Power Generation Analysis for High-Temperature Gas Turbine in Thermodynamic Process

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A thermodynamic process of power generation has been developed for a gas turbine, a steam turbine, and a compressor, supported by adiabatic expansion or compression processes. Recently, the possible inlet temperature of gas turbines has been increased to 1500°C or more, and the inlet temperature of steam turbines also has been increased to 650°C. At these high-temperature conditions, turbine nozzles or blades have to be cooled by some devices. When we use the adiabatic expansion or compression process, it is necessary to check the fluid-dynamic friction caused by fluid flow between the nozzle and blade. However, considering the high-temperature gas or steam turbine, the nozzle and blade have a cooling effect and frictional heat generation. Furthermore, if we introduce a new concept of internal cooling devices for the compressor to reduce its driving power, the compression process has the cooling effect as well as frictional heat generation. In addition, another new concept for a continous reheating system for gas turbine may improve the thermal efficiency of the combined cycle. These thermodynamic processes of expansion and compression with heat sources and cooling devices to estimate the power generation of turbine and the driving power of a compressor are analyzed. Then, the analytical results are set up and differentiated by their adiabatic, cooled, and heated processes.

Introduction

RECENTLY, gas turbines have been operated at an inlet temperature of 1500°C or more supported by cooling devices applied to keep higher thermal efficiency. Steam turbines have been also operated at the inlet temperature of 650°C, which is higher than usual level. However, a theoretical analysis of power generation with a cooling effect has not been developed by any researcher or manufacturing group. The internal efficiency of an adiabatic turbine can be calculated by a ratio of the actual enthalpy drop to the reversible adiabatic enthalpy drop. The cooled turbine has to be checked by the same analytical procedure as the adiabatic turbine for an estimation of its internal efficiency. When the cooling effect is exactly same as the frictional effect, the enthalpy drop will have the same value of reversible adiabatic enthalpy drop, and the internal efficiency seems to be unity. However, when the cooling effect is introduced, the outlet temperature of the turbine is decreased, but its internal efficiency is not increased. Therefore, we have to study the thermodynamics of the cooling effect for a high-temperature turbine. Furthermore, if we can apply a continous reheating system for the gas turbine, the analysis of heated expansion process has to be checked by another procedure because its reheating system may increase the thermal efficiency of the combined cycle. Therefore, it is necessary to investigate the cooling, heating, and frictional effects for the high-temperature turbine. In this study, we investigate a basic concept of the expansion or compression process and introduce a

definition of the heat-exchange rate in which the cooling, frictional and heating effects can be checked. Each expansion or compression process will be affected by the frictional effect as well as by the cooling or heating effect. If we introduce the heat-exchange rate, it is possible to make a correction of cooling and heating effects for an estimation of the power output in the expansion process and the driving power in the compression process. When the heating effect is zero, the expansion or compression process is defined by the adiabatic process. Thus, we have to apply the heat-exchange rate for our study on power generation of high-temperature gas turbines with cooling, heating, and frictional effects.

Basic Concept of Expansion and Compression Processes in Gas Turbines

The power generation of a gas turbine and driving power of a compressor are estimated by thermodynamic processes of expansion and compression, which can be calculated and checked by a Temperature-Entropy (T-S) diagram or Enthalopy-Entropy (H-S) diagram. The H-S diagram is acceptable for easy calculation of power generation or driving power in each process, as shown in Fig. 1, because the enthalpy difference shows its power output or input in the adiabatic process. If each process involves fluid-dynamic friction, its enthalpy difference is decreased in the expansion process and increased in the compression process by the frictional heat-generating effect, as shown in Figs. 2 and 3. Thus, the actual thermodynamic process with irreversible phenomena will have a smaller enthalpy drop in expansion and a larger enthalpy increase in compression compared with the reversible adiabatic process. However, the other actual thermodynamic processes with the cooling, heating, and frictional heat-generating effects have to be checked by a certain concept that can be applied to correct the enthalpy difference of cooled or heated process. Sometimes the enthalpy drop of a cooled expansion process seems to be larger than that of the adiabatic process because the cooled process has the smaller entropy increase. Then we cannot agree that the power

