Fall 2017   CIS 262

Automata, Computability and Complexity
Jean Gallier

Homework 8

November 14, 2017; Due November 21, 2017, beginning of class

Problem B1 (40 pts). Give context-free grammars for the following languages:

(a) $L_5 = \{wcw^R \mid w \in \{a,b\}^*\}$ (w^R denotes the reversal of w)

(b) $L_6 = \{a^m b^n \mid 1 \leq m \leq n \leq 2m\}$

(c) For any fixed integer $K \geq 2$,

$L_7 = \{a^m b^n \mid 1 \leq m \leq n \leq Km\}$

(d) $L_8 = \{xcy \mid |x| = 2|y|,\, x, y \in \{a,b\}^*\}$

In each case, give a (very) brief justification of the fact that your grammar generates the desired language.

Problem B2 (60 pts). Given a context-free language L and a regular language R, prove that $L \cap R$ is context-free.

Do not use PDA's to solve this problem!

Use the following method. Without loss of generality, assume that $L = L(G)$, where $G = (V, \Sigma, P, S)$ is in Chomsky normal form, and let $R = L(D)$, for some DFA $D = (Q, \Sigma, \delta, q_0, F)$. Use a kind of cross-product construction as described below. Construct a CFG G_2 whose set of nonterminals is $Q \times N \times Q \cup \{S_0\}$, where S_0 is a new nonterminal, and whose productions are of the form:

$$S_0 \to (q_0, S, f),$$

for every $f \in F$;

$$(p, A, \delta(p,a)) \to a \quad \text{iff} \quad (A \to a) \in P,$$

for all $a \in \Sigma$, all $A \in N$, and all $p \in Q$;

$$(p, A, s) \to (p, B, q)(q, C, s) \quad \text{iff} \quad (A \to BC) \in P,$$

for all $p, q, s \in Q$ and all $A, B, C \in N$;

$$S_0 \to \epsilon \quad \text{iff} \quad (S \to \epsilon) \in P \text{ and } q_0 \in F.$$

Prove that for all $p, q \in Q$, all $A \in N$, all $w \in \Sigma^+$, and all $n \geq 1$,

$$(p, A, q) \overset{n}{\underset{lm}{\Longrightarrow}}_{G_2} w \quad \text{iff} \quad A \overset{n}{\underset{lm}{\Longrightarrow}}_{G} w \quad \text{and} \quad \delta^*(p, w) = q.$$

Conclude that $L(G_2) = L \cap R$.

TOTAL: 100 points.