Methods of Inference and Learning for Performance Modeling of Parallel Applications

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Outline

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- Motivation & Background
  - Modeling Challenges
  - Empirical Modeling Paradigm
  - Applications
- Parameter Characterization
  - Measurement Sampling
  - Hierarchical Clustering
  - Association & Correlation
- Performance Modeling
  - Piecewise Polynomial Regression
  - Artificial Neural Networks
  - Model Evaluation
- Conclusion

Modeling Challenges

- **Increasing Complexity**
  - System :: memory architectures, interconnect topologies
  - Algorithm :: data decomposition, communication costs
  - Expensive combinatorial optimization
- **Limits of Analytical Modeling**
  - Simplifying assumptions mitigate complexity
  - Performance bounds, scalability trends

Empirical Modeling Paradigm

- **Comprehensively understand parameter space**
  - Specify large, high-resolution parameter space
  - Consider all parameters simultaneously
- **Selectively measure configurations**
  - Sample randomly from space for measurement
  - Decouple resolution of space and measurement
- **Efficiently leverage measured data with inference**
  - Reveal trends, trade-offs from sparse sampling
  - Enable predictions for metrics of interest

Semicoarsening Multigrid (SMG2000)

- Solve discretized 3-D differential equations
- Coarsens only z-dimension, relaxes xy planes

	Set	Parameters	Measure	Range	$\lvert S_i \rvert$
S_1	N_x	x-dim working set	grid points	10 - 510	501
S_2	N_y	y-dim		10 - 510	501
S_3	N_z	z-dim		10 - 510	501
S_4	P_x	x-dim processors	log_2(procs)	0 - 9	10
S_5	P_y	y-dim		0 - 9	10
S_6	P_z	z-dim		0 - 9	10

High-Performance Linpack (HPL)

- Solve dense linear system
- LU decomposition, backward substitution

	Set	Parameters	Measure	Range	$\lvert S_i \rvert$
S_1	Matrix Size	N	sq. matrix dim	1000	1
S_2	Block Size	NB	sq. block dim	10 - 80	8
S_3	Processor Distribution	rows (P) columns (Q)	log_2(procs) log_2(procs)	0 - 9 9-P	10
S_4	Panel Factor	$PFACT$	algorithm	L,R,C	3
S_5	Recursive Factor	$RFACT$	algorithm	L,R,C	3
S_6	Recursive Base	$NBMIN$	block size	1 - 8	8
S_7	Recursive Sub-Panels	$NDIV$	sub-panels	2 - 4	3
S_8	Broadcast	$BCAST$	algorithm	1rg, 1rM, 2rg, 2rM, Lng, LnM	6

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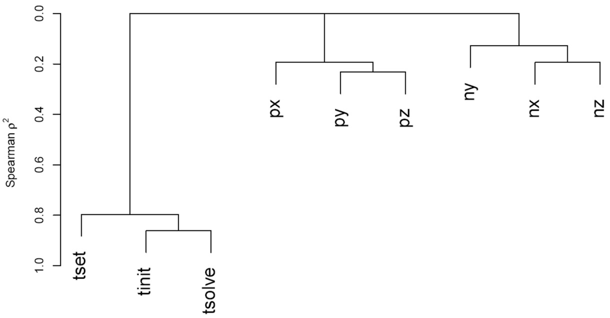
Measurement Sampling

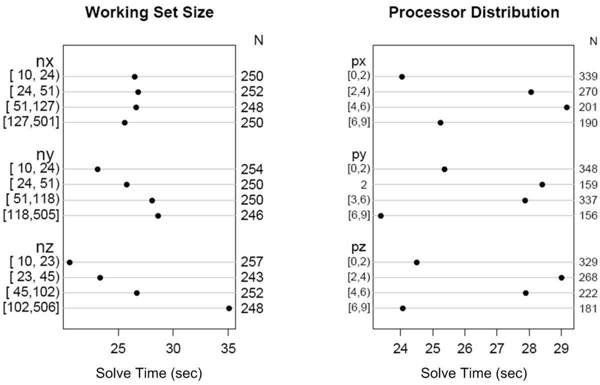
- **Uniform at Random**
  - Unbiased samples from full range of parameter values
- **Stratification**
  - Additional weight to samples with small measured values
  - Mitigates least squares bias toward large measured values
- **Regional Sampling**
  - Per-query models using samples most similar to query
  - Similarity quantified by Euclidean distances