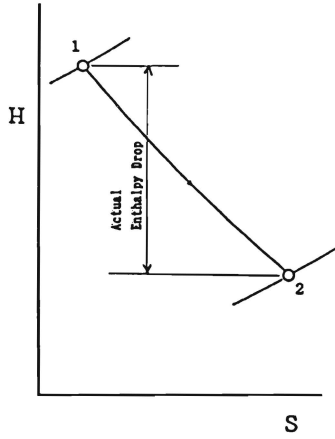**Fig. 1 Thermodynamic path of expansion process.**

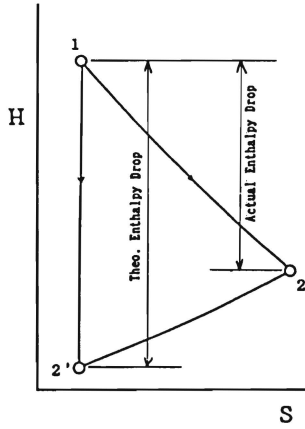**Fig. 2 Thermodynamic path of adiabatic expansion process.**

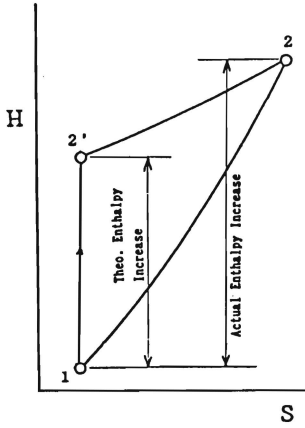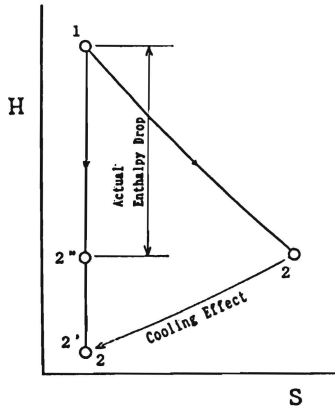**Fig. 3 Thermodynamic path of adiabatic compression process.**

Fig. 4 Thermodynamic path of cooled expansion process.

generation and driving power may be estimated by a simplified usual procedure based on enthalpy difference only. Therefore, we have to introduce a factor of heat-exchange rate, which is applied to make a correction of cooling, heating, and frictional effects.[1] If the heat-exchange rate becomes zero, the expansion and compression processes are defined by the adiabatic process. When the heating effect is introduced in each process, the heat-exchange rate is increased by its effect. Next, if the cooling effect in expansion process has the same value as the frictional heat-generating effect, its enthalpy drop in the H-S diagram is just equal to the reversible adiabatic, as shown in Fig. 4. However, it is clear that the power generation of the cooled expansion process has to be smaller than that of the reversible adiabatic process and to be corrected by the heat loss from the cooling device.

Thermodynamic Analysis of Expansion and Compression Processes with Cooling and Heating Effects

We investigate a thermodynamic analysis of expansion and compression processes in a gas turbine with cooling and heating effects. The ideal gas is introduced to simulate a fluid flow between nozzles and blades in which the power generation of the turbine and the driving power of the compressor can be calculated by theoretical formulas. If we estimate each value of the adiabatic, cooled, or heated process, the thermodynamic path between the inlet and outlet can be divided by infinitesimally small paths, as shown in Fig. 5. Each small path is constructed by reversible adiabatic, constant pressure with a cooling or heating effect and throttling for a theoretical approach of smoothly changed type I. However, an usual simplified concept will be applied for the other paths between the same inlet and outlet, which can be represented by four stepwise types II–V, as shown in Fig. 6. Stepwise type II in Fig. 6 is constructed by heated constant pressure path from 1 to 5, throttling path from 5 to 5″, and reversible adiabatic path from 5″ to 2. Another stepwise type, III,

