

CS/ECE 250: Computer Architecture

Logic Design: Tristate Buffers, Finite State Machines

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Slides are derived from work by
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Admin

- **Homework #3 assigned**
- **Readings**
 - **Pragmatic Logic**
 - **Combinational Circuits Ch 4.1-4.2, Ch 5.3**
 - **Sequential Circuits Ch 6**
 - **Also if you want appendix C of H&P**

Finite State Machine

- $\mathbf{S} = \{ s_0, s_1, \dots, s_{n-1} \}$ is a finite set of states.
- $\mathbf{I} = \{ i_0, i_1, \dots, i_{k-1} \}$ is a finite set of input values.
- $\mathbf{O} = \{ o_0, o_1, \dots, o_{m-1} \}$ is a finite set output values.

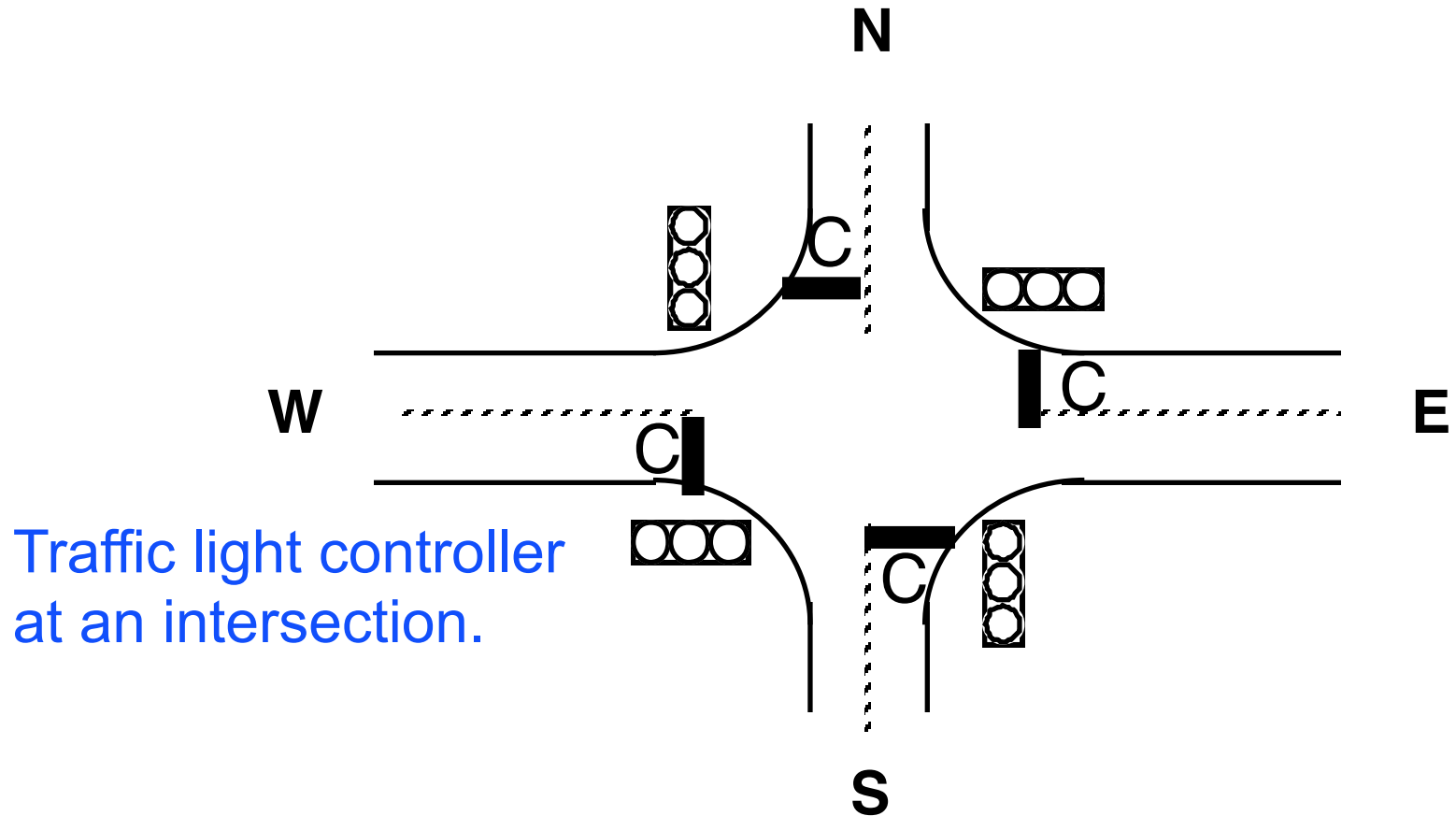
Definition: A finite state machine is a function $F: (\mathbf{S} \times \mathbf{I}) \rightarrow (\mathbf{S} \times \mathbf{O})$ that gets a sequence of input values $i_k \in \mathbf{I}$, $k = 0, 1, 2, \dots$ and it produces a sequence of output values $O_k \in \mathbf{O}$, $k = 1, 2, \dots$ such that:

