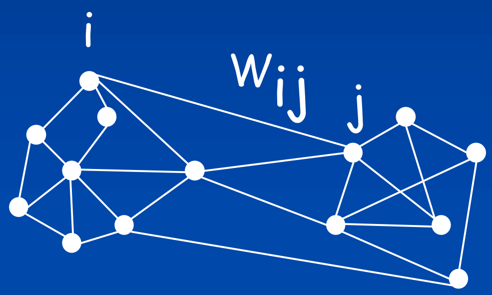
MULTISCALE SEGMENTATION

Florence BENEZIT , Jianbo SHI, 2004

Introduction

Graph Based Object Segmentation



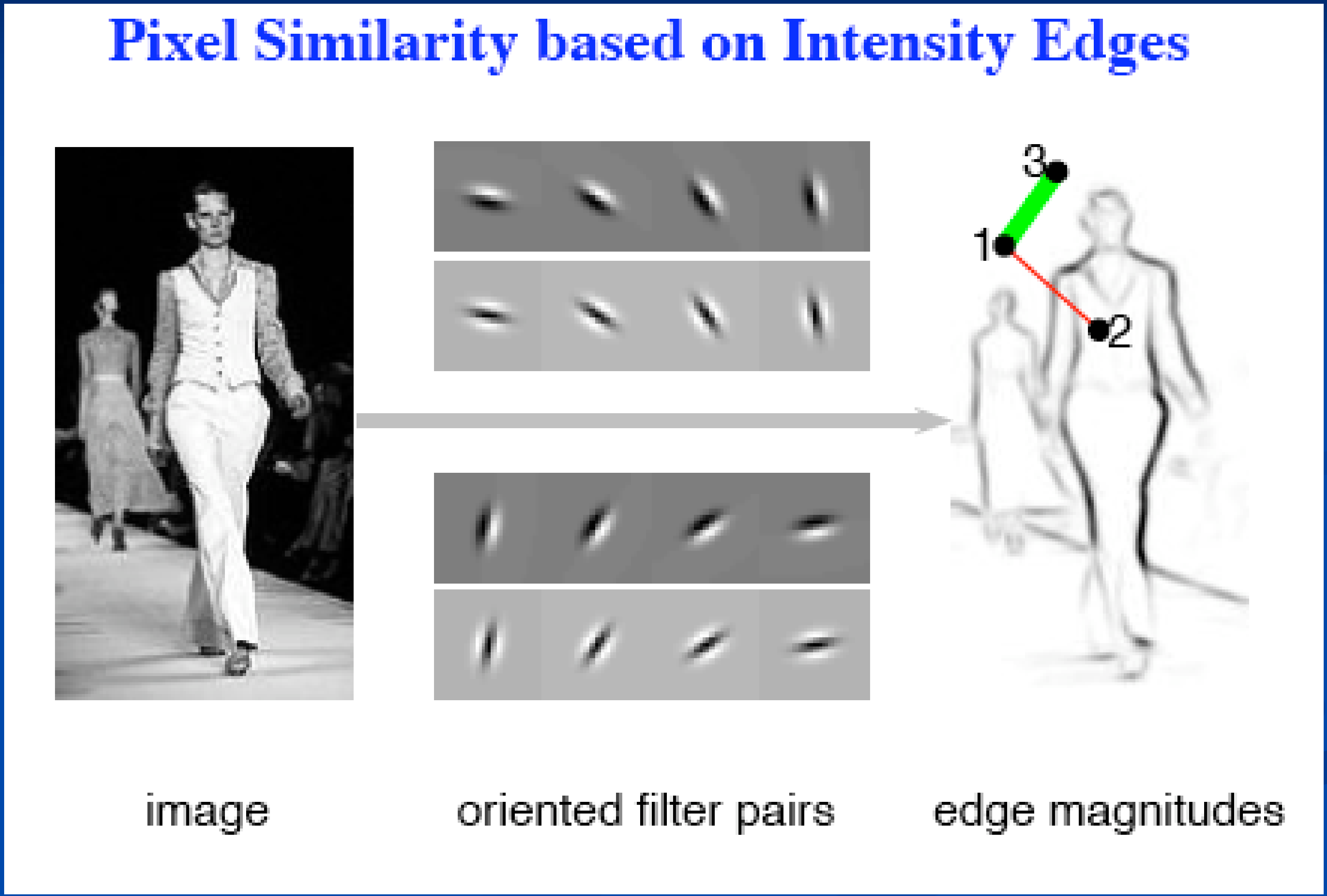
$$G = \{V, E\}$$

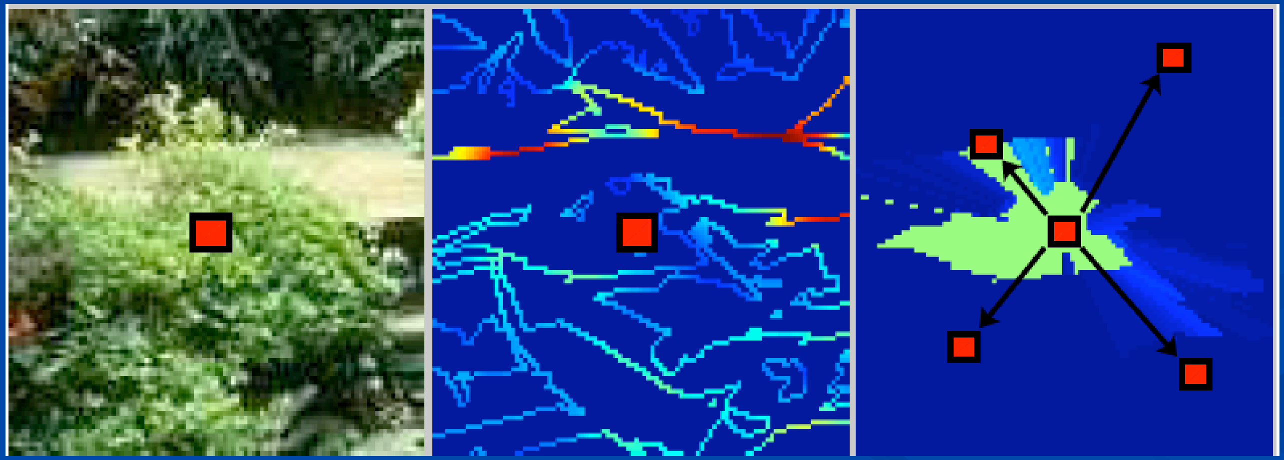
V: graph nodes
E: edges connection nodes

Image = { pixels }
Pixel similarity

Segmentation = Graph partition

Pixel Affinity graph





Normalized Cut As Generalized Eigenvalue problem

$$X = [X_1, \dots X_K]$$

Program PNCX:

$$\text{maximize } \varepsilon(X) = \frac{1}{K}\sum_{l=1}^{K}\frac{X_l^T W X_l}{X_l^T D X_l}$$

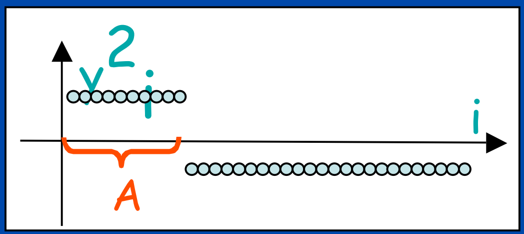
$$\text{subject to } X \in \{0,1\}^{N\times K}, \quad X 1_K = 1_N.$$

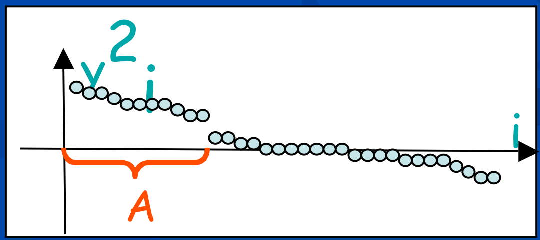
$$Z = X(X^T D X)^{-\frac{1}{2}}$$

Relaxed to program PNCZ:

$$\text{maximize } \varepsilon(Z) = \frac{1}{K}\text{tr}(Z^T W Z)$$

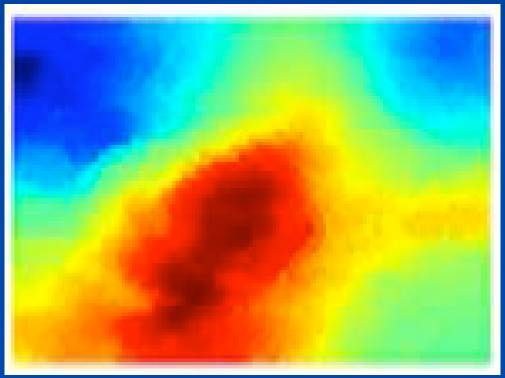
$$\text{subject to } Z^T D Z = I_K.$$



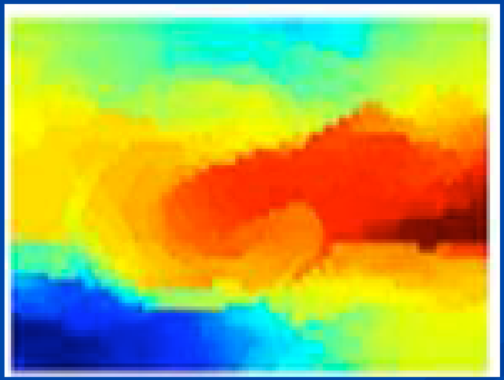


How big connection radius?

