SECOND-LAW ANALYSIS OF AN IDEAL OTTO CYCLE

NOAM LIOR and GEORGE J. RUDY†

Department of Mechanical Engineering and Applied Mechanics, University of Pennsylvania,
Philadelphia, PA 19104-6315, U.S.A.

Abstract—The system input and output exergy at each portion of an ideal Otto cycle, as well as the process effectiveness, are calculated for compression ratios of 3.0:9.0 and air/fuel equivalence ratios of 0.25:1.0. For comparison, the energy-based efficiencies are also calculated. It was found that both effectiveness and efficiency increase with compression ratio and with air-to-fuel ratio (but at different rates), and that the effectiveness does not vary much when the air-to-fuel ratio increases above 200%. It was also shown that exergy analysis represents the losses in the cycle, such as in the combustion and exhaust processes, much more realistically than the conventional first-law analysis and serves as a good guide for cycle improvements. Several recommendations are made in that direction.

NOMENCLATURE

C_p = Specific heat at constant pressure
C_v = Specific heat at constant volume
h = Enthalpy of one species
H = Enthalpy of mixture
p = Pressure
Q = Heat
R = Gas constant
S = Entropy
T = Absolute temperature
v = Specific volume
W = Work output
x = Mole fraction
γ = Ratio of specific heats, C_p/C_v
ϵ = Effectiveness [equation (30)]
η = Efficiency [equations (31, 32)]
ϕ = Exergy
Φ = Equivalence ratio

Subscripts

i = Species index
0 = Dead state
1, 2, 3, 4, 5 = Cycle state points (Figs 1, 2)

INTRODUCTION

The ubiquitous spark–ignition internal combustion engine, the primary power source for automobiles, consumes about 20% of the total annual energy used in the U.S. Continuous efforts have been expended since the invention of the Otto cycle, which describes it thermodynamically, to improve its performance and efficiency [1–3]. Consistent with these objectives, the value of exergy analysis [4, 5] was recognized as a very instructive method for identifying cycle irreversibilities which destroy the ability to extract useful work. A simple example of exergy calculations for one set of Otto cycle conditions, based on the property charts from Hershey *et al.* [6] was presented by Hatsopoulos and Keenan [7]. More recently, a num-

ber of studies were published on the application of exergy analysis to advanced diesel cycles and their components [8–10]. This paper presents an exergy analysis of the Otto cycle, which was performed to improve the understanding of exergy utilization and variations as a function of compression ratio (CR) and air/fuel ratio (the equivalence ratio, Φ) and, consequently, to identify possible ways for improving cycle effectiveness. The working fluid in this analysis is a hydrocarbon fuel (octane) mixed with air.

Figure 1 shows the ideal Otto cycle in two thermodynamic planes. It can be seen that the cycle consists of the following steps.

5 → 1. Intake stroke

Mixture of fuel and air is drawn into the cylinder as the piston moves to increase the volume in the cylinder.

1 → 2. Compression stroke

Intake valve is closed, and the piston moves back, compressing the mixture (isentropically in the ideal cycle shown).

2 → 3. Combustion

At the end of compression, a spark is introduced which causes ignition of the mixture; this combustion step is assumed to be instantaneous and isochoric in the ideal cycle.
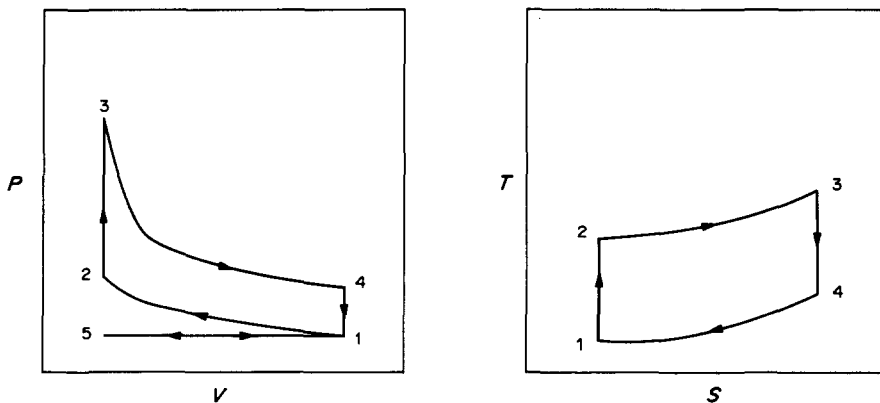
3 → 4. Expansion stroke