$$F(s_k, i_k) = (s_{k+1}, o_{k+1}) \quad k=0, 1, 2, \dots$$

Finite State Machine

- **Finite State Machine is:**
 - A machine with a finite number of possible **states**.
 - A machine with a finite number of possible **Inputs**.
 - A machine with a finite number of possible different **outputs**.
 - At each period (clock cycle) the machine receives an **input** and it produces an **output**.
 - The **output** is a function of the **input** and **current state**.
 - After each period the machine changes **state**.
 - The **new state** is a function of the **input** and **current state**.

Example: Traffic Light Controller



Finite State Machine (cont.)

- **Example: Traffic lights controller:**
 - There are four states:
 - NG: Green light in the north-south direction.
 - NY: Yellow light in the north-south direction.
 - EG: Green light at the East-West direction.
 - EY: Yellow light at the East-West direction.
 - There are four outputs:
 - (G;R): North-South **green light**, East-West **red light**
 - (Y;R): North-South **yellow light**, East West **red light**
 - (R;Y): North-South **red light**, East-West **yellow light**
 - (R;G): North-South **red light**, East-West **green light**
 - There are four input values:
 - (c, c): Car at the North-South, Car at East-West
 - (c, nc) Car at North-South, No-car at East-West
 - (nc, c): No-car at North-South, Car at East-West
 - (nc, nc): No-car at North-South, No-car at East-West

FSM Example: Traffic Light

State Transitions:

State	Input	Next-State	Output
NG	(-; NC)	NG	(G; R)
NG	(-; C)	NY	(G; R)
NY	-	EG	(Y; R)
EG	(NC; -)	EG	(R; G)
EG	(C; -)	EY	(R; G)
EY	-	NG	(R; Y)