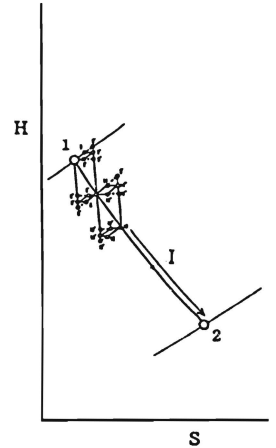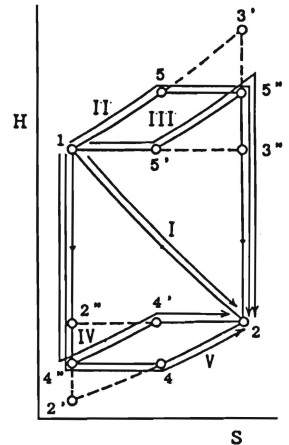**Fig. 5 Thermodynamic path of expansion process (type I).**

Fig. 6 Thermodynamic path of expansion process (types I–V).

is constructed by the throttling path from 1 to 5', heated constant pressure path from 5' to 5'', and reversible adiabatic path from 5'' to 2. Next, stepwise type IV is constructed by the reversible adiabatic path from 1 to 4'', heated constant pressure path from 4'' to 4', and throttling path from 4' to 2. Finally stepwise type V is constructed by the reversible adiabatic path from 1 to 4'', throttling path from 4'' to 4, and heated constant pressure path from 4 to 2. Thus, the smoothly changed type I can be summerized by infinitesimal small paths for each stepwise type I–V.[4,5]

The power output for each of types I–V can be calculated by the following (refer to the Appendix).

Smoothly changed type I

$$Lt = \{x[n/(n-1)] + (1-x)[k/(k-1)]\}RT_1\left[1 - \gamma^{(n-1)/n}\right] \quad (1)$$

Stepwise type II:

$$Lt = [k/(k-1)]RT_1\left\{(1-x) + x\gamma^{(n-1)/n-(k-1)/k} - \gamma^{(n-1)/n}\right\} \quad (2)$$

Stepwise type III:

$$Lt = [k/(k-1)]RT_1\left\{(1-x) + x\gamma^{(n-1)/n-(k-1)/k} - \gamma^{(n-1)/n}\right\} \quad (3)$$

Stepwise type IV:

$$Lt = [k/(k-1)]RT_1\left\{1 - x\gamma^{(k-1)/k} - (1-x)\gamma^{(n-1)/n}\right\} \quad (4)$$

Stepwise type V:

$$Lt = [k/(k-1)]RT_1\left\{1 - x\gamma^{(k-1)/k} - (1-x)\gamma^{(n-1)/n}\right\} \quad (5)$$

where Lt is the power output; k is the specific heat ratio; R is the gas constant; T is the absolute temperature; x is the heat-exchange rate defined by a ratio of $Qh/(Qh + Qf)$, where Qh is heat input or output and Qf is frictional heat generation; γ is the pressure ratio of $P2/P1$, where $P2$ is outlet pressure and $P1$ is inlet pressure; and n is the polytropic index. From the formulas for types I–V, with the same inlet and outlet, we can realize that the values of power output will be different from each other.

The heat input for each of types I–V can be calculated by the following (refer to the Appendix).

Smoothly changed type I:

$$Qh = [n/(n-1) - k/(k-1)]RT_1x\left[1 - \gamma^{(n-1)/n}\right] \quad (6)$$

Stepwise type II:

$$Qh = [k/(k-1)]RT_1x\left[\gamma^{(n-1)/n-(k-1)/k} - 1\right] \quad (7)$$

Stepwise type III:

$$Qh = [k/(k-1)]RT_1x\left[\gamma^{(n-1)/n-(k-1)/k} - 1\right] \quad (8)$$

Stepwise type IV:

$$Qh = [k/(k-1)]RT_1x\left[\gamma^{(n-1)/n} - \gamma^{(k-1)/k}\right] \quad (9)$$

Stepwise type V:

$$Qh = [k/(k-1)]RT_1x\left[\gamma^{(n-1)/n} - \gamma^{(k-1)/k}\right] \quad (10)$$

The values of heat input or output are caused by heat sources or cooling devices but are also different from each other.

