Accurate & Efficient Regression Modeling for Microarchitectural Performance & Power Prediction

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Outline

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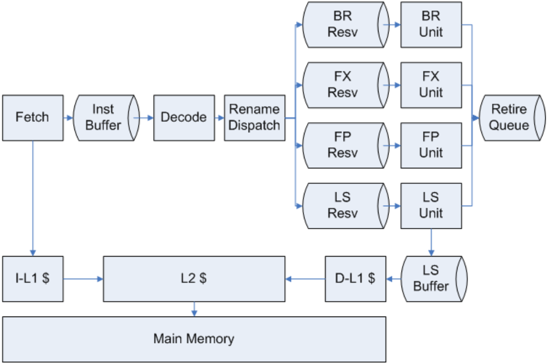
Microarchitectural Design Space



- Increasing diversity of interesting, viable designs
- Examples :: Power 4, Pentium 4, UltraSPARC T1
- Tractably quantify trends across comprehensive design space

Microarchitectural Simulation Challenges

- **Cycle-Accurate Simulation**
  - Accurately identifies trends in design space
  - Tracks instructions' progress through microprocessor
  - Estimates performance, power, temperature, ...
- **Simulation Costs**
  - Long simulation times (minutes,hours per design)
  - Number of potential simulations scale exponentially (m^p)
    - p :: parameter count
    - m :: parameter resolution

Microarchitectural Sampling

- **Temporal Sampling**
  - Sample from instruction traces in time domain
  - Reduce simulation costs via size of inputs
  - Synthetic traces from profiled workloads [1]
  - Sampled traces from phase analysis [2]
- **Spatial Sampling**
  - Sample from design space
  - Reduce simulation costs via number of simulations

[1] Eeckhout [ISPASS'00]
[2] Sherwood [ASPLOS'02], Wunderlich [ISCA'03]

Simulation Paradigm

- **Comprehensively understand design space**
  - Specify large, high-resolution design space
  - Consider all design parameter simultaneously
- **Selectively simulate modest number of designs**
  - Sample points randomly from design space for simulation
  - Decouple resolution of design space and simulation
- **Efficiently leverage simulation data with inference**
  - Reveal trends, trade-offs from sparse sampling
  - Enable predictions for metrics of interest

Regression Theory

- **Statistical Inference**
  - Models approximate solutions to intractable problems
  - Requires initial data to train, formulate model
  - Leverages correlations from initial data for prediction
- **Regression Models**
  - Low formulation costs (1K samples from 1B designs)
  - Accurate inference ($4 - 7\%$ median error)
  - Efficient computation (100's of predictions per second)

Model Formulation

- **Notation**
  - n observations ▷ {*simulated design samples*}
  - Response :: $\vec{y} = y_1, \ldots, y_n$ ▷ {*e.g., performance, power*}
  - Predictor :: $\vec{x}_i = x_{i,1}, \ldots, x_{i,p}$ ▷ {*e.g., depth, cache*}
  - Regression Coefficients :: $\vec{\beta} = \beta_0, \ldots, \beta_p$
  - Random Error :: $\vec{e} = e_1, \ldots, e_n$ where $e_i \sim N(0, \sigma^2)$
  - Transformations :: $f, \vec{g} = g_1, \ldots, g_p$
- **Model**

$$f(y) = \beta_0 + \sum_{j=1}^{p} \beta_j g_j(x_j) + e$$

Predictor Interaction

- **Modeling Interaction**
  - Suppose effects of predictors x_1, x_2 cannot be separated
  - Construct predictor $x_3 = x_1 x_2$

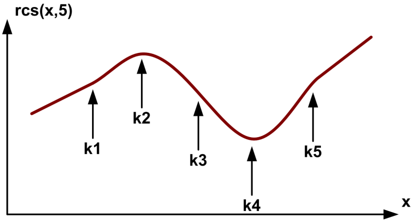
$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + e_i$$

- **Example**
  - Let x_1 be pipeline depth, x_2 be L2 cache size
  - Performance impact of pipelining affected by cache size

$$Speedup = \frac{Depth}{1 + Stalls/Inst}$$

Predictor Non-Linearity I

- **Restricted Cubic Splines**
  - Divide predictor domain into intervals separated by knots
  - Piecewise cubic polynomials joined at knots
  - Higher order polynomials provide better fits [3]