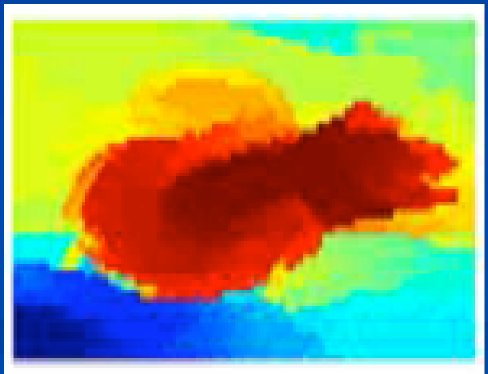
BAD AND GOOD EIGENVECTORS



r=1

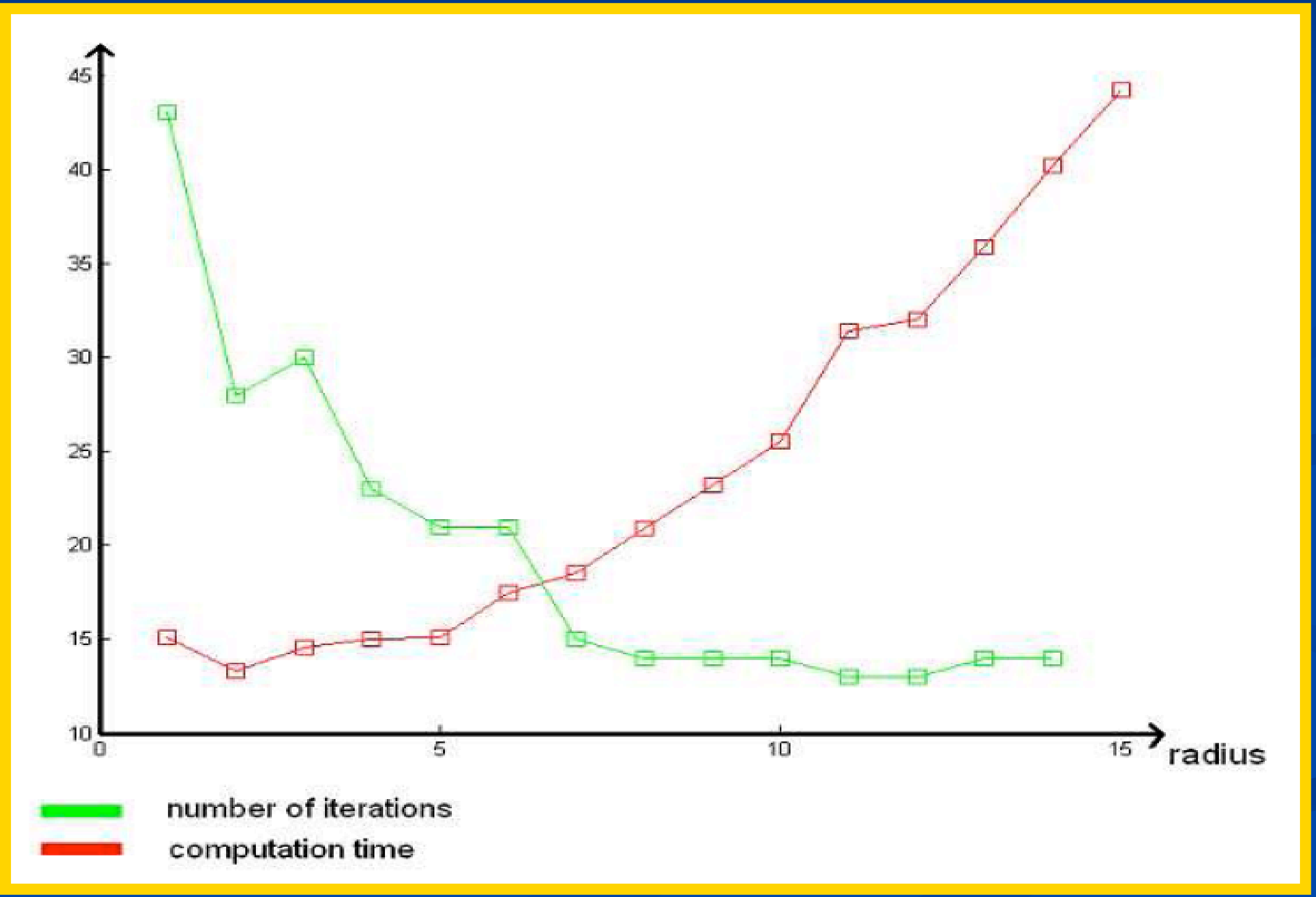


r=3



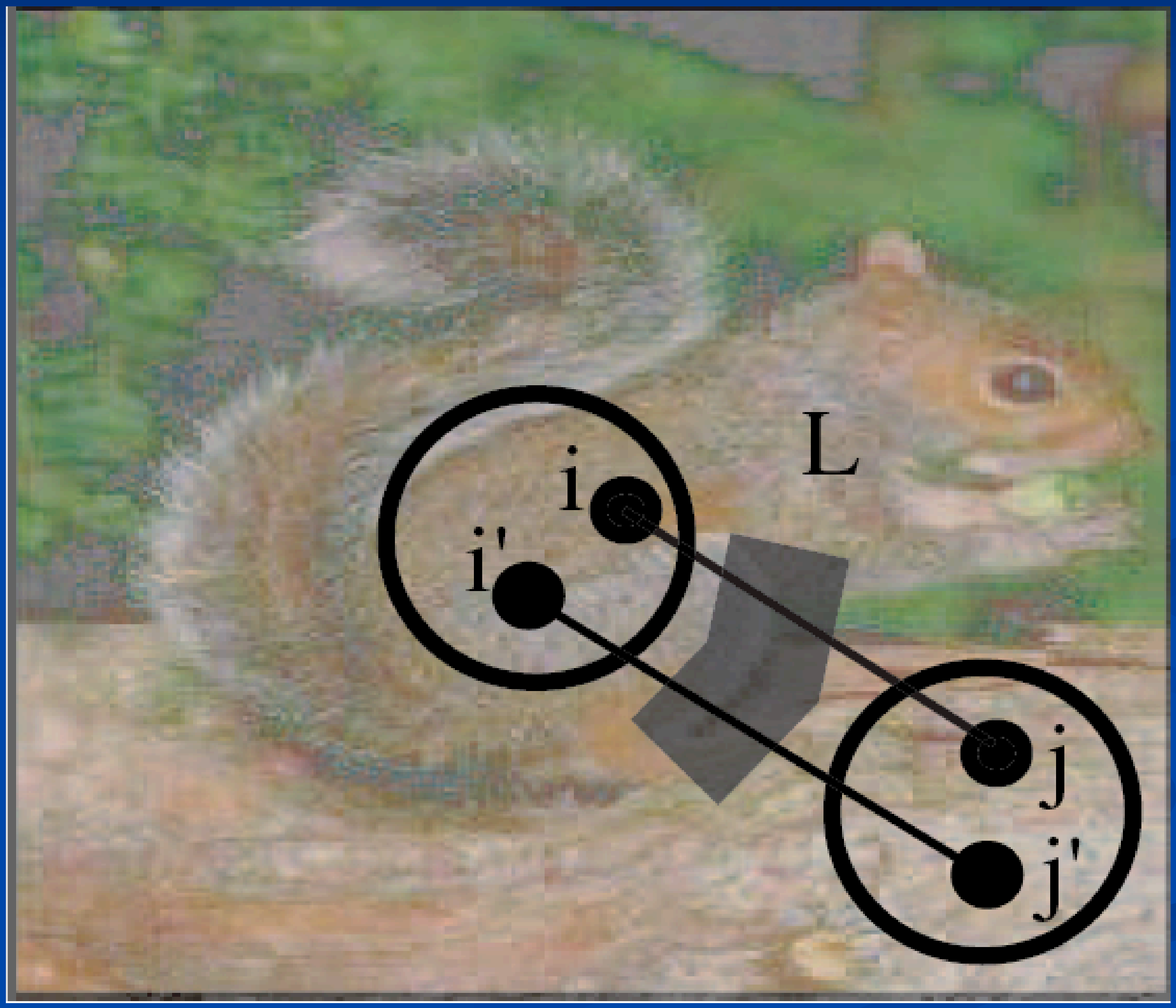
r=14

ACCURACY VERSUS COMPUTATION TIME



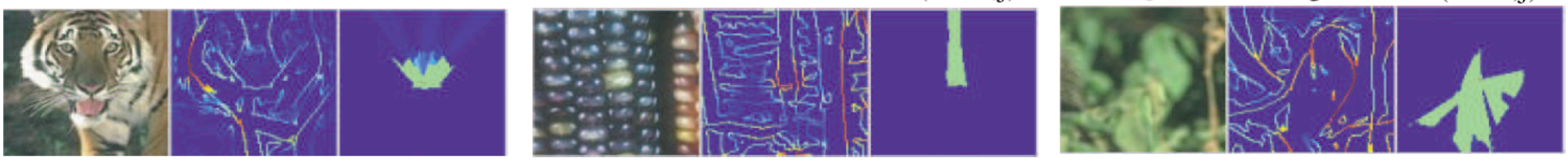
MULTISCALE AFFINITY MATRIX AND LAYER CONSTRAINTS

Long range connections

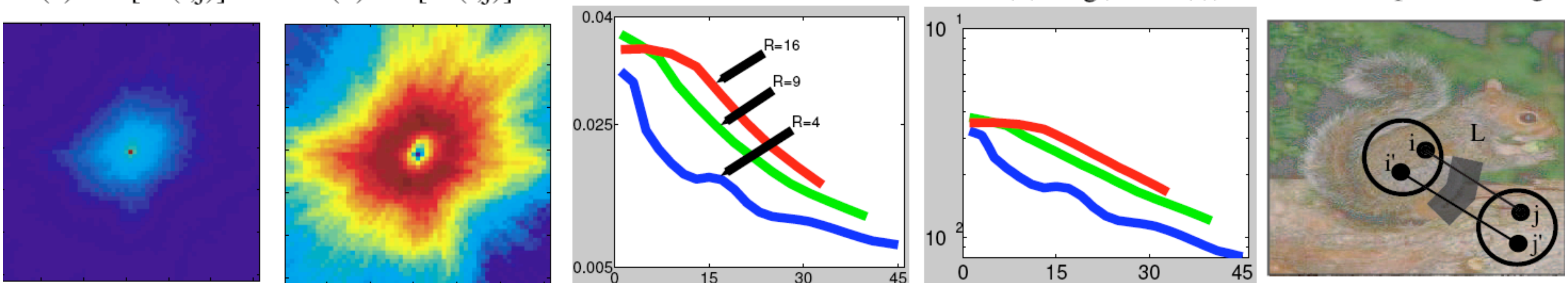


Statistics of W on natural images

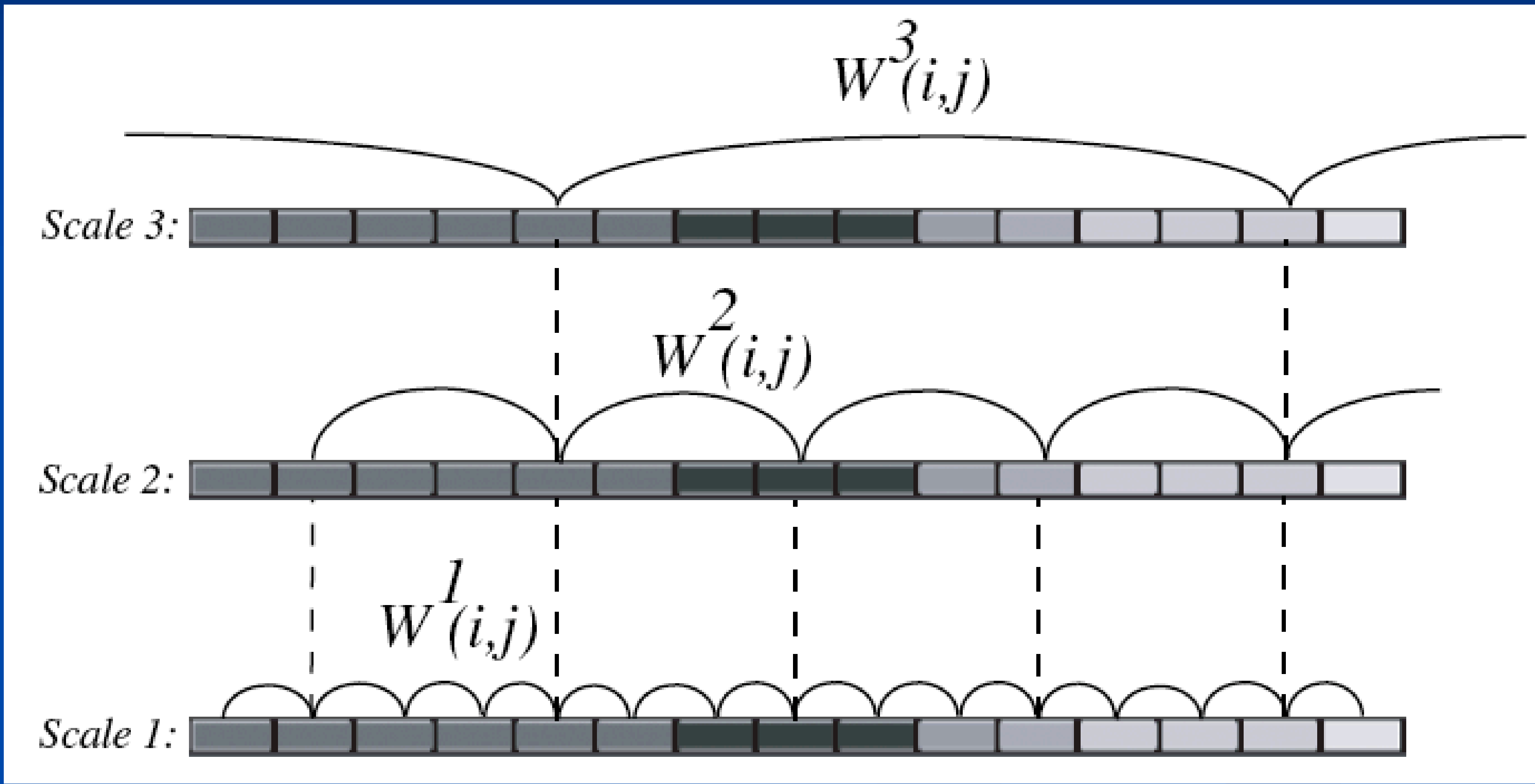
Image | Edge | W(icenter,j) | Image | Edge | W(icenter,j) | Image | Edge | W(icenter,j)