The combustion products, which reached pressure higher than the ambient in the previous step, expand (here isentropically) and move the piston which thus performs mechanical work.

4 → 5. Exhaust

The exhaust valve opens and the trapped gases escape.

†Present address: General Electric Co., Valley Forge, PA 19101, U.S.A.

Fig. 1. The Otto cycle in the P-V and T-S planes.

In reality, none of the processes involved are reversible, isobaric, or isochoric. The actual cycle resembles more closely the cycle shown in Fig. 2, and methods for its rigorous modeling and analysis are under intensive development [11–13]. Nevertheless, we shall, as an essential introductory step to the second-law analysis of real cycles, analyze in this paper only the ideal cycle. Although not exact, this approach allows an easier preliminary identification and evaluation of the major factors in the process.

ANALYSIS

Determination of the thermodynamic state points

To begin the analysis, the exergy of the incoming (trapped) fuel/air mixture must be evaluated. In an open system, this would involve,

$$\phi_{\text{mix}} = \sum_i x_i (\phi_0^i + \Delta\phi_{0\to1}^i) \tag{1}$$

where "i" represents the individual components. Although the Otto cycle does have mass flow across its boundaries, it only occurs during the intake and exhaust phases. More simply, once a control mass of fuel and air is trapped, the system is "closed" for the remainder of one cycle. Therefore, it is appropriate to use the closed system exergy [4, 5],

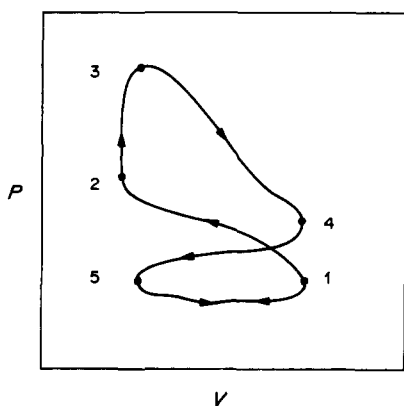$$\phi_{\text{closed mix}} = \phi_{\text{open mix}} - (p - p_0)v, \tag{2}$$

Fig. 2. A real Otto cycle.

which, with equation (1), becomes

$$\phi_{\text{closed mix}} = \sum_i x_i [\phi_0^i + \Delta\phi_{0\to1}^i - (p_i - p_0)v], \tag{3}$$

where

x_i is the mole fraction of i,
p_i is the partial pressure of i,
ϕ_0^i is the standard chemical exergy of i,

and $\Delta\phi_{0\to1}^i$ is the correction for nonstandard conditions of i at **state 1**, expressed by Sussman [4] as:

$$\Delta\phi_{0\to1} = (H_1 - H_0) - T_0(\bar{S}_1 - \bar{S}_0), \tag{4}$$

or

$$\Delta\phi_{0\to1} = C_p(T_1 - T_0) - T_0\left[C_p \ln\left(\frac{T_1}{T_0}\right) + R \ln\left(\frac{P_0}{P_i}\right)\right]. \tag{5}$$

But

$$T_1 = T_0,$$

so

$$\Delta\phi_{0\to1} = -T_0 R \ln(p_0/p_i). \tag{6}$$

We can calculate the partial pressures of the reactants in the mixture, p_i, by

$$p_i = x_i p, \tag{7}$$

for each of the components (C_8H_{18}, O_2, N_2), and since all the parameters on the right-hand side of equation (6) are then either given or calculated by equation (7), one can calculate the correction term $\Delta\phi_{0\to1}$ for use in the exergy expression (3).