[3] Stone [SS'86]

Predictor Non-Linearity II

- **Location of Knots**
  - Location of knots less important than number of knots [4]
  - Place knots at fixed predictor quantiles
- **Number of Knots**
  - Flexibility, risk of over-fitting increases with knot count
  - 5 knots or fewer are often sufficient
  - 4 knots balances flexibility, risk of over-fitting

[4] Harrell [Springer'01]

Prediction

- **Expected Response**
  - β are known from least squares
  - $x_{i,1}, \ldots, x_{i,p}$ are known for a given query i
  - Expected response is weighted sum of predictor values

$$
\begin{aligned}
E\left[y\right] &= E\left[\beta_0 + \sum_{j=1}^{p} \beta_j x_j\right] + E\left[e\right] \\
&= \beta_0 + \sum_{j=1}^{p} \beta_j x_j
\end{aligned}
$$

Outline

- Motivation & Background
  - Simulation Challenges
  - Simulation Paradigm
  - Regression Theory
- Model Derivation
  - Experimental Methodology
  - Derivation Overview
  - Model Specification
- Model Evaluation
  - Performance
  - Power
- Conclusion

Tools and Benchmarks

- **Simulation Framework**
  - Turandot :: a cycle-accurate trace driven simulator
  - PowerTimer :: power models derived from circuit analyses
  - Baseline simulator models POWER4/POWER5 architecture
- **Benchmarks**
  - SPEC2kCPU :: compute-intensive benchmarks
  - SPECjbb :: Java server benchmark
- **Statistical Framework**
  - R :: software environment for statistical computing
  - Hmisc and Design packages[5]

[5] Harrell [Springer,'01]

Spatial Sampling

- **Design Space**
  - S_i :: set of values for parameter x_i, $i \in [1, p]$
  - $S = \prod_{i=1}^{p} S_i$ :: design space
  - B :: set of benchmarks
  - $|S| \approx 10^9$ and $|B| = 22$
- **Sampling Uniformly at Random (UAR)**
  - Sample $n = 4,000$ designs and benchmarks for simulation
  - Decouple resolution of design space and simulation
  - Unbiased observations from full range of parameter values

Predictors :: Microarchitecture

	Set	Parameters	Measure	Range	$\lvert S_i \rvert$
S_1	Depth	depth	FO4	9::3::36	10
S_2	Width	width L/S reorder queue store queue functional units	insn b/w entries entries count	4,8,16 15::15::45 14::14::42 1,2,4	3
S_3	Physical Registers	general purpose (GP) floating-point (FP) special purpose (SP)	count count count	40::10::130 40::8::112 42::6::96	10
S_4	Reservation Stations	branch fixed-point/memory floating-point	entries entries entries	6::1::15 10::2::28 5::1::14	10
S_5	I-L1 Cache	i-L1 cache size	log_2(entries)	7::1::11	5
S_6	D-L1 Cache	d-L1 cache size	log_2(entries)	6::1::10	5
S_7	L2 Cache	L2 cache size L2 cache latency	log_2(entries) cycles	11::1::15 6::2::14	5
S_8	Control Latency	branch latency	cycles	1,2	2
S_9	FX Latency	ALU latency FX-multiply latency FX-divide latency	cycles cycles cycles	1::1::5 4::1::8 35::5::55	5
S_{10}	FP Latency	FPU latency FP-divide latency	cycles cycles	5::1::9 25::5::45	5
S_{11}	L/S Latency	Load/Store latency	cycles	3::1::7	5
S_{12}	Memory Latency	Main memory latency	cycles	70::5::115	10

Predictors :: Application-Specific

- **Application Characteristics**
  - Collect program characteristics on baseline architecture
  - Instruction throughput
  - Cache access patterns
  - Branch patterns
  - Sources of pipeline stalls
- **Application Effects**
  - Significant interactions with microarchitectural predictors
  - Example :: Impact of d-L1 cache affected by access rates

