be found in [5], illustrates the connection between the inverses of nonsingular M-matrices and resistance distance in graphs.

Theorem IV.6 *[5], [1] Let $M = sI - A$ with $A \geq 0$ irreducible and $s \geq \rho(A)$ be a symmetric nonsingular M-matrix (i.e., a Stieltjes matrix). Then $R_{ij} \geq 0$, the resistance distance between node i and node j of the connected graph whose adjacency matrix is given by A can be written as*

$$R_{ij} = M_{ii}^{-1} + M_{jj}^{-1} - 2M_{ij}^{-1} \qquad i,j = 1,\dots,n.$$

Furthermore, If the matrix M is not invertible, the inverse can be replaced by any generalized inverse.

The consequence of the above theorem is that the inverse of $(D-A+B)$ is a Gramm matrix (i.e., the positive definite matrix of inner products of points defined based on resistance distances) in some $n-m$-dimensional space. As a result, when we calculate the virtual coordinates, we are just finding an embedding of the graph with a given resistance distance in a 2-dimensional space. Therefore, the routing algorithm in [17] is indirectly using resistance distance information instead of Euclidean or graphic distance.

V. Discussion and concluding remarks

We proved that the value of the geographic coordinates is determined by a convex combination of the peripheral nodes, which can be calculated explicitly. To find out the intuitive meaning of these values, we note that the term $[b_1 \dots b_m]x_p = b_1x_{p1} + \dots + b_mx_{pm}$, is the value of the coordinates injected to the one-hop neighbor of the peripheral nodes. The location information is propagated among the internal nodes, first getting into the neighbors of the peripheral nodes and then propagating through the graph according to the resistance distance between the neighbors of the peripheral nodes and the rest of the internal nodes. Alternatively, One could also consider the following mechanical interpretation for the final value of virtual coordinates [13]: Since all virtual coordinates converge to a convex combination of the peripheral nodes, the final values of these coordinates can be thought of as points in an interior of an m-simplex in R^{m-1}, in which each peripheral node is an extreme point of such a simplex. Now assume that the edges of the graph (including the peripheral nodes) are made out of ideal rubber bands [13] and the peripheral nodes are glued to the extremal points of the m simplex, then the final value of virtual coordinates would be the final location of the remaining nodes, when the rubber band stops oscillating. In the end, the weights of the convex combination determine how "close" each node has been (in the sense of the resistance distance) to each of the peripheral nodes. Therefore, even though the nodes are not aware of distances with one another, the virtual coordinates have a certain distance information encoded in them. As a result, routing using these coordinates is likely to be more successful when there are obstacles in the network. This is due to the fact that the measure of proximity is not in terms of Euclidean distance, but is given in terms of resistance distance (which would be less than the graphic distance, if there are multiple paths between two nodes). The important point is that this distance information represents "the logical distance", as opposed to "geographic distance". Finally, we note that the above algorithm for calculating virtual coordinates is the same as the leader follower coordination algorithm in [8], with two differences: first difference is that the positions are averaged with neighbors as opposed to the headings. Secondly, the fixed peripheral nodes (which can be interpreted as formation leaders) are more than 1. More importantly, here the topology of the network is fixed, but in [8], the topology changes with time. Also, note that including each node's position in the averaging will speed up the convergence drastically, as it will increase the mixing rate of the corresponding Markov chain with transition matrix T with m ergodic classes as in equation (2).

In [8], we also showed that under certain connectivity assumptions, all nodes converge to the state of the fixed

node, if there is only one fixed node. This was true even if the topology of the network changed with time. It can be proved that nodes converge to a convex hull of peripheral nodes, even if the topology changes but a certain notion of "connectivity through time" is maintained. A next step would be to explore the possibility of routing when the network topology changes, but some weak notion of connectivity is maintained. Also, it is important to characterize the exact relationship between the Euclidean distance of the nodes using virtual coordinates and the resistance distances.

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