To determine the last term in the parentheses on the right-hand side of equation (3), the required volume is calculated by assuming ideal gas behavior of the fuel–air mixture:

$$\hat{v} = \frac{RT}{p}, \tag{8}$$

which then allows the calculation of the exergy of the reactants (ϕ_i) and of the reactant mixture ($\phi_1 = \Sigma_i x_i \phi_i$) at **state 1** [by using equation (3)].

Assuming the ideal gas, for the isentropic process $1 \to 2$,

$$T_1 v_1^{\gamma-1} = T_2 v_2^{\gamma-1} \qquad (9)$$

and

$$p_2 = p_1 \left(\frac{v_1}{v_2}\right)\left(\frac{T_2}{T_1}\right) \qquad (10)$$

where $\gamma = C_p/C_v$ is the ratio of specific heats.

In an attempt to compensate for the large temperature variations in the Otto cycle, the specific heats are expressed as a function of temperature by Campbell [2],

$$\text{For } C_8H_{18}, \quad \hat{C}_p = 25.3 + 0.162T$$

$$\hat{C}_v = 16.98 + 0.162T$$

$$\text{For } O_2, \quad \hat{C}_p = 27.0 + 0.0079T$$

$$\hat{C}_p = 18.68 + 0.0079T$$

$$\text{For } N_2, \quad \hat{C}_p = 27.6 + 0.0051T$$

$$\hat{C}_v = 19.28 + 0.0051T, \qquad (11)$$

where the specific heats are in (kJ/kg.mole.K) and T, the temperature, is in (K). Then, for the mixture,

$$C_p = \Sigma x_i \hat{C}_{pi} = (0.0165)(25.3 + 0.162T) + (0.2066)$$

$$\times (27 + 0.0079T) + (0.7769)(27.6 + 0.0051T)$$

$$C_p = 27.438 + 0.00827T,$$

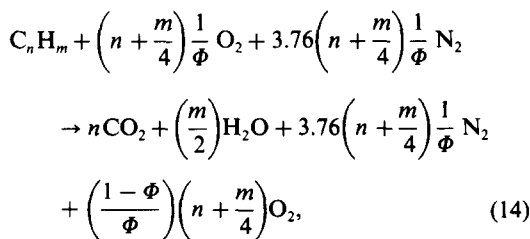and, similarly,

$$C_v = 19.121 + 0.00827T. \qquad (12)$$

Then,

$$\gamma = \frac{\hat{C}_p}{\hat{C}_v} = \frac{27.438 + 0.00827T}{19.121 + 0.00827T}. \qquad (13)$$

An iterative procedure is then employed where γ is evaluated at the midpoint, between T_2 and T_1. Thus, knowing γ from equation (13), T_2 is calculated from equation (9), and since the compression ratio (v_1/v_2) is known, equation (10) serves then to determine p_2. Knowledge of T_2 and P_2 allows the calculation of the specific heats [equations (11, 12)], partial pressures, and exergies of the reactants and of the reactant mixture at state 2 by using equations (3), (5) and (8) as outlined above.

The combustion process $2 \to 3$ is assumed to be adiabatic, at constant volume, complete and instantaneous.

The basic chemical balance equation for the combustion of a hydrocarbon–air mixture is:

$$C_nH_m + \left(n + \frac{m}{4}\right)\frac{1}{\Phi} O_2 + 3.76\left(n + \frac{m}{4}\right)\frac{1}{\Phi} N_2$$

$$\to nCO_2 + \left(\frac{m}{2}\right)H_2O + 3.76\left(n + \frac{m}{4}\right)\frac{1}{\Phi} N_2$$

$$+ \left(\frac{1-\Phi}{\Phi}\right)\left(n + \frac{m}{4}\right)O_2, \qquad (14)$$

where Φ is the "equivalence ratio", defined as the ratio of the actual fuel/air ratio to the stoichiometric fuel/air ratio [3–5]. Equation (14) only holds for
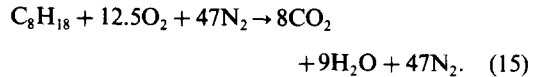
$$0 < \Phi \leqslant 1.$$