The frictional heat-generation for each of types I–V can be calculated by the following (refer to the Appendix).

Smoothly changed type I:

$$Qf = [n/(n-1) - k/(k-1)]RT_1(1-x)\left[1 - \gamma^{(n-1)/n}\right] \quad (11)$$

Stepwise type II:

$$Qf = [k/(k-1)]RT_1\left\{x\gamma^{(k-1)/k} + (1-x)\gamma^{(n-1)/n}\right\}$$
$$\times \log e\left\{x + (1-x)\gamma^{(n-1)/n-(k-1)/k}\right\} \quad (12)$$

Stepwise type III:

$$Qf = [k/(k-1)]RT_1\gamma^{(n-1)/n} \log e\left\{x + (1-x)\gamma^{(n-1)/n-(k-1)/k}\right\} \quad (13)$$

Stepwise type IV:

$$Qf = [k/(k-1)]RT_1\left\{x\gamma^{(k-1)/k} + (1-x)\gamma^{(n-1)/n}\right\}$$
$$\times \log e\left\{x + (1-x)\gamma^{(n-1)/n-(k-1)/k}\right\} \quad (14)$$

Stepwise type V:

$$Qf = [k/(k-1)]RT_1\gamma^{(n-1)/n} \log e\left\{x + (1-x)\gamma^{(n-1)/n-(k-1)/k}\right\} \quad (15)$$

These frictional heat generations are also different from each other. When we try to check the expansion process, the simplified types II or III may be sometimes applied for thermodynamic analysis. However, the actual phenomenon of expansion process will be smoothly changed by type I and differ from types II or III. If we consider the adiabatic condition of the expansion process, the heat-exchange rate becomes zero and the power output is converted to the same well-known expression of polytopic expansion process. That is, for smoothly changed and stepwise types I–V,

$$Lt = [k/(k-1)]RT_1\left[1 - \gamma^{(n-1)/n}\right] \quad (16)$$

Analytical Results of Power Output or Input, Heat-Exchange Rate, and Frictional Heat Generation

In this study, we introduce some nondimensional parameters of polytrope index, heat-exchange rate, and pressure ratio for cycle analysis. If we try to set the cycle condition of gas and steam turbines, the power generation and driving power can be calculated by the preceding formulas in which the cooling, heating, and frictional heat generation are introduced by using the heat-exchange rate. Furthermore, the other heat input or output and frictional heat generation can be also calculated by the same formulas. Thus, the calculated results may be set up in nondimensional expressions of power-exchange ratio, heat-exchange ratio and pressure ratio for gas turbine.[7] The parameters are defined by

$$\phi = Lt/(Lt + Lf) = Lt/(Lt + Qf)$$

$$x = Qh/(Qh + Qf), \qquad \gamma = P2/P1 \quad (17)$$

where ϕ is power-exchange ratio and Lf is frictional loss in fluid flow. When we consider the combined cycle of gas and steam turbines, each cycle is connected by a heat exchanger and each thermal level of both turbines has to be checked by a T-S diagram, as shown in Fig. 7. Recently, an inlet temperature of steam turbine has increased to 650°C, which is very important to keep the higher thermal efficiency of steam cycle and to have a possibility for higher thermal efficiency of the combined cycle. Therefore, an exhaust temperature of a gas turbine has to keep up a high enough level of steam inlet
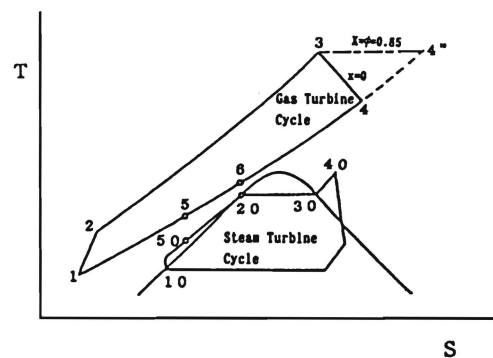

Fig. 7   T-S diagram of combined cycle.

temperature and to be controlled by a reheating device in expansion process. Furthermore, an inlet temperature of a gas turbine may be increased up to 1700°C in the near future, because this research target has been cleared by some applications of high-quality materials and cooling devices. Therefore, the thermodynamic analysis of the expansion process with heat sources or cooling devices can be supported by introducing the heat-exchange rate, which can be applied to the separate effects of both heat input and frictional heat generation.