- means don't care

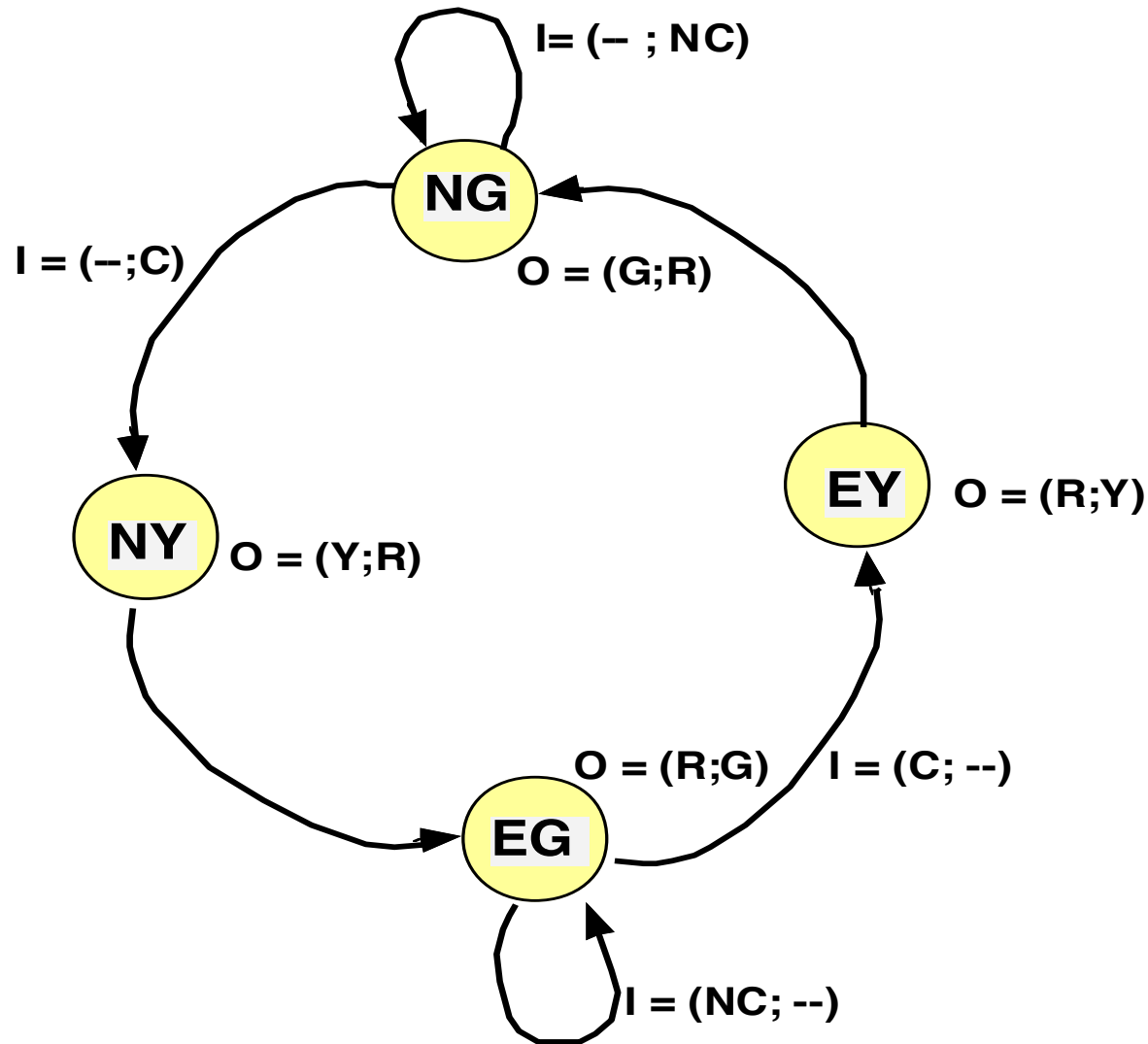
Format
(North/South; East/West)

Finite State Machine (cont.)

- Finite State Machines can be represented by a graph.
- The graph is called a **state diagram**.
- The states are the nodes in the graph.
- The directed edges in the graph represent **state transitions**.
- Each directed edge is labeled with the **inputs** that cause the transition
- Nodes are labeled with the **outputs**.