Derivation Overview

- **Hierarchical Clustering**
- **Performance Associations and Correlations**
  - qualitative scatterplots, quantitative ρ^2
- **Model Specification**
  - predictor interaction, non-linearity
- **Assessing Fit**
  - R^2 statistic
- **Residual Analysis**
  - normality (quantile-quantile), randomness (scatterplots)
- **Significance Testing**
  - hypothesis testing, F-statistic, p-values

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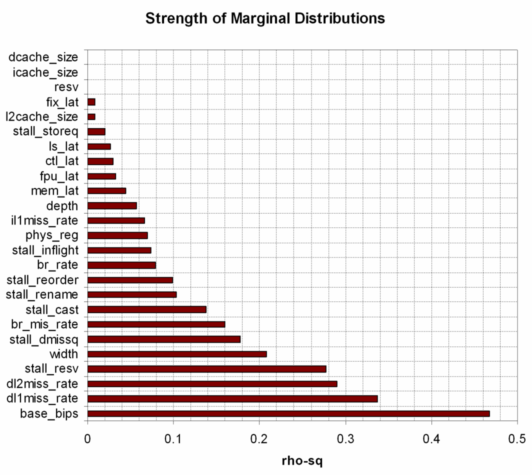
Performance Correlations



Model Specification

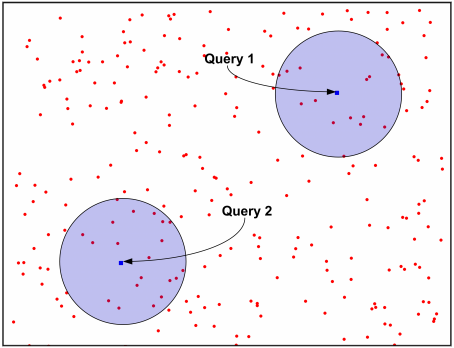
- **Interactions**
  - Pipeline width/depth interact with
    - instruction bandwidth structures (queues, register file)
    - cache hierarchy
  - Cache hierarchy sizes interact with
    - adjacent levels in hierarchy
    - application-specific access rates
  - Baseline performance interacts with resource sizings
- **Restricted Cubic Splines**
  - Weaker relationships (latencies, caches, queues) :: 3 knots
  - Stronger relationships (depth, registers) :: 4 knots
  - Baseline performance :: 5 knots

Outline

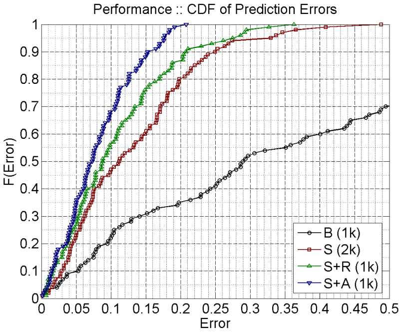
Validation Approach

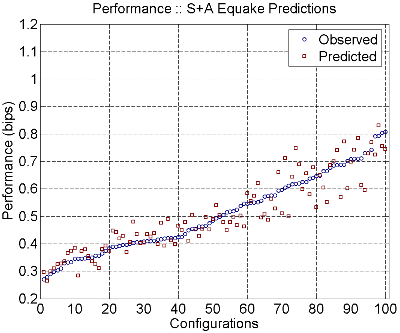
- **Framework**
  - Formulate models with $n < 4,000$ samples
  - Obtain 100 additional random samples for validation
  - Quantify percentage error, $100 * |\hat{y}_i - y_i|/y_i$
- **Model Variants**
  - **Baseline (B) ::** Non-transformed response
  - **Stabilized (S) ::** Square-root of response
  - **Regional (S+R) ::** Per query with similar samples
  - **Application (S+A) ::** Per benchmark with similar samples

