

**Typos and Errors In  
Linear Algebra and Optimization  
with Applications to Machine Learning, Vol. I**

by  
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10, middle of the page, change “First, we quantity” to “First, the quantity”.

13, top of page, change “more equations that variables” to

“more equations than variables”.

67, Problem 2.3, change  $(x_1 + x_2, y_1, +y_2)$  to  $(x_1 + x_2, y_1 + y_2)$ .

103, line -2, “Section 4.2” should be “Section 4.1”.

108, line 3, the next to the last entry in the vector is  $-2$ , not  $2$ .

147, change “to far afield ” to “too far afield”.

162, the second row of the permutation should be  $2\ 3\ 1$ .

167, line -2, missing  $+$  sign before  $a_{11}$ .

167, line -1, the expression should be

$$a_{11}a_{22} - a_{12}a_{21}.$$

168, line 1, the expression should be

$$a_{11}a_{22} - a_{12}a_{21}.$$

193, the reference to Hoffman and Kunze is wrong.

197, in Problem 6.10, last line, in the rightmost expression,  $dx$  should be  $-dx$ .

217, the statement that  $L^{-1}$  is obtained from  $L$  by flipping some signs is incorrect. The  $k$ th column of  $L$  is the  $k$ th column of  $E_k^{-1}$ .

239, last line, change  $-1/\sqrt{5}$  to  $-1/\sqrt{2}$

241, change line 2 to “of the  $1 \times 1$  matrix  $(a_{11})$  in  $A$ ”.

245, line 1, change “methods” to “method”

245, line -4, change “Cholsesky” to “Cholesky”.

257, in Proposition 7.13, the statement  $E_p \cdots E_1 = F_q \cdots F_1$  is false unless  $A$  has rank  $m$ . The proof needs some corrections and also has some typos.

261, in the example from Kumpel et al., in the matrix  $N$ , change  $-3$  to  $3$ , and the general solution is wrong. Rows 2 and 3 should be permuted and  $-3$  should be  $3$ .

275, Problem 7.3, change “your” to “you”.

281, in Problem 7.12, missing period at the end of the equation.

282, change “Question (c)” to “Part (3)”.

282, in Problem 7.14(3), bad alignment in the matrix  $A$ .

283, in Part (4), bad alignment in the matrix  $R$ .

282, in Part (5), change “fuction” to “function”.

283, Problem 7.15, the second formula should be

$$\bar{b}_{ij} = \left( a_{ij} - \sum_{k=1}^{j-1} b_{ik} \bar{b}_{jk} \right) / b_{jj}.$$

287, in Definition 8.1, delete  $z$  in  $x, y, z \in E$ .

293, line 2,  $\|u\|_q^q$  should be  $\|v\|_q^q$ .

295, in the Minkowski identity, the exponents  $q$  and  $1/q$  in the third term are  $p$  and  $1/p$ .

299, change  $\bar{a}_{ij}$  to  $\overline{a_{ij}}$  in the definition of the conjugate.

302, line 1, change “eigenvalue” to “eigenvalues”.

304, In (3), change to “It shown in Section 14.1 that the trace ...”.

306, “the Frobenius norm is not a subordinate matrix norm for  $n \geq 2$ .”

310, delete  $=$  after  $\|A\|_2$

315, top of page, change “relative order of the order” to “relative error of the order”.

319, in Definition 8.11,  $A = V\Sigma U^*$ .

329, in Definition 8.16, change “real of complex” to “real or complex”.

333, Problem 8.1, in line 2, the matrix is  $A$ , not  $B$ .

337, just above Problem 8.15, change to  $\text{cond}_2(A_m) \approx 39, 206$ .

341, bottom of page, change  $\|B^k\|$  to  $\|B_\epsilon^k\|$

341, The beginning of the proof assumes prematurely that the limit exists. In fact, this follows from the second step. Also,  $\|B_\epsilon\| < 1$  should be  $\rho(B_\epsilon) < 1$ .

343, In the proof of (1), all the  $B_1$  should be  $B$ .

345, the description of the entries in  $E$  in  $F$  uses incorrect signs.

351, Line 4, change “at lot” to “a lot”.

358, on line 10, add “starting with the matrix  $\lambda^2 D - \lambda E - \lambda F$ , we get”

358, change (2) to “For any  $\alpha \neq 0$ ,  $\alpha$  is an eigenvalues of  $J$  iff  $-\alpha$  is an eigenvalues of  $J$ , and if  $\alpha$  is an eigenvalues of  $J$ , then  $\alpha^2$  is an eigenvalue of  $\mathcal{L}_1$ .”

358, delete the last sentence of Proposition 9.6.

359, change  $\max_{\lambda} |p_J(\lambda)|=0$  to  $\max_{\alpha} |p_J(\alpha)|=0$ .

At the end of proof change “if  $\neq 0$ ” to “since  $\neq 0$ ”.

364, in Problem 9.5, change the second sentence to “That is, if  $A$  is an invertible Hermitian matrix with the splitting  $A = M - N$  where  $M$  is invertible, if the Hermitian matrix  $M^* + N$  is positive definite and if  $\rho(M^{-1}N) < 1$ , then  $A$  is positive definite.”