FSM State Diagram

Example: Traffic light Controller



State Coding

State	Code
NG	00
NY	01
EG	10
EY	11

Input	Code
(C;C)	11
(C;NC)	10
(NC;C)	01
(NC;NC)	00

One bit for each
Input
Input is either
true or false

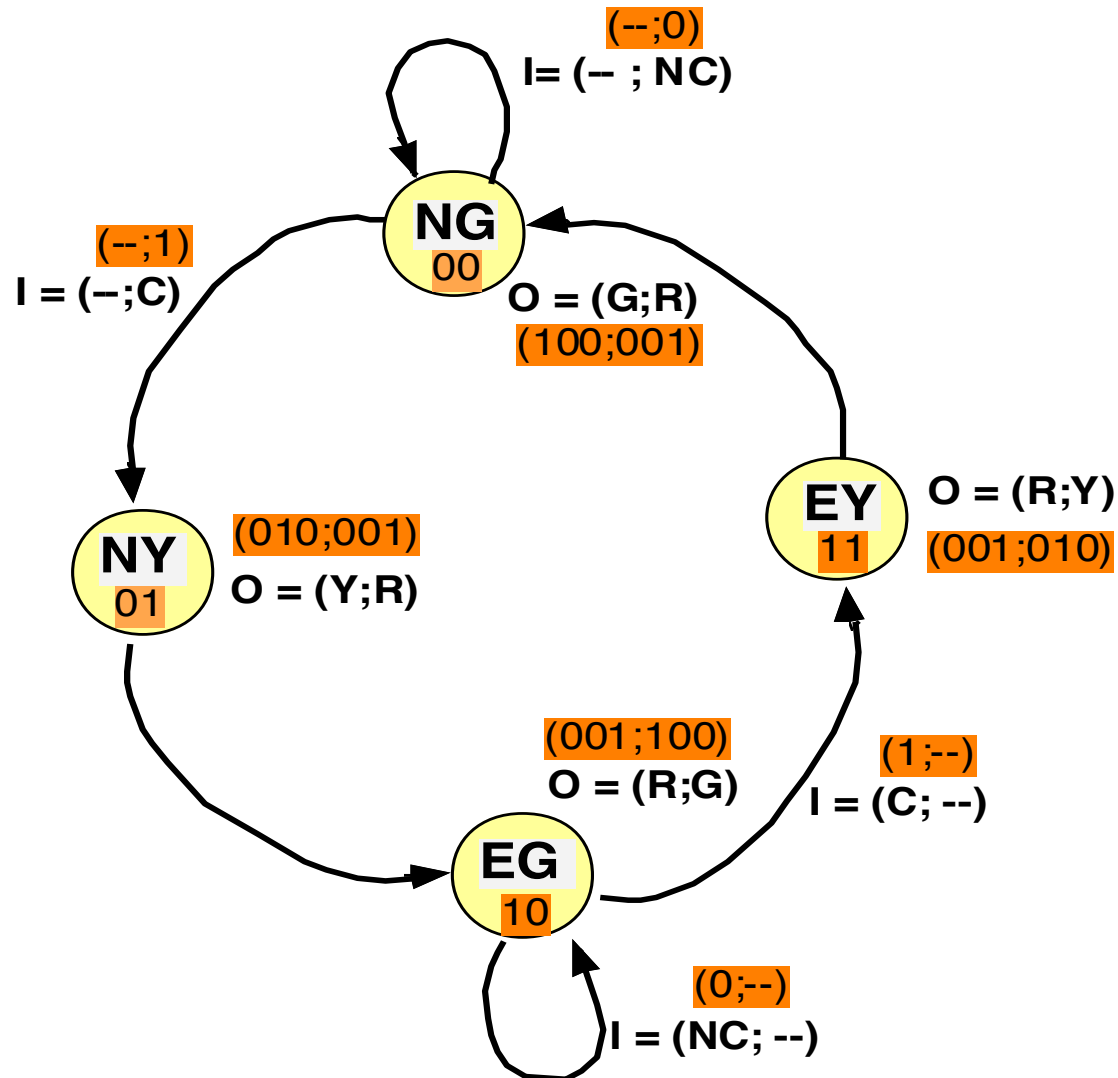
Enumerate States

Output	Code
(R;G)	001100
(G;R)	100001
(Y;R)	010001
(R;Y)	001010

One bit per color for each
light GYRGYR

(North; East)

Coded State Diagram



Example: Traffic Light Controller

S = State, bits are S0 and S1

NS = Next State, bits are NS0 and NS1

IN	S	NS	OUT
01	01	01	012345
0-	00	00	100001
1-	00	01	100001
--	01	10	010001
-0	10	10	001100
-1	10	11	001100
--	11	00	001010

$$\begin{aligned} NS1 &= S0' * S1' * I0 + S0 * S1' * I1 \\ &= S1' * (S0' * I0 + S0 * I1) \end{aligned}$$

$$\begin{aligned} NS0 &= S0' * S1 + S0 * S1' * I1' + S0 * S1' * I1 \\ &= S0' * S1 + S0 * S1' \end{aligned}$$