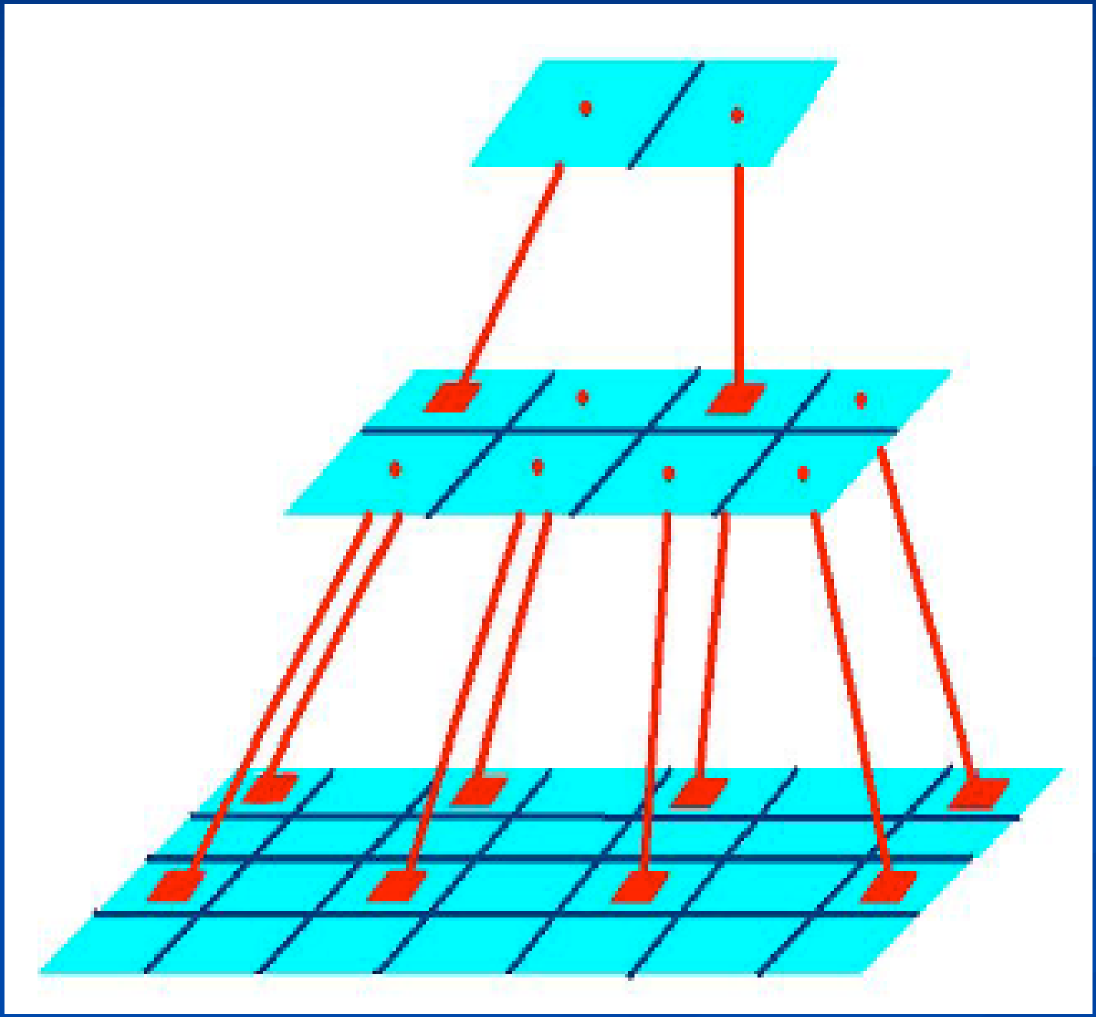
(a) Ave[W(i,j)] (b) Var[W(i,j)] (c) VarW(r) (d) Log(VarW(r)) (e) Graph Coarsing



Multiscale graph decomposition



Multi-scale graph decomposition



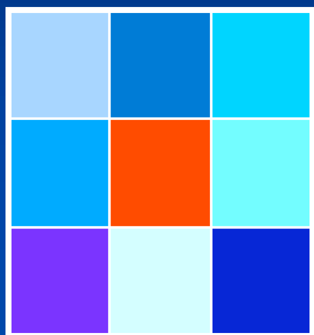
Original graph weight:
O(N^2)

Multi-scale graph weights:
O(N)

MULTISCALE MATRIX

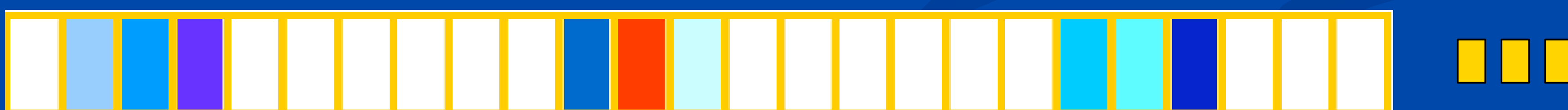
$$\begin{bmatrix} W1 & C(1,2)^T \\ C(1,2) & W2 \end{bmatrix}$$

AVERAGE MATRIX



: pixel i from coarse level

row number i :



0	1/9	1/9	1/9	0	0	0	0	0	0	1/9	1/9	1/9	0	0	0	0	0	0	1/9	1/9	1/9	0	0	0	...