For example, in the case of a stoichiometric mixture,

$$\Phi = 1,$$

and for n-octane

$$n = 8, \quad m = 18,$$

so then

$$C_8H_{18} + 12.5O_2 + 47N_2 \to 8CO_2$$

$$+ 9H_2O + 47N_2. \qquad (15)$$

Chemical balance equation (14) is used in all of the calculations related to the combustion reaction and products in this paper.

To evaluate the combustion reaction, the method described by Reynolds and Perkins [14] is adopted. It is noted that

$$\hat{H}_{\text{reactants}} = \hat{H}_{\text{products}} \qquad (16)$$

for the specified conditions, where $\hat{H}$ is the mixture enthalpy, then evaluating the enthalpies of the reactants [14] by

$$\hat{h}_i = \hat{h}_{i_0} + \hat{C}_{p_i}(T - T_0), \qquad (17)$$

and then, for the mixture,

$$\hat{H}_{\text{reactants}} = \sum_i x_i h_i. \qquad (18)$$

For the products, from equation (14),

$$\hat{H}_{\text{products}} = \left(\frac{8}{8 + 9 + 47}\right)\hat{h}_{CO_2}$$

$$+ \left(\frac{9}{64}\right)\hat{h}_{H_2O} + \left(\frac{47}{64}\right)\hat{h}_{N_2}. \qquad (19)$$

Using equation (16) to equate (18) and (19), and having the enthalpies of the combustion products (CO_2, H_2O and N_2) as a function of temperature [15, 16], produces an equation with a temperature T_3 as the single unknown. T_3 is calculated therefrom, and using

$$p_3 = p_2\left(\frac{T_3}{T_2}\right)\left(\frac{N_{\text{products}}}{N_{\text{reactants}}}\right) \qquad (20)$$

(cf. [2]), allows the determination of p_3. Knowing the temperature and pressure, it is possible to evaluate the exergy following combustion. First, as in equation (12), the specific heats are evaluated. From further data in Campbell [2],

$$\hat{C}_p(CO_2) = 27.4 + 0.0058T$$

$$\hat{C}_v(CO_2) = 19.08 + 0.0058T$$

$$\hat{C}_p(H_2O) = 30.5 + 0.0103T$$

$$\hat{C}_v(H_2O) = 22.18 + 0.0103T$$

$$\hat{C}_p(N_2) = 27.6 + 0.0051T$$

$$\hat{C}_v(N_2) = 19.28 + 0.0051T, \qquad (21)$$

(all specific heats are in kJ/kg.mole.K, T in K). It should be noted that the linear temperature dependence represented by equations (11) and (21) is strictly correct only up to 1000 K. It was used, due to its simplicity, introducing an error of <10% in the results. More exact property values can be found in other works [16–18].

The partial pressures and exergies of the individual combustion products, and the specific volume and exergy of the mixture of the combustion products (assumed to behave as a perfect gas) are thus calculated for **state point 3**, just as indicated above in the calculation of state point 2.

The final part of the ideal cycle, **process 3 → 4**, involves an isentropic expansion of the combustion products. It is this expansion which is the work producing phase of the cycle. Continuing to assume ideal gas behavior,

$$T_3 v_2^{\gamma-1} = T_4 v_1^{\gamma-1}, \tag{22}$$

and

$$p_4 = p_3 \left(\frac{v_2}{v_1}\right)\left(\frac{T_4}{T_3}\right), \tag{23}$$

equation (21) is used to determine the specific heats of the mixture of combustion products, resulting, for the mixture, in

$$\hat{C}_p = 27.953 + 0.00591T,$$

and

$$\hat{C}_v = 19.644 + 0.00591T. \tag{24}$$

Then,

$$\gamma = \frac{\hat{C}_p}{\hat{C}_v} = \frac{27.953 + 0.00591T}{19.644 + 0.00591T}, \tag{25}$$

and an iterative evaluation, similar to that employed in process 1 → 2 (described above), is done to determine γ at the midpoint temperature between T_3 and T_4. Just as in the calculation for determining state point 2, this is then used in equation (22) to determine T_4; equation (23) is then used to determine p_4; and the ideal gas relation [such as equation (8)] is used to determine $\hat{v}_4$. Having thus determined **state 4** fully, the exergies of the individual combustion products, as well as of their mixture, can be calculated by the same methods described above for the preceding portions of the process.