A simplified system of combined cycle constructed by gas and steam turbines, as shown in Fig. 8, is connected by the heat exchanger.[2,3] If we try to have the heat input in the expansion process, this gas turbine cycle may have a tendency of an increasing exhaust temperature.[6] Furthermore, the thermal efficiency of combined cycle will be affected by both inlet temperatures of the gas
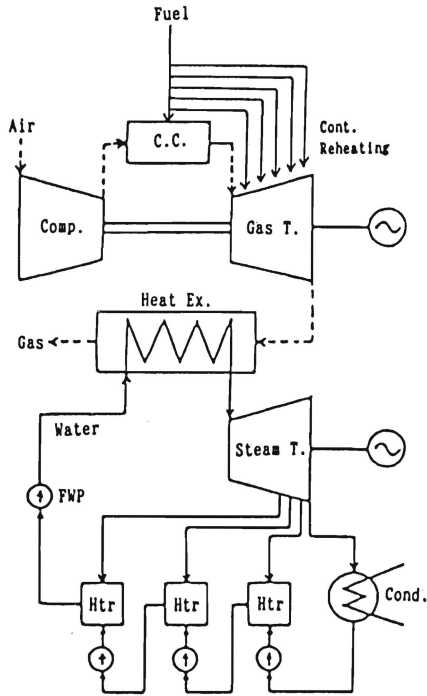

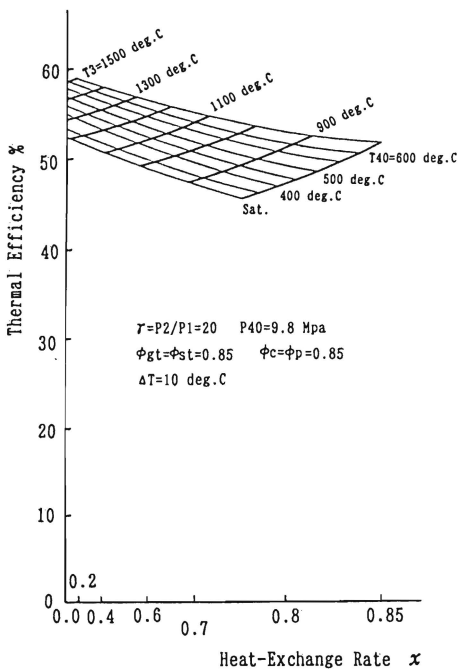Fig. 8   Arrangement of combined cycle.

Fig. 9   Thermal efficiency of combined cycle.

and the steam turbines. An example of the calculated result for the combined cycle is shown in Fig. 9, and it is recognized that the inlet temperature and heat-exchange rate of the gas turbine will be very important factors to keep the higher thermal efficiency. Furthermore, the higher inlet temperature of the steam turbine will be also supported by increasing heat-exchange rate under the constant inlet temperature of the gas turbine. Thus, the effect of heat source or cooling device has to be checked by introducing the heat-exchange rate and to be compared with frictional heat generation, because it is important to realize the difference between them.

Conclusion

A new concept of gas turbine analysis has been introduced to consider the cooling and heating effects in the expansion and compression processes. The heat-exchange rate is applied to differentiate between adiabatic, cooled, and heated processes for thermodynamic analysis. Furthermore, some parameters of power-exchange rate, heat-exchange rate, and pressure ratio are applied to set up the nondimensional expressions of combined cycle analysis. If we introduce the heat-exchange rate for this analysis, it is possible to estimate the power generation of the turbine and the driving power of the compressor with heat sources and cooling devices. Therefore, it is recommended that the thermal behaviors of high-temperature gas and steam turbines have to be analyzed by using the new concept presented.