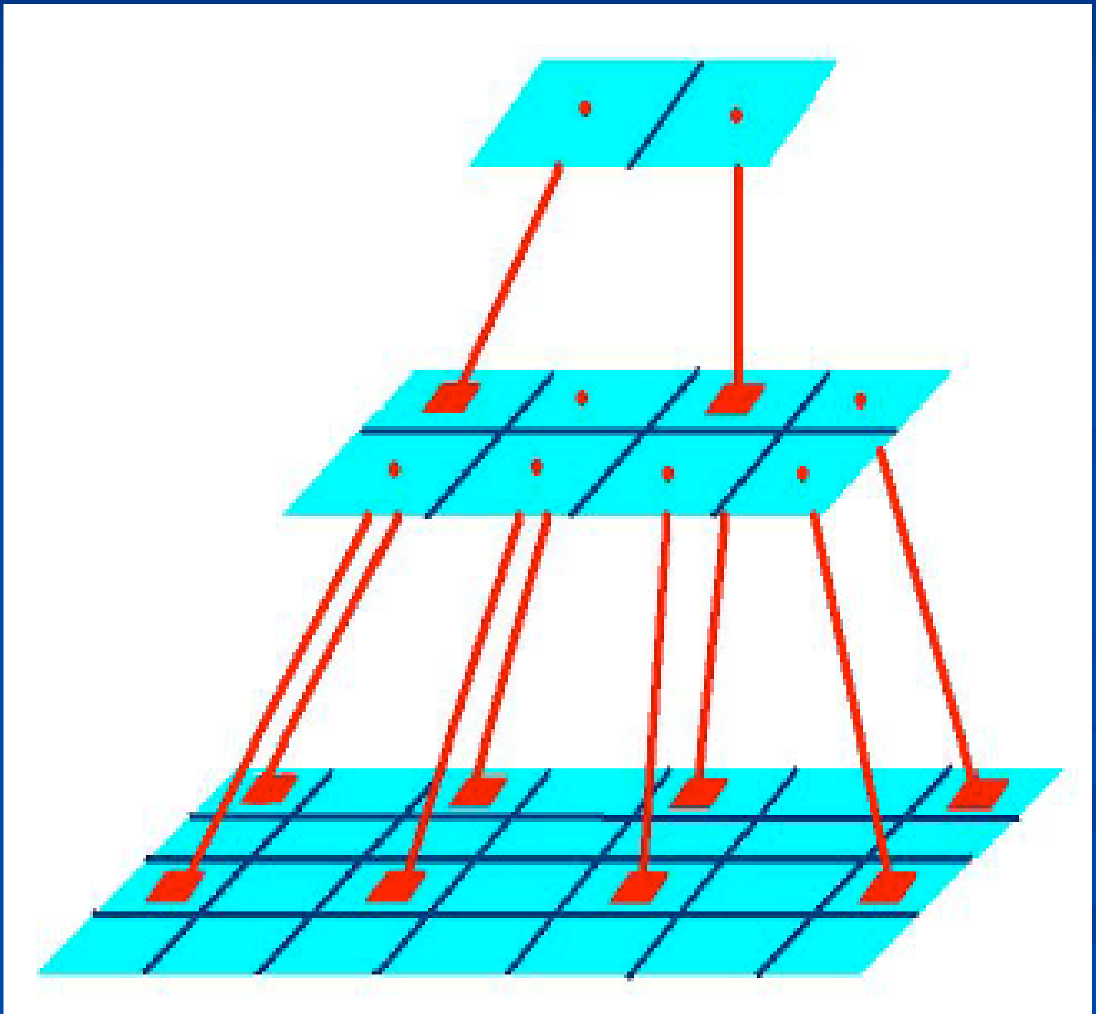
AVERAGE MATRIX

number of pixels in layer s

number of pixels in layer s+k

C(s,s+k)

Con-current partitioning of Multi-scale graph



MULTISCALE REPRESENTATION AND CONSTRAINTS

$$\begin{bmatrix} C(1,2) & -Id \end{bmatrix} * \begin{bmatrix} X1 \\ X2 \end{bmatrix} = 0$$

PROBLEM

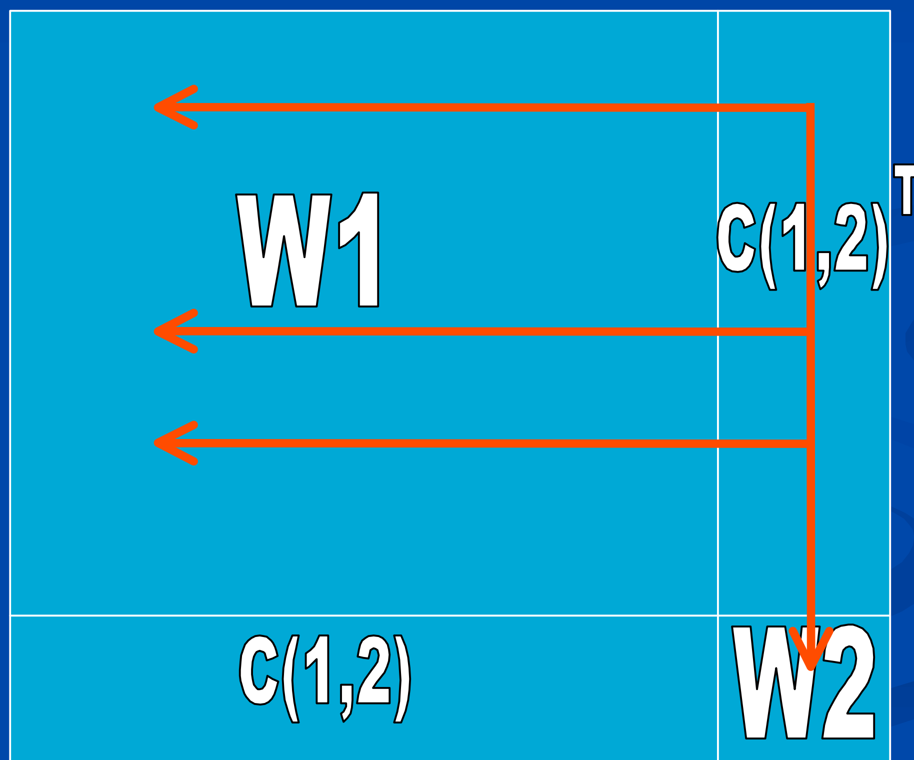
$$\text{maximize } \varepsilon(X) = \frac{1}{K}\sum_{l=1}^{K}\frac{X_l^T W X_l}{X_l^T D X_l}$$