It is assumed throughout that the intake conditions are:

$$p_1 = 1 \text{ atm}; \quad T_1 = 25°C, \tag{26}$$

the same as the dead-state conditions chosen for this analysis.

For these conditions, Ref. [1] gives the following data,

$$\phi_{0, C_8H_{18}} = 5.30 \text{ MJ/g.mole}$$

$$\phi_{0, O_2} = 0,$$

$$\phi_{0, N_2} = 0. \tag{27}$$

Process exergy and energy changes, effectiveness and efficiency

If it is assumed that no work is done by the exhaust gases, and they return to the environment, the following changes in exergy can be found for the process:

Exergy added during compression = $\phi_2 - \phi_1$
Exergy lost during combustion = $\phi_2 - \phi_3$
Exergy extracted during expansion = $\phi_3 - \phi_4$
Exergy lost from exhaust = ϕ_4. (28)

Since this is a closed system with only one control mass, the system effectiveness can be found from

$$\epsilon = \frac{\text{net exergy extracted for intended use}}{\text{decrease in input fuel exergy}} \tag{29}$$

or

$$\epsilon_{system} = \frac{(\phi_3 - \phi_4) - (\phi_2 - \phi_1)}{\phi_{in}}, \tag{30}$$

since no use is made of the exhaust gases.

From the classical **energy** method of analysis for this ideal cycle,

$$\eta = \frac{W_{net}}{Q_{in}} = \frac{C_v(T_3 - T_2) - C_v(T_4 - T_1)}{C_v(T_3 - T_2)}. \tag{31}$$

If constant specific heats are assumed (as is often done), this reduces to $\eta = \eta(\text{CR}, \gamma)$. This analysis, however, assumes constant specific heats for individual processes [at the average temperature of each process, using equations (12) and (24)] but not for the system. Assuming constant $\hat{C}_p$ and $\hat{C}_v$ would result in an efficiency defined by

$$\eta_{const} = 1 - \frac{1}{(\text{CR})^{\gamma-1}}, \tag{32}$$

which will obviously yield efficiency values that are different from those obtained from (31), and more in error.