Regional Sampling

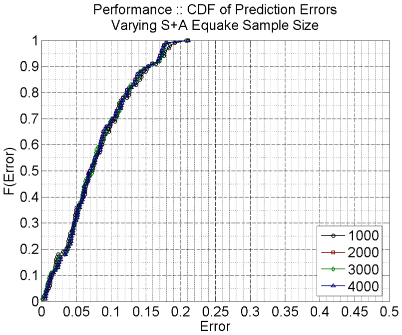


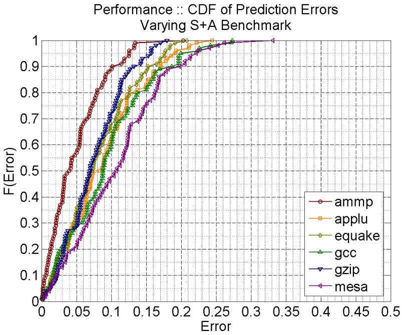
Performance Prediction



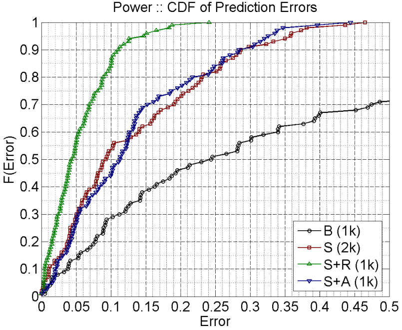


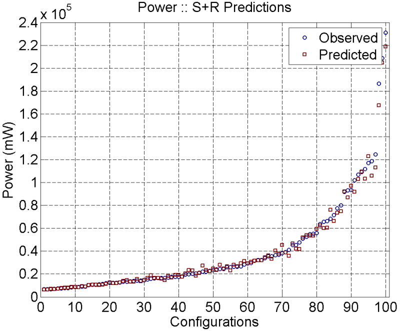
Performance Sensitivity :: S+A



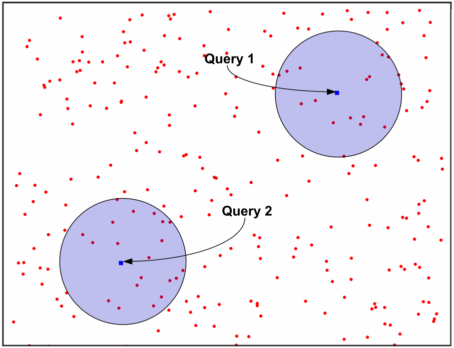


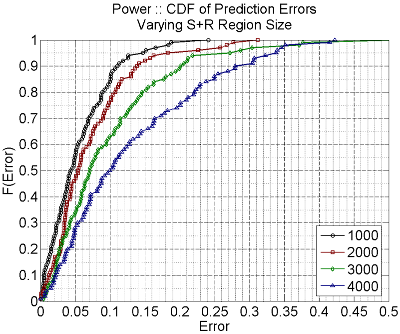
Power Prediction





Power Sensitivity :: S+R Region





Outline

Conclusion

- **Simulation Paradigm**
  - Comprehensively understand design space
  - Selectively simulate modest number of designs
  - Efficiently leverage simulation data with inference
- **Model Evaluation**
  - 7.4%, 4.3% median errors for performance, power
  - S+A, S+R more accurate for performance, power
- **Future Directions**
  - Demonstrate for comprehensive design studies [6]
  - Expand design space and benchmark suite
  - Extend to CMP's and interconnect modeling

[6] Lee [HPCA'07] :: www.deas.harvard.edu/∼bclee

Appendix

www.deas.harvard.edu/∼bclee

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  In *International Symposium on Performance Analysis of Systems and Software*, Austin, Texas, March 2006.

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  Automatically characterizing large scale program behavior.
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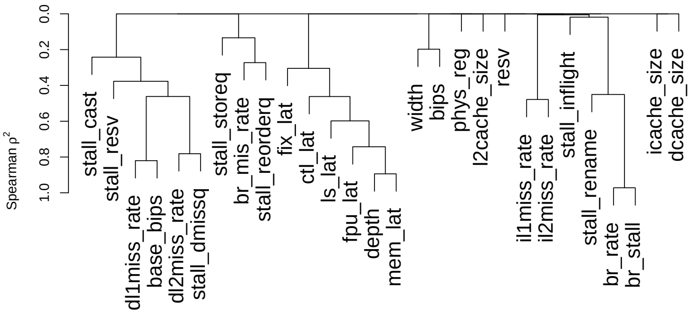
Controlling Simulation Costs