$$\text{subject to} \qquad X \in \{0,1\}^{N\times K}, \quad X1_K = 1_N$$

$$CX = 0$$

MAXIMIZING ENERGY

$$e(X)=X_1^T W_1 X_1 + X_2^T W_2 X_2 + 2\, X_1^T C X_2$$

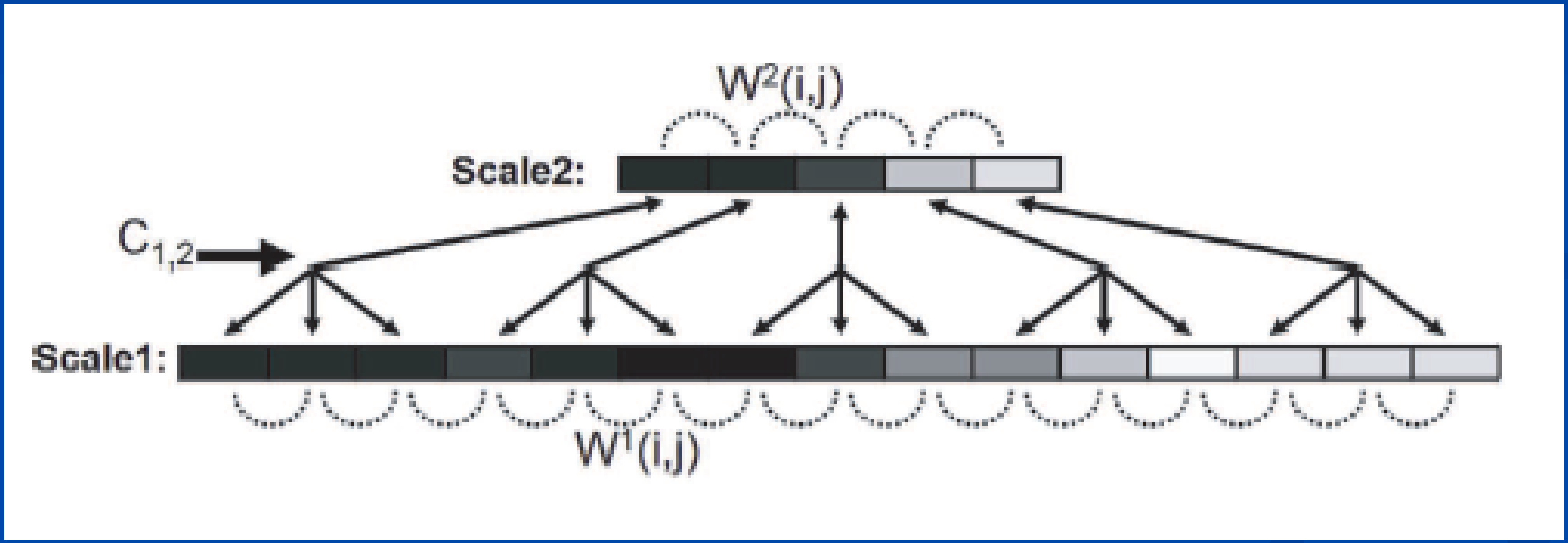


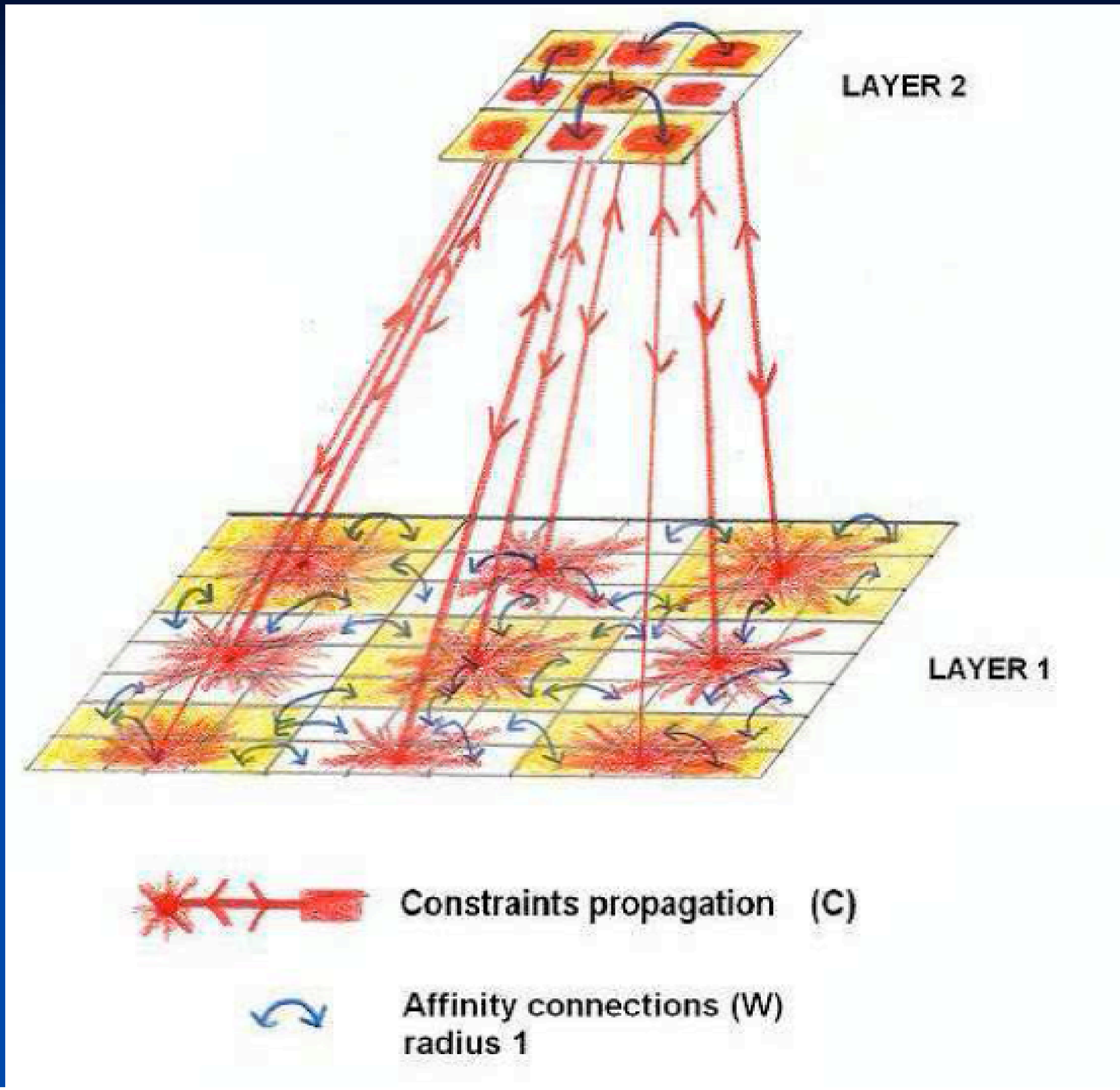
$$\text{maximize } \varepsilon(X) = \frac{1}{K}\sum_{l=1}^{K} \frac{X_l^T W X_l}{X_l^T D X_l}$$

$$\text{subject to} \quad X \in \{0,1\}^{N\times K}, \quad X1_K = 1_N$$

$$CX = 0$$

Cross scale constraints





LAYER 2
LAYER 1
Constraints propagation (C)
Affinity connections (W)
radius 1

RELAXED PROBLEM

$$\text{maximize } \varepsilon(Z) = \frac{1}{K}\operatorname{tr}(Z^T W Z)$$

$$\text{subject to} \qquad Z^T D Z = I.$$

$$CZ = 0$$

$P = D^{-\frac{1}{2}} W D^{-\frac{1}{2}}$ be the normalized affinity matrix, and Q be the projector onto the feasible solution space:

$$Q = I - D^{-\frac{1}{2}} C^T (C D^{-1} C^T)^{-1} C D^{-\frac{1}{2}}. \qquad (17)$$

Let $V = (V_1, ..., V_K)$ be the first K eigenvectors of matrix QPQ. Then the solutions to Eq. (15) are given by scaling any rotation of the K eigenvectors $V = (V_1, ..., V_K)$:

$$\arg\max_{Z} \varepsilon(Z) = \{D^{-\frac{1}{2}} V R : R \in O(K)\}. \qquad (18)$$

$P = D^{-\frac{1}{2}} W D^{-\frac{1}{2}}$ be the normalized affinity matrix, and Q be the projector onto the feasible solution space:

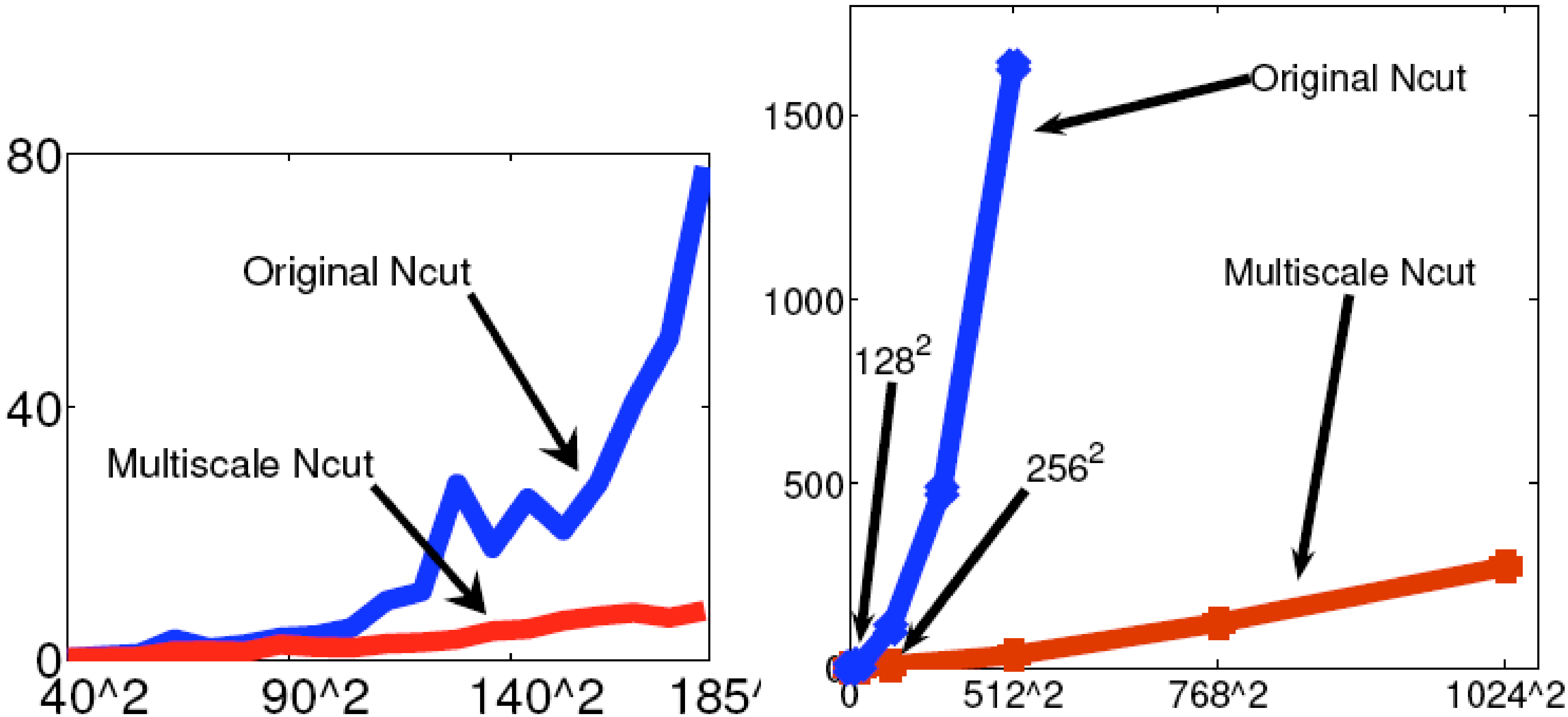
Can be done in O(N) operations.