RESULTS AND DISCUSSION

The thermodynamic properties, including exergy, at each of the state points, and the effectiveness and efficiencies [using equations (30)–(32)] were calculated for compression ratios (CR) of 3.0, 6.2 and 9.0 at the fuel/air equivalence ratio of 1.0 (stoichiometric), and for equivalence ratios of 0.25, 0.5 and 1.0 at a compression ratio of 6.2. The thermodynamic properties at the state points, and the exergy "flow" charts are shown in Figs 3–7 (in MJ/g·mole fuel), and the exergy "flows" are also shown in Table 1. Table 2 summarizes the effectiveness and efficiency results for these cases. A value of $\gamma = 1.356$ was used in the efficiency calculation which assumed constant specific heats [equation (32)]. Figures 8 and 9 show the efficiency and effectiveness as a function of the compression ratio and the equivalence ratio, respectively. As expected, both the effectiveness and the efficiency increase with the compression ratio, but the effectiveness increases at a rate which is roughly double. This is primarily because first-law (energy)
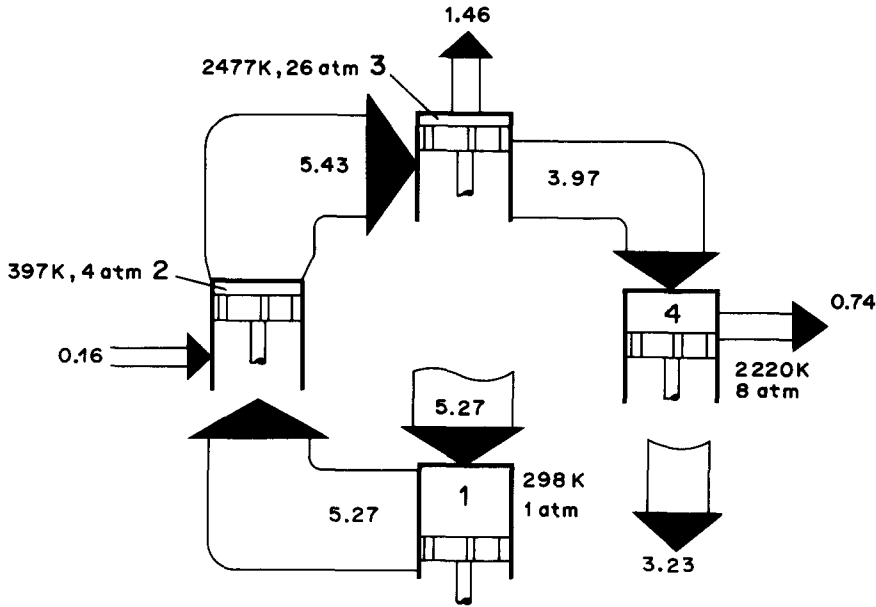
Fig. 3. Exergy flow chart: CR = 3.0, Φ = 1.0.

analysis does not account well for the fact that the losses in the potential to do useful work both in the combustion and exhaust processes decrease with CR as well, thereby augmenting the improving work output.

It is also noteworthy that any increases in the compression work invested with the increase of CR are more than compensated by the increase in the work extracted.

Both the effectiveness and the efficiency decrease with the equivalence ratio (i.e., as the air-to-fuel ratio decreases). This decrease is approximately linear for the efficiency and approximately at a similar rate for

the effectiveness at equivalence ratios larger than about 0.5. For equivalence ratios between 0.25 and 0.5 the effectiveness is approximately constant. Figures 3–7 provide an explanation: increasing the air-to-fuel ratio indeed lowers the top (combustion) pressure and temperature and, thus, reduces the exergy input during the work-producing expansion process (3 → 4), but it also reduces significantly the exergy of the exhaust gases, the two phenomena having opposite effects on the work output.

First-law analysis does not reveal the magnitude of work-potential lost in the combustion process. The exergy analysis results shown above indicate that
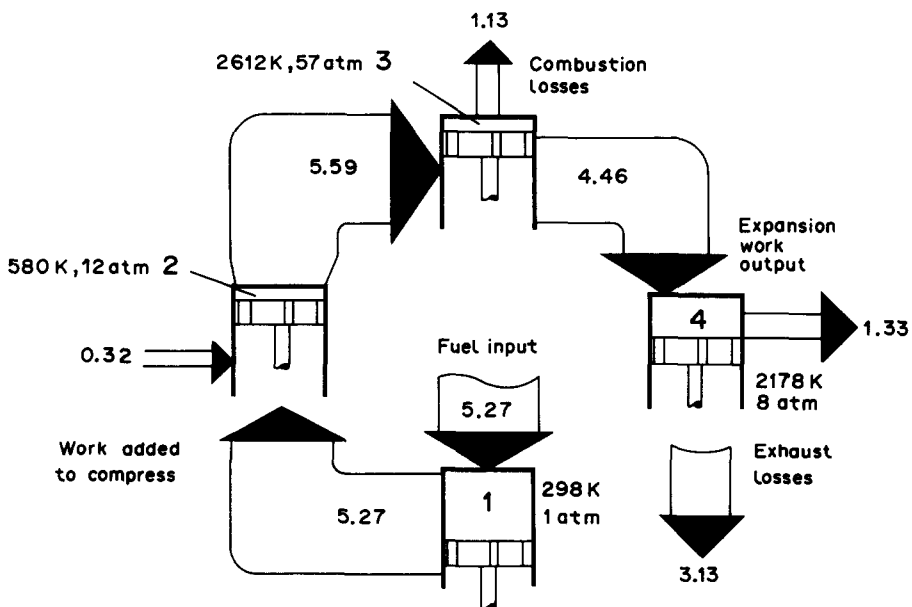