Appendix: Smoothly Changed Type I with Small Stepwise Types II–V

The thermodynamic path of smoothly changed type I has been divided by infinitesimally small paths to obtain the power output, heat input, and frictional heat generation. If the number of small paths are increased to infinity, each value of the II above II may be converged to the following.

Small stepwise type II:

$$Lt = \lim_{m \to \infty} \frac{k}{k-1} RT_1 \left\{ (1-x) + x\gamma^{(1/m)[(n-1)/n-(k-1)/k]} \right.$$

$$\left. - \gamma^{(1/m)[(n-1)/n]} \right\} \frac{\gamma^{(n-1)/n} - 1}{\gamma^{(1/m)[(n-1)/n]} - 1}$$

$$= \left\{ x\frac{n}{n-1} + (1-x)\frac{k}{k-1} \right\} RT_1 \left[1 - \gamma^{(n-1)/n} \right] \quad (A1)$$

Small stepwise type III:

$$Lt = \lim_{m \to \infty} \frac{k}{k-1} RT_1 \left\{ (1-x) + x\gamma^{(1/m)[(n-1)/n-(k-1)/k]} \right.$$

$$\left. - \gamma^{(1/m)[(n-1)/n]} \right\} \frac{\gamma^{(n-1)/n} - 1}{\gamma^{(1/m)[(n-1)/n]} - 1}$$

$$= \left\{ x\frac{n}{n-1} + (1-x)\frac{k}{k-1} \right\} RT_1 \left[1 - \gamma^{(n-1)/n} \right] \quad (A2)$$

Small stepwise type IV:

$$Lt = \lim_{m \to \infty} \frac{k}{k-1} RT_1 \left\{ 1 - x\gamma^{(1/m)[(k-1)/k]} \right.$$

$$\left. - (1-x)\gamma^{(1/m)[(n-1)/n]} \right\} \frac{\gamma^{(n-1)/n} - 1}{\gamma^{(1/m)[(n-1)/n]} - 1}$$

$$= \left\{ x\frac{n}{n-1} + (1-x)\frac{k}{k-1} \right\} RT_1 \left[1 - \gamma^{(n-1)/n} \right] \quad (A3)$$

Small stepwise type V:

$$Lt = \lim_{m \to \infty} \frac{k}{k-1} RT_1 \left\{ 1 - x\gamma^{(1/m)[(k-1)/k]} \right.$$

$$\left. - (1-x)\gamma^{(1/m)[(n-1)/n]} \right\} \frac{\gamma^{(n-1)/n} - 1}{\gamma^{(1/m)[(n-1)/n]} - 1}$$

$$= \left\{ x\frac{n}{n-1} + (1-x)\frac{k}{k-1} \right\} RT_1 \left[1 - \gamma^{(n-1)/n} \right] \quad (A4)$$

Small stepwise type II:

$$Qh = \lim_{m \to \infty} \frac{k}{k-1} RT_1 x \left\{ \gamma^{(1/m)[(n-1)/n-(k-1)/k]} - 1 \right\}$$

$$\times \frac{\gamma^{(n-1)/n} - 1}{\gamma^{(1/m)[(n-1)/n]} - 1}$$

$$= \left(\frac{n}{n-1} - \frac{k}{k-1} \right) RT_1 x \left[1 - \gamma^{(n-1)/n} \right] \qquad (A5)$$

Small stepwise type III:

$$Qh = \lim_{m \to \infty} \frac{k}{k-1} RT_1 x \left\{ \gamma^{(1/m)[(n-1)/n-(k-1)/k]} - 1 \right\}$$

$$\times \frac{\gamma^{(n-1)/n} - 1}{\gamma^{(1/m)[(n-1)/n]} - 1}$$

$$= \left(\frac{n}{n-1} - \frac{k}{k-1} \right) RT_1 x \left[1 - \gamma^{(n-1)/n} \right] \qquad (A6)$$