400, in Problem 10.2, change “hyperlane” to “hyperplane”.

423, line 5, change to “since our inner product is positive definite”.

432, line 11, change  $W(x) \geq 0$  to  $W(x) > 0$ .

434, change line 8, change to “is a map satisfying Condition (2), and that (2) implies (3).”

442, In the Remark, Part (2), change  $R_2 = DR_1$  to  $R_1 = DR_2$ .

445, in Proposition 11.15, the last sentence should be “Moreover, equality holds iff either  $A$  has orthogonal rows in the left inequality or orthogonal columns in the right inequality.”

452, in Problem 11.8, line 4, change “and” to “to”.

455, Problem 11.13 is identical to Problem 11.6.

467, in Definition 12.3, change “if” to “is”.

505, bottom of page, change “but by its conjugate” to “but by the conjugate of the original matrix”.

508, in Proposition 13.16, the last sentence should be “Moreover, equality holds iff either  $A$  has orthogonal rows in the left inequality or orthogonal columns in the right inequality.”

514, top of page, change “a linear involution is called a projection” to “a linear idempotent map is called a projection”.

515, line 2, change  $(\text{id}+f) \circ (\text{id}-f) = \text{id}-f^2$  to  $(\text{id}-f) \circ (\text{id}+f) = \text{id}-f^2$ .

520, in Proposition 13.28, change the beginning to “If  $p, q \geq 1$  and  $1/p + 1/q = 1$ , or  $p = 1$  and  $q = \infty$ , or  $p = \infty$  and  $q = 1$ ,”

540, in the proof of Theorem 14.1, change “eigenvalue in  $K$ ” to “eigenvalues in  $K$ ”.

541, line 6-7, change to “However, since the entries in the first column of  $U$  are all zero for  $i = 2, \dots, n$ , we get”

545, Figure 14.1 has the wrong scale.

547, Figure 14.2 has the wrong scale. In the caption, it should be  $|z - 1| \leq 11$ .

548, Figure 14.3 has the wrong scale.

567, in Definition 15.2, change “conjugate of  $q$ ” to “conjugate of  $X$ ”.

574, in Definition 15.2, change “homomorphim” to “homomorphism”.

591, line 5, delete  $= 0$  in  $v \in \text{Ker}(f) \cap \text{Im}(f)$ .

593, near bottom, missing space in “defined such”.

613, line 7, insert “as” before “the”.

615, In Proposition 16.14 and up to Section 16.7, change  $r$  to  $m$ .

616, in the proof of Theorem 16.13, line 6, change min to inf. At the end of the proof, explain that since the inf is achieved, it can be replaced by min.

616, line -1, change  $\lambda_1, \dots, \lambda_k$  to  $\alpha_1, \dots, \alpha_k$ .

618, Proposition 16.16, in the proof, change “form” to “from”.

619, in the proof of Proposition 16.16, line 12, change to “If  $i = k$  and  $j = 1$ , we obtain  $\lambda_k(A) + \lambda_1(B) \leq \lambda_k(A + B)$ ”. Line -9, change to  $\lambda_k(A) + \lambda_1(B) \leq \lambda_k(A + B)$ .

633, the bottom diagonal entry of the matrix should be 2.1206 (no extra period).

635, in Theorem 17.2, line 3, change  $H$  to  $U$ , and on line 6, change the first  $H$  to  $U$ .

640, line 11, insert “a” between “to” and “tridiagonal”.

647, line -5, insert “the” between “then” and “matrix”.

650, line 24, change “necessary” to “necessarily”.

652, line -6, change  $\alpha_n = u_n^* A U_n$  to  $\alpha_n = u_n^* A u_n$ .

652, line -6 change to “Arnoldi’s algorithm becomes”.

679, line 3, change “unnormalized” to “unnormalized”.

681, line -6, change  $\text{cut}(A_i, \overline{A_i})$  to  $\text{cut}(A_i)$ .

683, Problem 18.3. The correct result is that the eigenvalues are 0, 1 with multiplicity  $m + n - 2$ , and 2.

703, in Theorem 20.1, line 5, change to “singular values of  $A$ ”.

706, line 5, change  $\sigma_{k+1}, \dots, \sigma_r$  to  $\sigma_{k+1}, \dots, \sigma_r$  (missing comma).

710, in Theorem 20.3, change to “singular values of  $A$ ”.

716, in Problem 20.2, just before (2) the equation is

$$\det(zI_{m+n} - S) = z^{n-m} \det(z^2 I_m - AA^\top).$$

In Part (2), change to “the eigenvalues of  $S$  are  $\pm\sigma_1, \dots, \pm\sigma_r$ , with  $m + n - 2r$  additional zeros.”

718, in Problem 20.11, in Part (2), change  $Z^\top X$  to  $X^\top Z$ ,  $R^\top$  to  $R$ , and  $VU^\top$  to  $UV^\top$ .

723, line 10, in the proof of Theorem 21.1,  $A(v_2 - v_2)$  should be  $A(v_2 - v_1)$ .

757, the reference to Hoffman and Kunze is wrong.

757, the reference to Godement should be Godement (1963).

792, the reference Godement (1958) should be deleted.

792, the reference Kenneth and Ray should be Hoffman and Kunze.