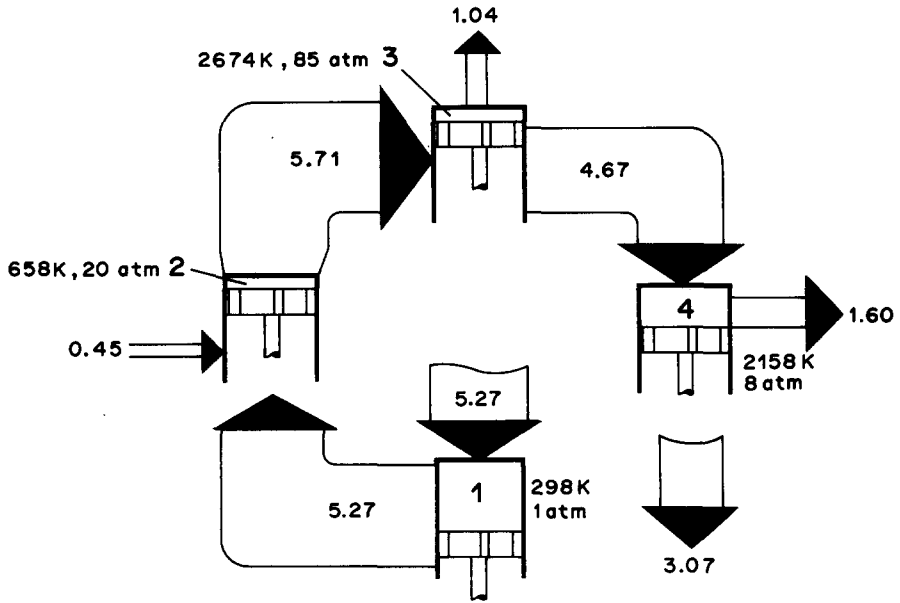Fig. 4. Exergy flow diagram: CR = 6.2, Φ = 1.0.

Fig. 5. Exergy flow diagram: CR = 9.0, Φ = 1.0.

Table 1. Exergy flows in the process

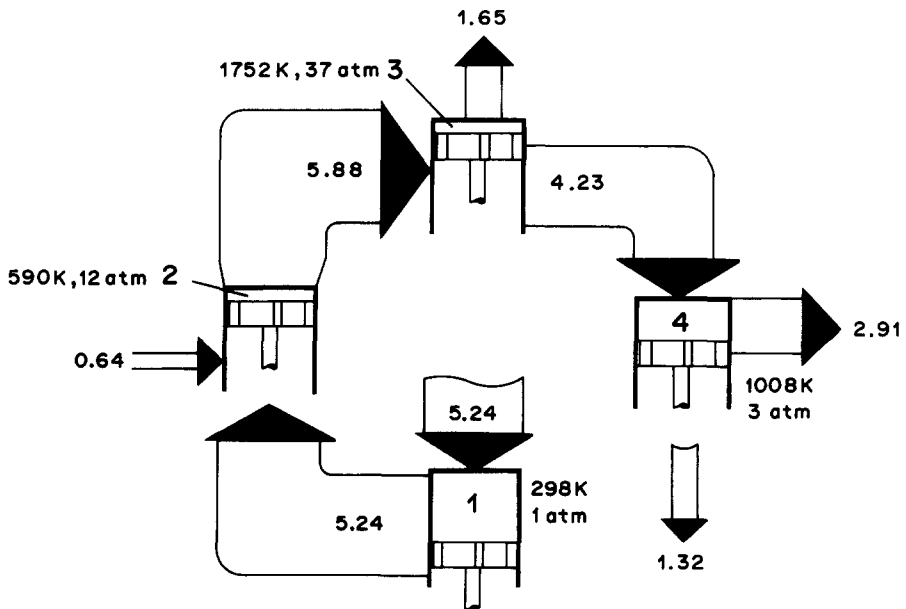| Exergy	CR:	3.0	6.2	9.0	6.2	6.2
(MJ/g.mole fuel)	Φ:	1.0	1.0	1.0	0.5	0.25
Added during compression (1 → 2)		0.16	0.32	0.45	0.64	1.16
Lost during combustion (2 → 3)		1.46	1.16	1.04	1.65	2.16
Extracted during expansion (3 → 4)		0.74	1.33	1.60	2.91	3.41
Lost in exhaust gases (4 → 1 → 5)		3.23	3.13	3.07	1.32	0.79