$$Q = I - D^{-\frac{1}{2}} C^T (C D^{-1} C^T)^{-1} C D^{-\frac{1}{2}}. \qquad (17)$$

Let $V = (V_1, ..., V_K)$ be the first K eigenvectors of matrix QPQ. Then the solutions to Eq. (15) are given by scaling any rotation of the K eigenvectors $V = (V_1, ..., V_K)$:

$$\arg\max_Z \varepsilon(Z) = \{D^{-\frac{1}{2}} V R : R \in O(K)\}. \qquad (18)$$

Computational speed up



(a) (b)

RESULTS

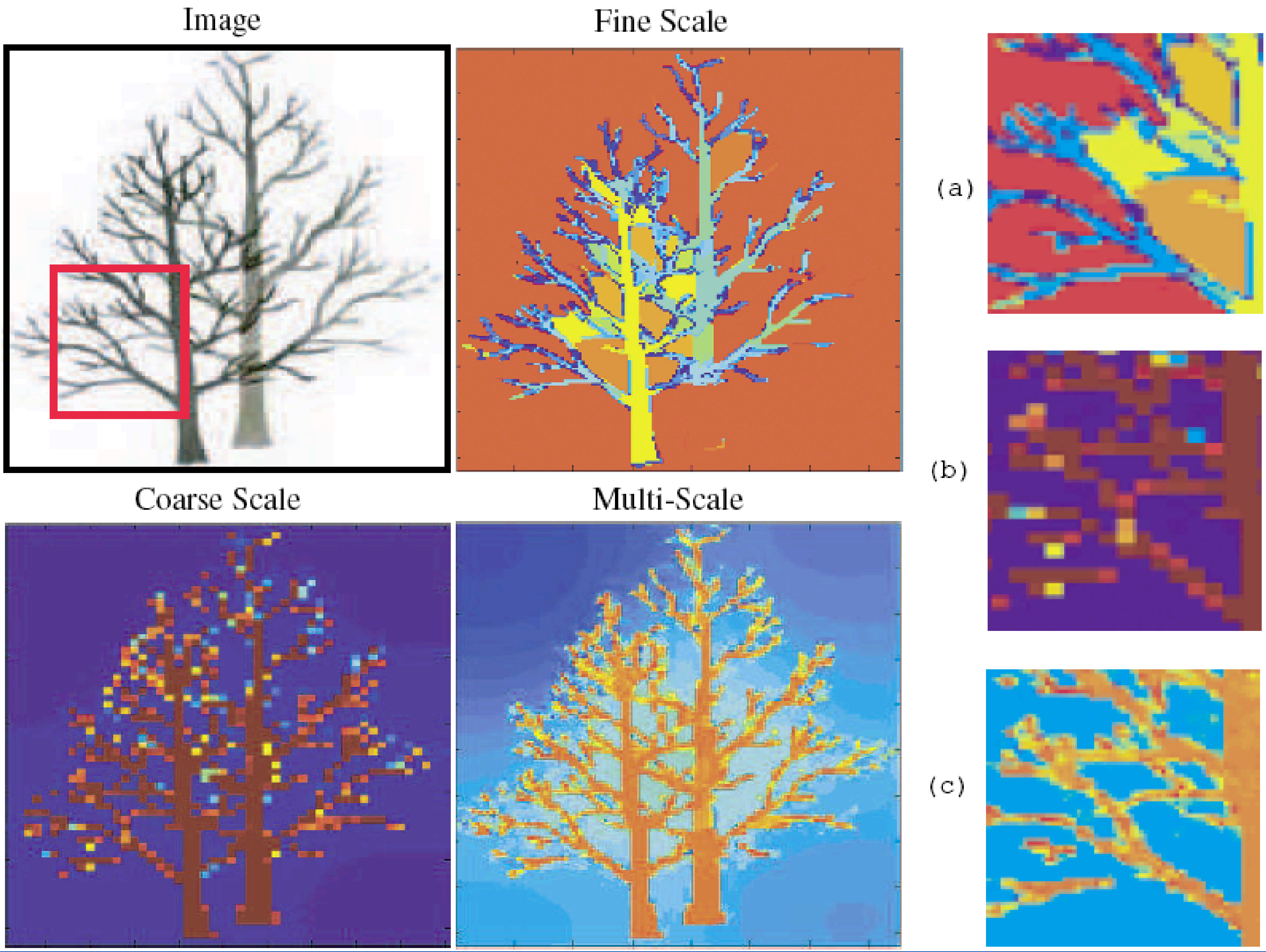
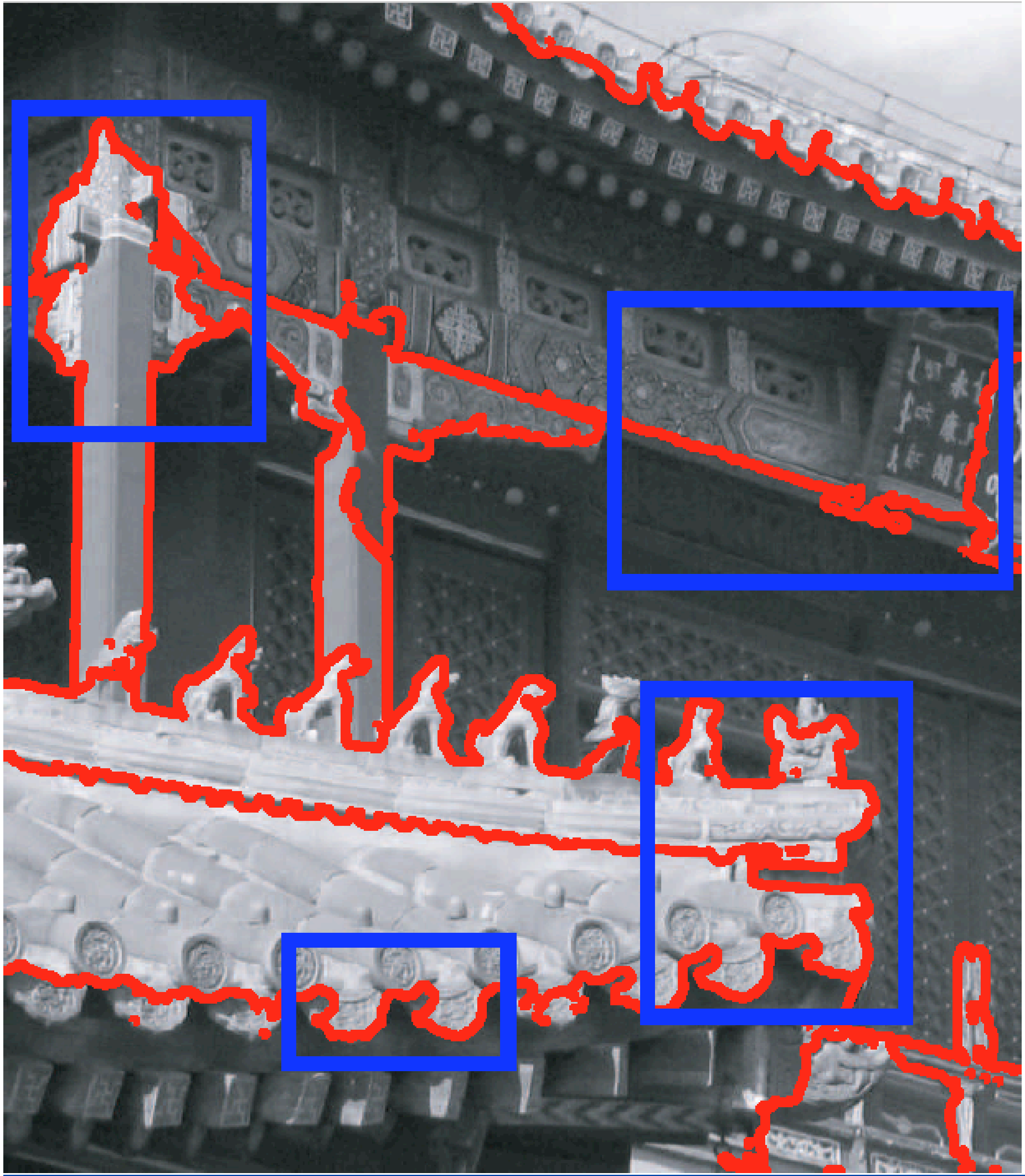
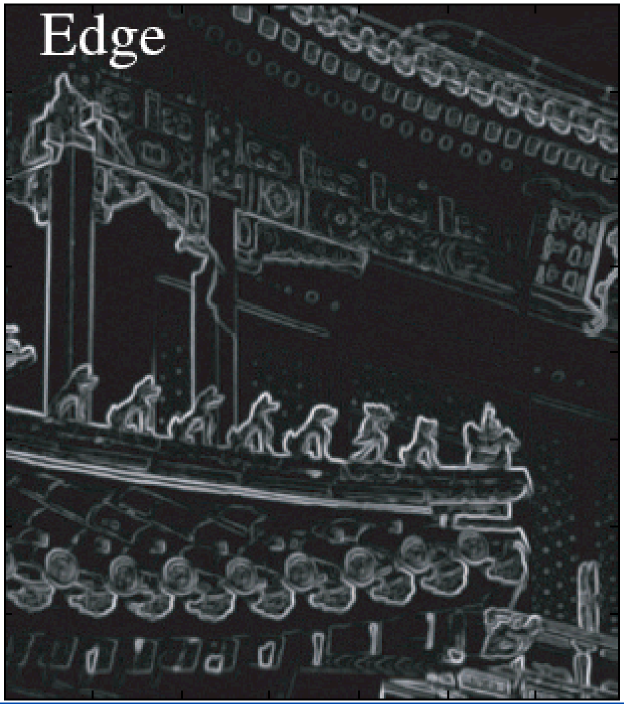
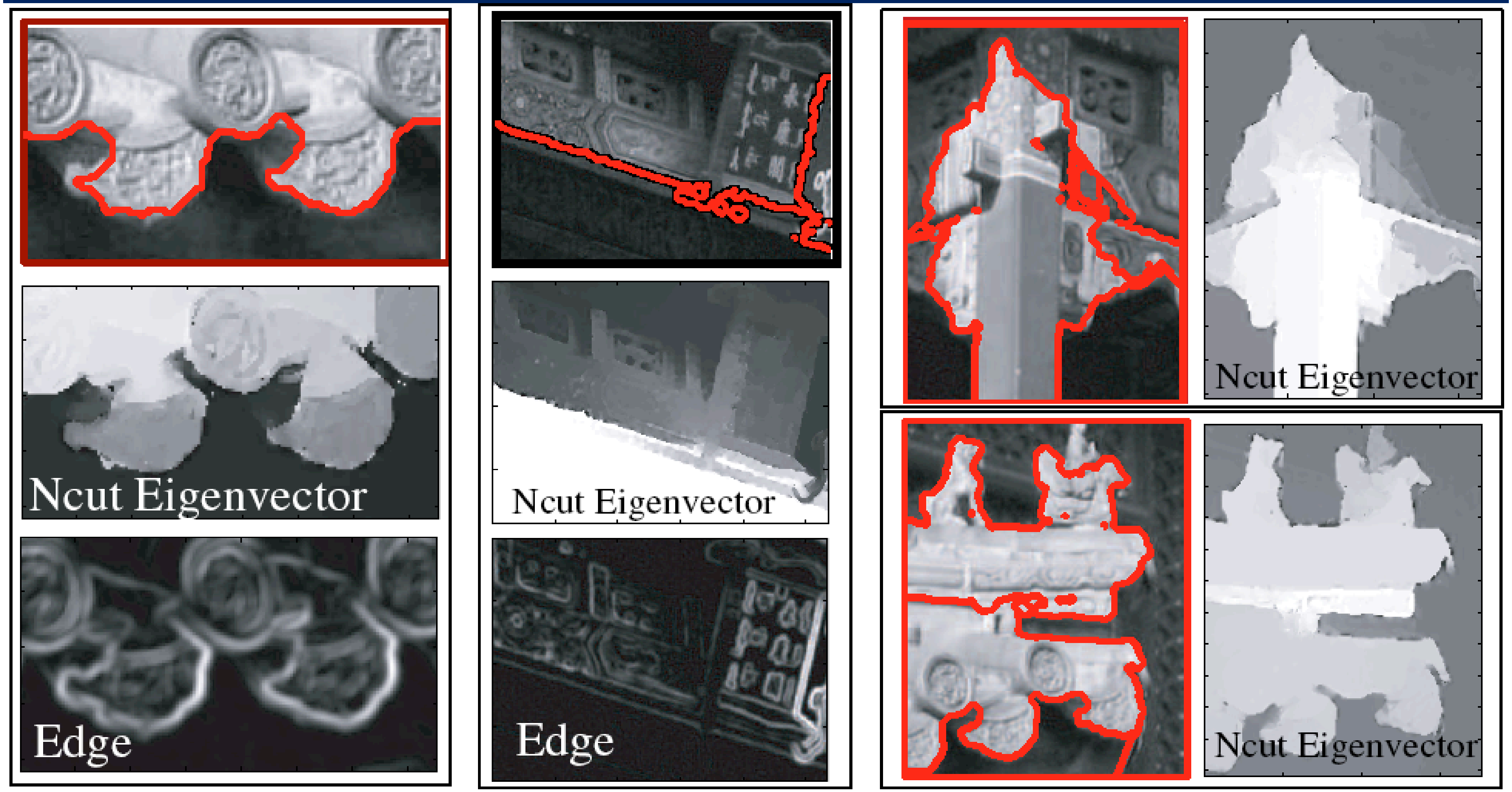