$$OUT0 = S0' * S1'$$

$$OUT1 = S0' * S1$$

$$OUT2 = S0 * S1' + S0 * S1 = S0$$

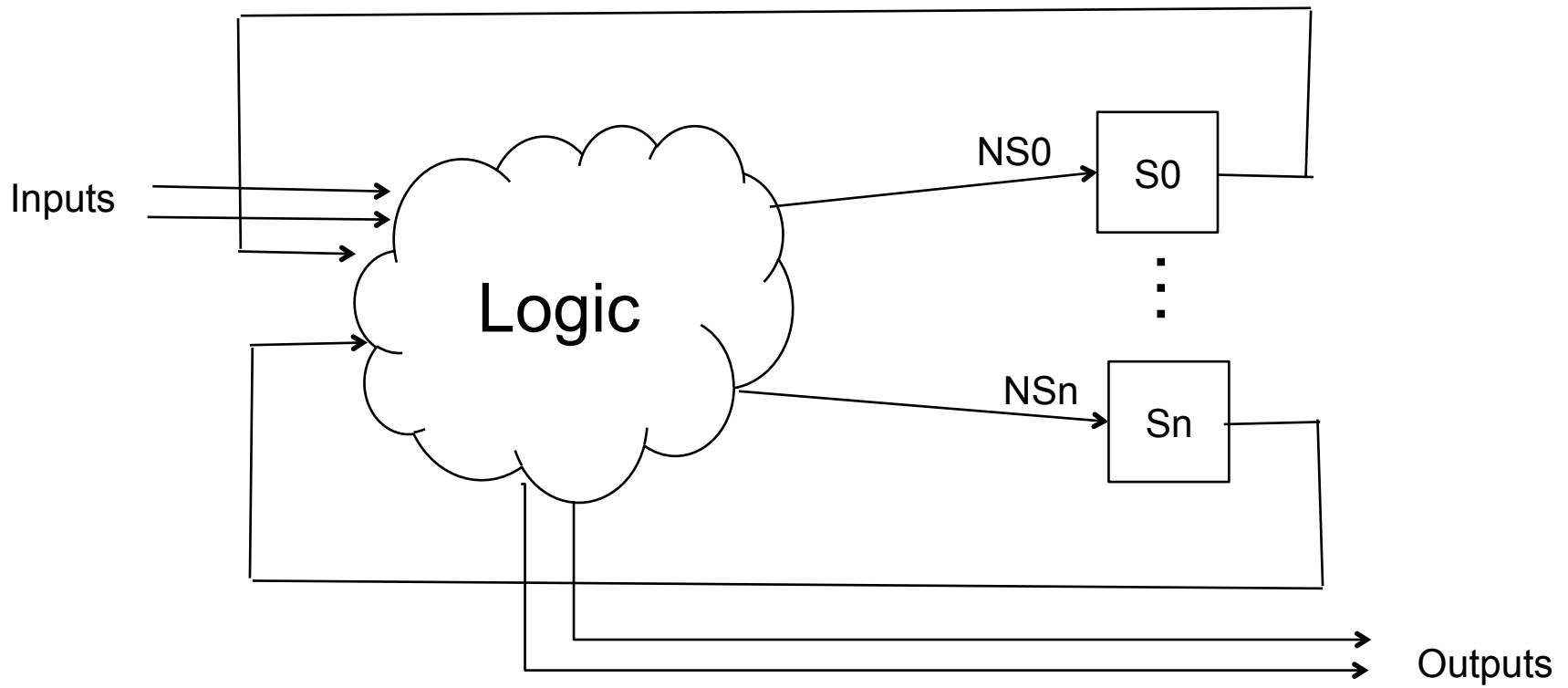
$$OUT3 = S0 * S1'$$

$$OUT4 = S0 * S1$$

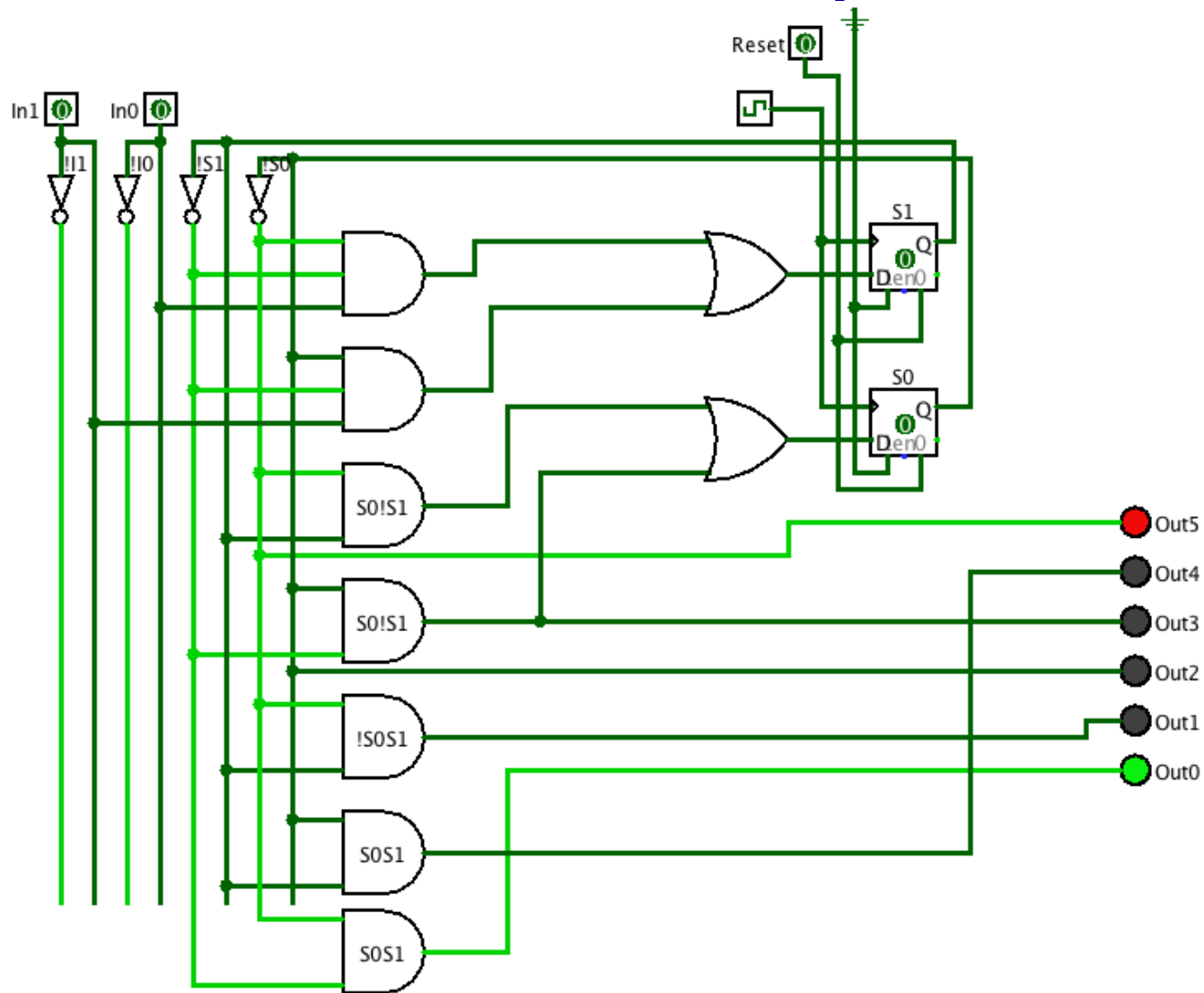
$$OUT5 = S0' * S1' + S0' * S1 = S0'$$

FSM Implementation

- **State is stored in D-Flip Flop**
- **Next State and Output are computed using combinational logic**



Traffic Controller FSM implementation



General Method for FSM design

- **Determine the problem:**
 1. Draw the state diagram,
 2. Write the truth table,
 3. Write sum-of-products equations
 4. Implement in Logic

A Simple Arrow FSM

- Consider those flashing arrow signs
- No light, one arrow, two arrows, three arrows
 > >> >>>
- Let's design the FSM to control this sign

Pattern Recognizer

- **A pattern recognizer examines a sequence of inputs to detect when it sees the pattern 101. When it sees this pattern its output is 1 forever.**
- **Let's design the FSM**

Summary

Can layout logic to compute things

Add, subtract,...

Now can store things

D flip-flops

Registers

Also understand clocks

Can build a finite state machine to control things.

Just about ready to make a processor datapath!