Hierarchical Clustering

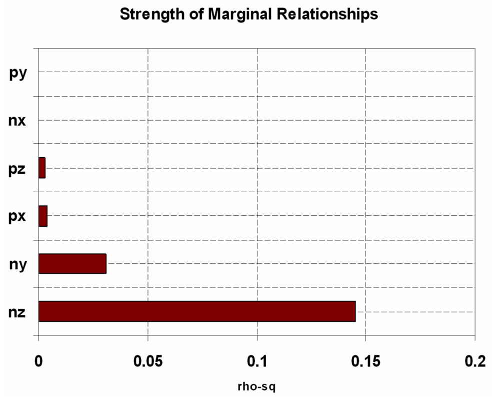


- Create a cluster for each parameter
- Merge most similar pair of clusters
- Repeat merge until single cluster obtained

Association Analysis



Correlation Analysis



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Regression Theory

- **Statistical Inference**
  - Models approximate solutions to intractable problems
  - Requires initial data to train, formulate model
  - Leverages correlations from initial data for prediction
- **Regression Approaches**
  - Piecewise Polynomial Regression (R)
  - Artificial Neural Networks (SNNS)

Piecewise Polynomial Regression

- **Notation**
  - n observations ▷ {*measured configurations*}
  - Response :: $\vec{y} = y_1, \ldots, y_n$ ▷ {*e.g., performance*}
  - Predictor :: $\vec{x}_i = x_{i,1}, \ldots, x_{i,p}$ ▷ {*e.g., matrix block size*}
  - Regression Coefficients :: $\vec{\beta} = \beta_0, \ldots, \beta_p$
  - Random Error :: $\vec{e} = e_1, \ldots, e_n$ where $e_i \sim N(0, \sigma^2)$
  - Transformations :: $f, \vec{g} = g_1, \ldots, g_p$
- **Model**

$$f(y) = \beta_0 + \sum_{j=1}^{p} \beta_j g_j(x_j) + e$$

Predictor Interaction

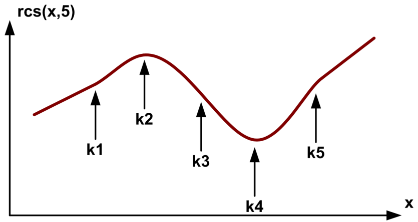
- **Modeling Interaction**
  - Suppose effects of predictors x_1, x_2 cannot be separated
  - Construct predictor $x_3 = x_1 x_2$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + e_i$$

- **Example**
  - Let $x_1 = P_z$ (processor count), $x_2 = N_z$ (working set size)
  - Performance impact of working set size depends on processors in the same dimension

Predictor Non-Linearity

- **Restricted Cubic Splines**
  - Divide predictor domain into intervals separated by knots
  - Piecewise cubic polynomials joined at knots
  - Higher knot counts for more significant parameters



Construction and Prediction

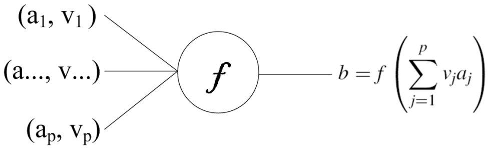
- **Least Squares**
  - Determines $\beta = \beta_0, \ldots, \beta_p$ to minimize $S(\beta)$
  - Solve system of $p+1$ partial derivatives $\delta S / \delta \beta$

$$
\begin{aligned}
S(\beta) &= \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \\
\hat{y}_i &= \beta_0 + \sum_{j=1}^{p} \beta_j x_{ij}
\end{aligned}
$$