Image
Fine Scale
Coarse Scale
Multi-Scale
(a)
(b)
(c)

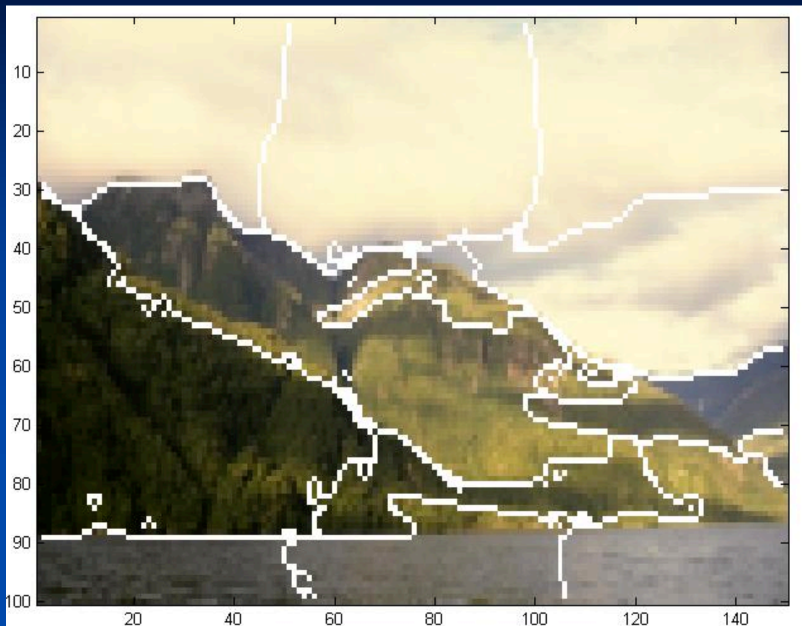


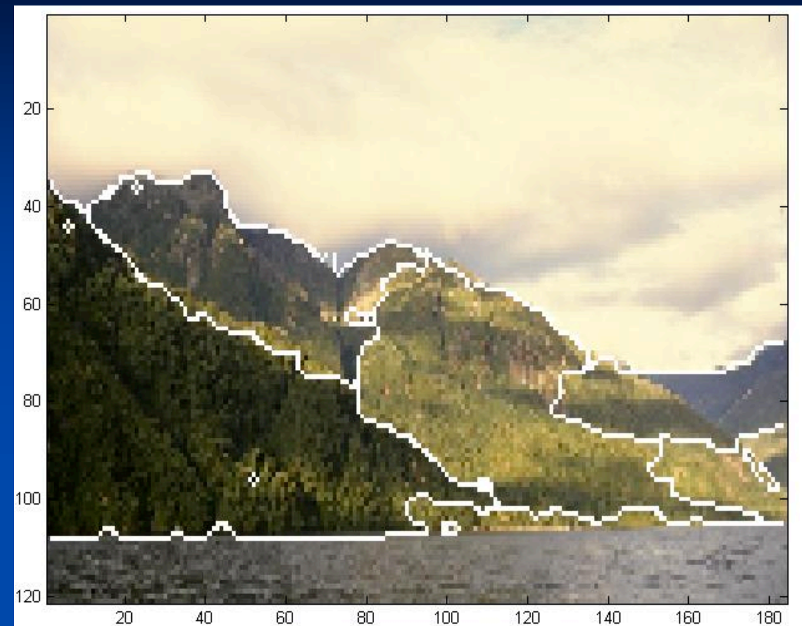

Segmentation

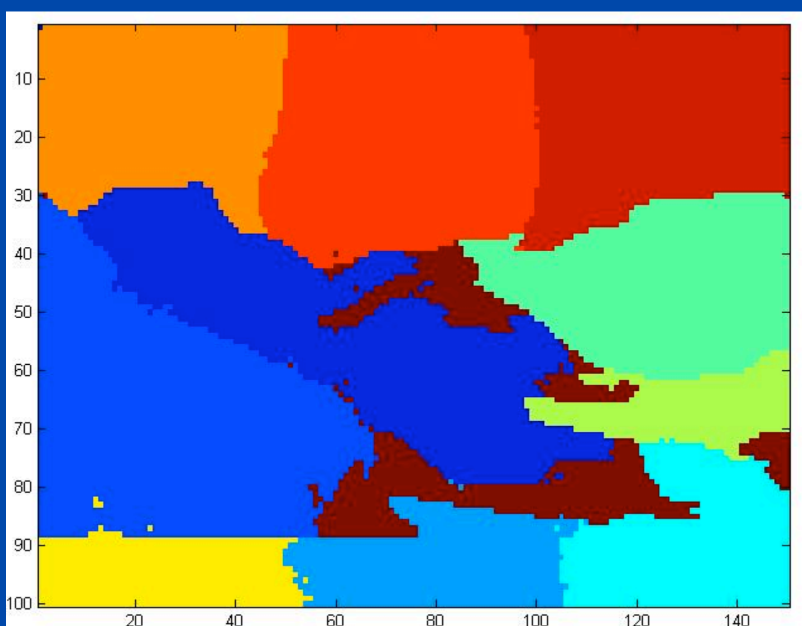

Edge

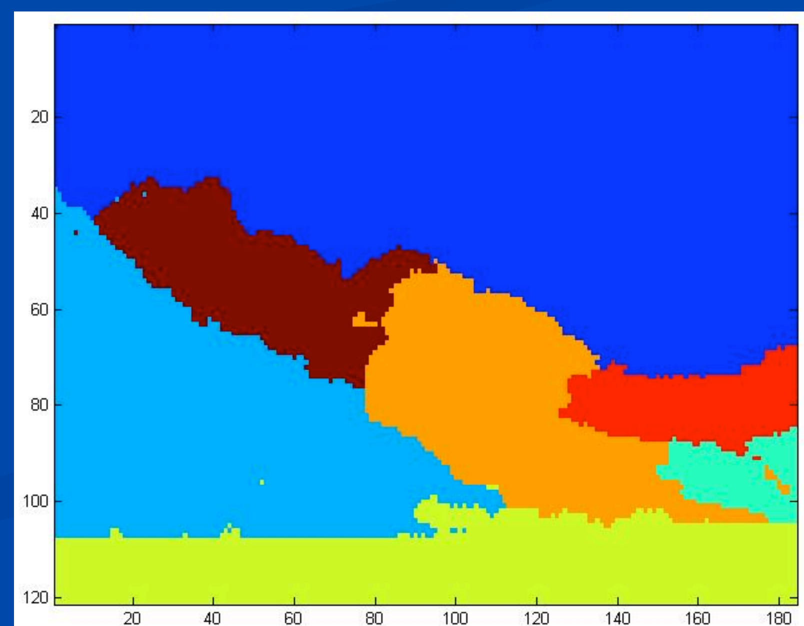

Ncut Eigenvector
Edge
Ncut Eigenvector
Edge
Ncut Eigenvector
Ncut Eigenvector

EXAMPLES

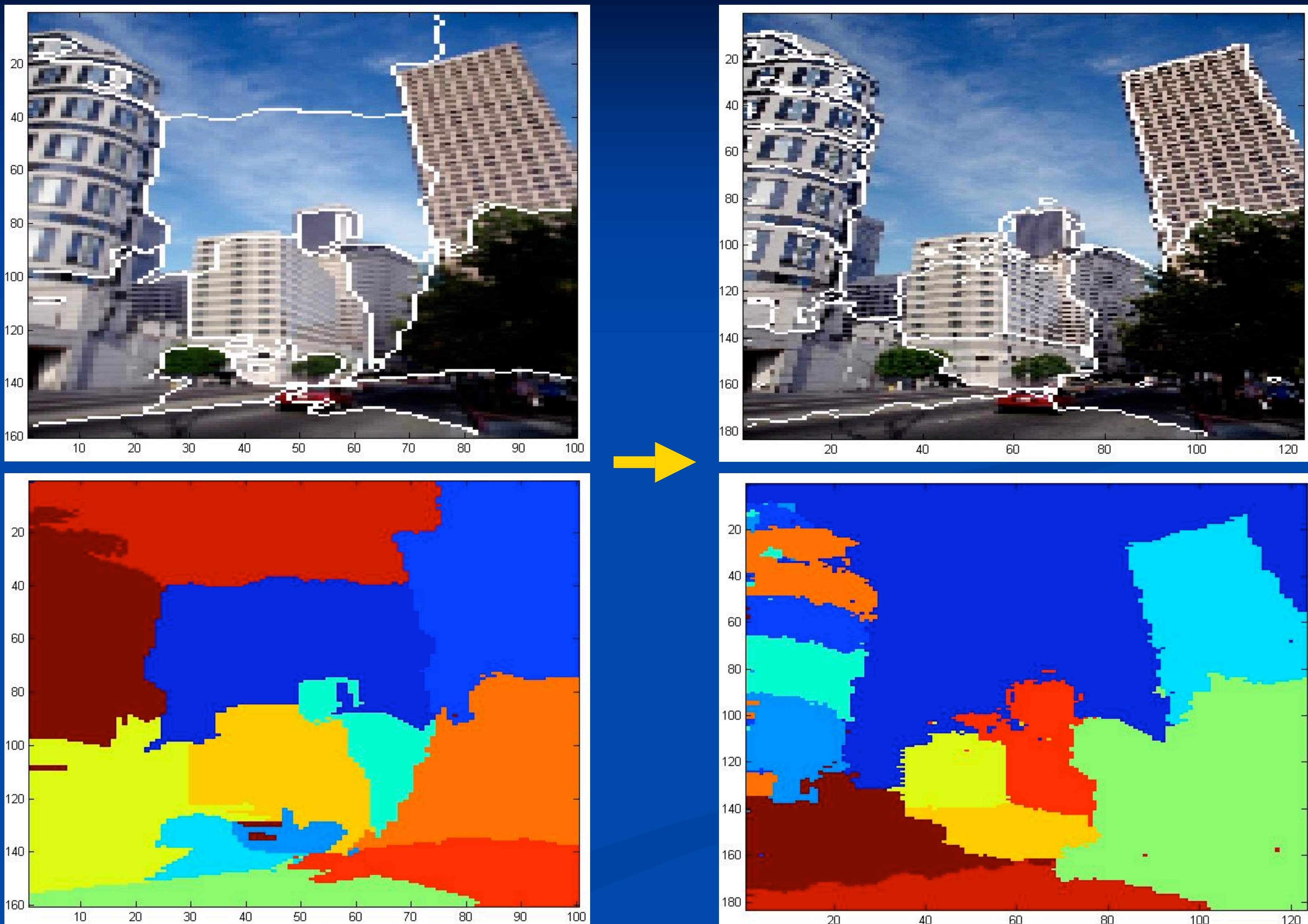

10 20 30 40 50 60 70 80 90 100
20 40 60 80 100 120 140