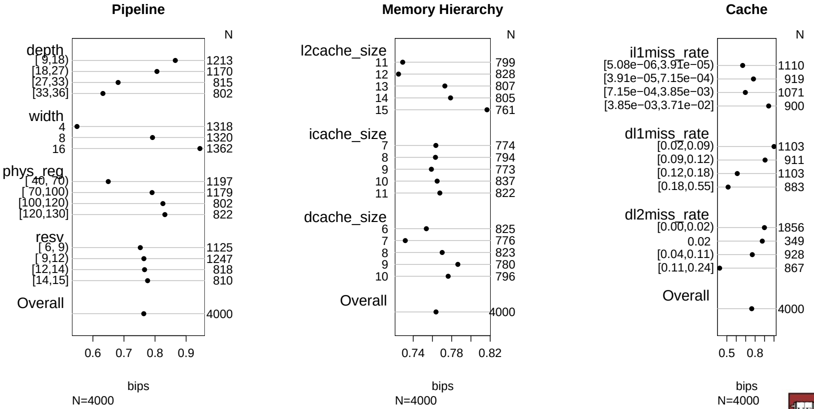
- **Hybrid Simulation**
  - Decouples simulation of microprocessor structures
  - Leverages fast, specialized simulators for particular units [7]
- **Trace Sampling/Compression**
  - Reduces redundant simulation
  - Simulate unique, representative instruction segments [8]
- **Synthetic Workloads**
  - Reduces size of simulator inputs
  - Profiles workload to construct smaller, synthetic traces [9]

[7] Li, Lee, Brooks, Hu, Skadron [HPCA'06]
[8] Liu, Asanovic [ISPASS'06], Sherwood, *et al.*, [ASPLOS'02]
[9] Eeckhout, Nussbaum, Smith, DeBosschre [IEEE Micro'03]

Variable Clustering



Performance Associations



Assessing Fit

- **Multiple Correlation Statistic**
  - R^2 is fraction of response variance captured by predictors
  - Large R^2 suggests better fit to observed data
  - $R^2 \rightarrow 1$ suggests over-fitting (less likely if $p < n/20$)

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \frac{1}{n}\sum_{i=1}^{n} y_i)^2}$$

- **Residual Distribution Assumptions**
  - Residuals are normally distributed, $e_i \sim N(0, \sigma^2)$
  - No correlation between residuals and response, predictors
  - Validate by scatterplots and quantile-quantile plots

$$\hat{e}_i = y_i - \hat{\beta}_0 - \sum_{j=0}^{p} \hat{\beta}_j x_{ij}$$

Predictor Non-Linearity I

- **Polynomial Transformations**
  - Undesirable peaks and valleys
  - Differing trends across regions
- **Linear Splines**
  - Piecewise linear regions separated by knots
  - Inadequate for complex, highly curved relationships
- **Restricted Cubic Splines**
  - Higher order polynomials provide better fits
  - Continuous at knots
  - Linear constraint on tails

Predictor Non-Linearity II

- **Location of Knots**
  - Location of knots less important than number of knots
  - Place knots at fixed predictor quantiles
- **Number of Knots**
  - Flexibility, risk of over-fitting increases with knot count
  - 5 knots or fewer are often sufficient [10]
  - 4 knots is a good compromise between flexibility, over-fitting
  - Fewer knots required for small data sets

[10] Stone [SS'86]

Significance Testing I

- **Approach**
  - Given two nested models, hypothesis H_0 states additional predictors in larger model have no response association
  - Test H_0 with F-statistics and p-values
- **Example**
  - Predictor interaction requires comparing nested models
  - Consider a model $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$.
  - Test significance of x_1 with null hypothesis $H_0 : \beta_1 = \beta_3 = 0$

Significance Testing II

- **F-Statistic**
  - Compare two nested models using their R^2 and F-statistic
  - R^2 is fraction of response variance captured by predictors

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \frac{1}{n}\sum_{i=1}^{n} y_i)^2}$$