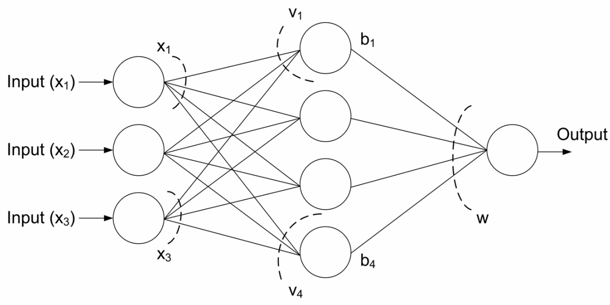
- **Prediction**
  - Weighted sum of predictors

Artificial Neural Networks

- **Notation**
  - Neuron Inputs :: $\vec{a} = a_1, \ldots, a_p$
  - Network Edges :: $\vec{v} = v_1, \ldots, v_p$
  - Neuron Output :: b
- **Neuron Model**



Network Model



$$\hat{y} = f\left(\sum_{j=1}^{H} w_j b_j\right) = f\left(\sum_{j=1}^{H} w_j\, f\left(\sum_{k=1}^{p} v_{j,k} x_k\right)\right)$$

Construction and Prediction

- **Gradient Descent**
  - Determine weights W to minimize $S(W)$
  - Iteratively update weights in direction of steepest descent

$$
\begin{aligned}
S(W) &= \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \\
W_{(t)} &= W_{(t-1)} - \eta \, \nabla S(W_{(t-1)})
\end{aligned}
$$

- **Prediction**
  - Nested weighted sum of predictors

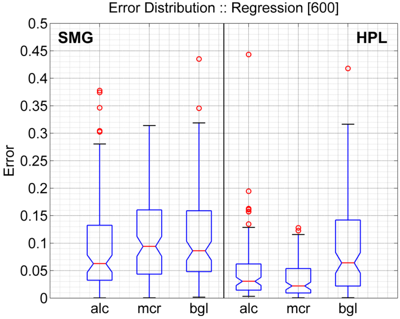
Comparison of Techniques

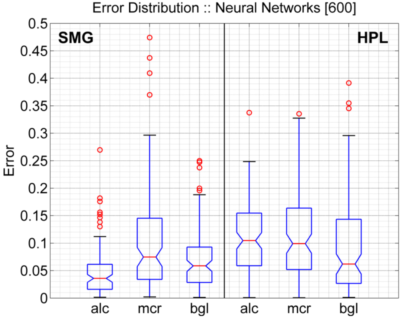
Regression	Neural Networks
Automation	
User provides model specification	Flexible model specification
Transparency	
Interactions, non-linearities exposed by specification	Network as black box, analysis through prediction
Computational Efficiency	
Linear solve $f\left(\beta_0 + \sum_{j=1}^{p} \beta_j g_j(x_j)\right)$	Gradient descent $f\left(\sum_{j=1}^{H} w_j f\left(\sum_{k=1}^{p} v_{j,k} x_k\right)\right)$

Evaluation Methodology

- **Framework**
  - Compare splines and neural networks
  - Formulate models with 600 random samples
  - Obtain 100 additional random samples for validation
- **Platforms**
  - IBM Blue Gene/L (bgl)
  - Intel Xeon (alc, mcr)
- **Applications**
  - Semicoarsening Multigrid (SMG)
  - High-Performance Linpack (HPL)

Predictive Accuracy





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Conclusion

- **Empirical Modeling Paradigm**
  - Comprehensively understand parameter space
  - Selectively measure configurations
  - Efficiently leverage measured data with inference
- **Characterization and Inference**
  - Statistical significance analyses
  - Regression via splines and neural networks
  - Performance predictions with median error <10 percent
- **ISCA 2007 Tutorial**
  - Inference and Learning for Large Scale Microarchitectural Analysis