Small stepwise type IV:

$$Qh = \lim_{m \to \infty} \frac{k}{k-1} RT_1 x \left\{ \gamma^{(1/m)[(n-1)/n]} \right.$$

$$\left. - \gamma^{(1/m)[(k-1)/k]} \right\} \frac{\gamma^{(n-1)/n} - 1}{\gamma^{(1/m)[(n-1)/n]} - 1}$$

$$= \left(\frac{n}{n-1} - \frac{k}{k-1} \right) RT_1 x \left[1 - \gamma^{(n-1)/n} \right] \qquad (A7)$$

Small stepwise type V:

$$Qh = \lim_{m \to \infty} \frac{k}{k-1} RT_1 x \left\{ \gamma^{(1/m)[(n-1)/n]} \right.$$

$$\left. - \gamma^{(1/m)[(k-1)/k]} \right\} \frac{\gamma^{(n-1)/n} - 1}{\gamma^{(1/m)[(n-1)/n]} - 1}$$

$$= \left(\frac{n}{n-1} - \frac{k}{k-1} \right) RT_1 x \left[1 - \gamma^{(n-1)/n} \right] \qquad (A8)$$

Small stepwise type II:

$$Qf = \lim_{m \to \infty} \frac{k}{k-1} RT_1 \left\{ x\gamma^{(1/m)[(k-1)/k]} + (1-x)\gamma^{(1/m)[(n-1)/n]} \right\}$$

$$\times \log e \left\{ x + (1-x)\gamma^{(1/m)[(n-1)/n-(k-1)/k]} \right\} \frac{\gamma^{(n-1)/n} - 1}{\gamma^{(1/m)[(n-1)/n]} - 1}$$

$$= \left(\frac{n}{n-1} - \frac{k}{k-1} \right) RT_1 (1-x) \left[1 - \gamma^{(n-1)/n} \right] \qquad (A9)$$

Small stepwise type III:

$$Qf = \lim_{m \to \infty} \frac{k}{k-1} RT_1 \gamma^{(1/m)[(n-1)/n]}$$

$$\times \log e \left\{ x + (1-x)\gamma^{(1/m)[(n-1)/n-(k-1)/k]} \right\} \frac{\gamma^{(n-1)/n} - 1}{\gamma^{(1/m)[(n-1)/n]} - 1}$$

$$= \left(\frac{n}{n-1} - \frac{k}{k-1} \right) RT_1 (1-x) \left[1 - \gamma^{(n-1)/n} \right] \qquad (A10)$$

Small stepwise type IV:

$$Qf = \lim_{m \to \infty} \frac{k}{k-1} RT_1 \left\{ x\gamma^{(1/m)[(k-1)/k]} + (1-x)\gamma^{(1/m)[(n-1)/n]} \right\}$$

$$\times \log e \left\{ x + (1-x)\gamma^{(1/m)[(n-1)/n-(k-1)/k]} \right\} \frac{\gamma^{(n-1)/n} - 1}{\gamma^{(1/m)[(n-1)/n]} - 1}$$

$$= \left(\frac{n}{n-1} - \frac{k}{k-1} \right) RT_1 (1-x) \left[1 - \gamma^{(n-1)/n} \right] \qquad (A11)$$

Small stepwise type V:

$$Qf = \lim_{m \to \infty} \frac{k}{k-1} RT_1 \gamma^{(1/m)[(n-1)/n]}$$

$$\times \log e \left\{ x + (1-x)\gamma^{(1/m)[(n-1)/n-(k-1)/k]} \right\} \frac{\gamma^{(n-1)/n} - 1}{\gamma^{(1/m)[(n-1)/n]} - 1}$$

$$= \left(\frac{n}{n-1} - \frac{k}{k-1} \right) RT_1 (1-x) \left[1 - \gamma^{(n-1)/n} \right] \qquad (A12)$$

Each formula of Lt, Qh, or Qf has an equal expression of Eqs. (1), (6), or (11). Therefore, the thermodynamic path of expansion or compression can be analyzed by introducing the concept of smoothly changed type I instead of the simplified stepwise types II–V.

Acknowledgments

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