20 40 60 80 100 120
20 40 60 80 100 120 140 160 180


10 20 30 40 50 60 70 80 90 100
20 40 60 80 100 120 140

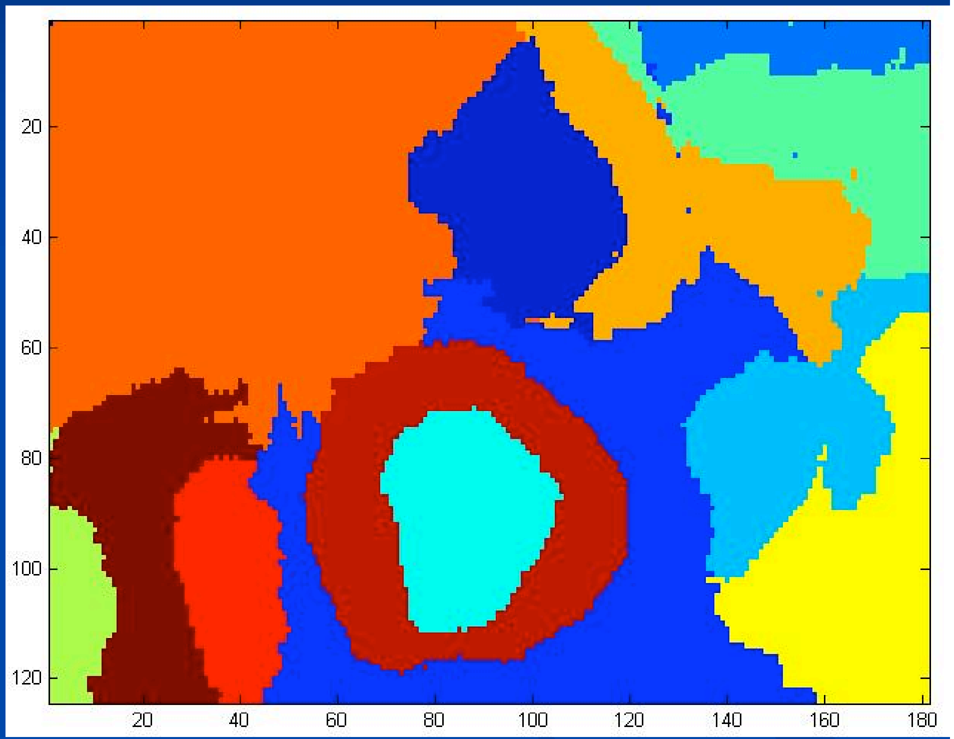

20 40 60 80 100 120
20 40 60 80 100 120 140 160 180

EXAMPLES


20 40 60 80 100 120 140 160
10 20 30 40 50 60 70 80 90 100
20 40 60 80 100 120 140 160 180
20 40 60 80 100 120

EXAMPLES


20
40
60
80
100
120
20
40
60
80
100
120
140
160
180


20
40
60
80
100
120
20
40
60
80
100
120
140
160
180