  - F-statistic of two nested models follows F distribution

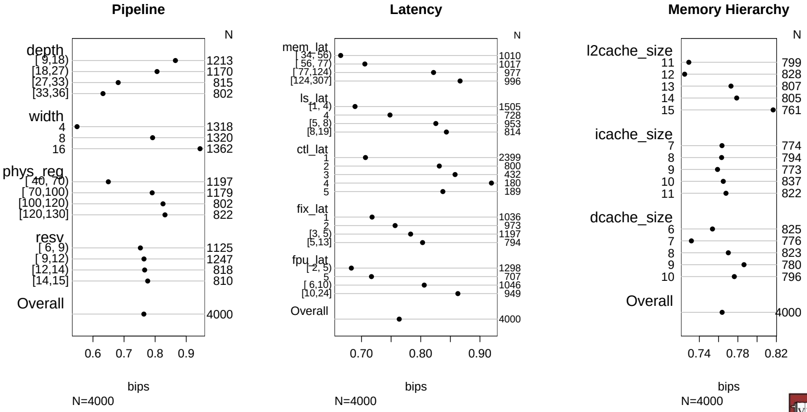
$$F_{k,n-p-1} = \frac{R^2 - R^2_*}{k} \times \frac{n-p-1}{1-R^2}$$

- **P-Values**
  - Probability F-statistic greater than or equal to observed value would occur under H_0
  - Small p-values cast doubt on H_0

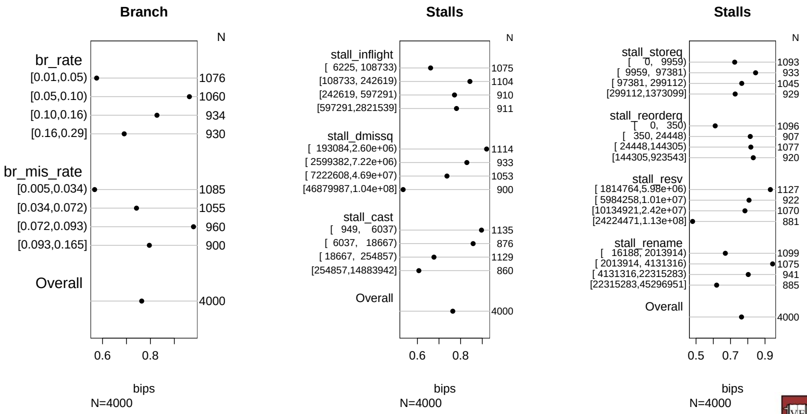
Treatment of Missing Data

- **Missing Completely at Random (MCAR)**
  - Treat unobserved design points as missing data
  - Sampling UAR ensures observations are MCAR
  - Data is missing for reasons unrelated to characteristics or responses of the configuration
- **Informative Missing**
  - Data is more likely missing if their responses are systematically higher or lower
  - “Missingness” is non-ignorable and must also be modeled
  - Sampling UAR avoids such modeling complications

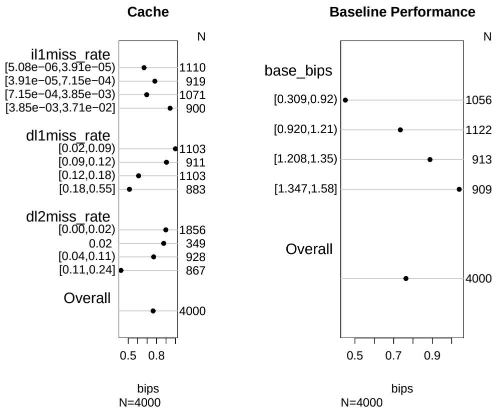
Performance Associations I



Performance Associations II



Performance Associations III



Significance Tests

- **Microarchitectural Predictors**
  - Majority of F-tests imply significance (p-values $< 2.2E - 16$)
  - Several predictors were less significant
    - Control latency (p-value $= 0.1247$)
    - Reservation station size (p-value $= 0.1239$)
    - L1 instruction cache size (p-value $= 0.02941$)
- **Application-Specific Predictors**
  - Majority of F-tests imply significance (p-values $< 2.2E - 16$)
  - Pipeline stalls classified by structure are less significant
    - Completion and reorder queue stalls (p-values > 0.4)

Related Work

- **Statistical Significance Ranking**
  - Yi :: Plackett-Burman, effect rankings
  - Joseph :: Stepwise regression, coefficient rankings
  - Bound parameter values to improve tractability
  - Require simulation for estimation
- **Synthetic Workloads**
  - Eeckhout :: Profile workloads to obtain synthetic traces
  - Nussbaum :: Superscalar and SMP simulation
  - Obtain distribution of instructions and data dependencies
  - Require simulation with smaller traces for estimation