Fig. 6. Exergy flow diagram: CR = 6.2, Φ = 0.5.

Table 2. Effectiveness and efficiency results

| | CR: | 3.0 | 6.2 | 9.0 | 6.2 | 6.2 |
	Φ:	1.0	1.0	1.0	0.5	0.25
Effectiveness (ϵ)		11.0	19.1	21.9	43.3	43.2
Efficiency (η)		11.6	15.5	17.3	40.9	50.7
η for constant specific heats		33.0	47.5	55.2	47.5	47.5

these losses are high, of the order of the useful work extracted. At the same time, first-law analysis over-estimates the significance of the exhaust gas losses: although the amount of energy lost is large ($\sim 84.5\%$ for CR = 6.2 and $\Phi = 1.0$), the exergy of the exhaust is much lower, amounting to about 60% of the fuel input for the same conditions.

The exergy analysis performed points to several possible ways for improving cycle performance:

(1) To reduce exergy losses in the combustion process: improved combustion chambers, fuel–air mixing, and ignition, as well as either adiabatic combustion or use of the exergy loss there to drive a secondary, lower temperature power-producing cycle.

(2) To reduce the exergy losses in the exhaust process: allow piston expansion past the intake volume, use the exhaust gases as the exergy source for
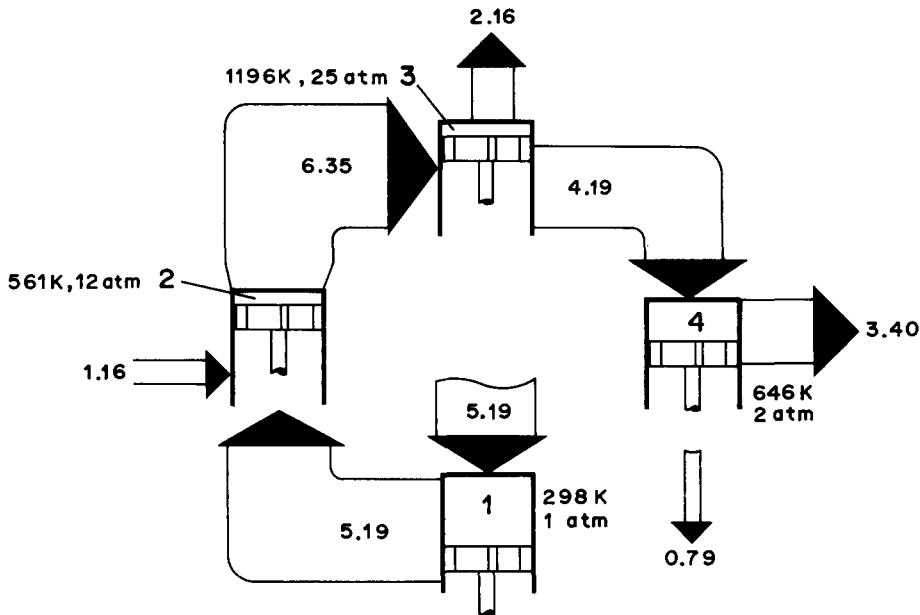

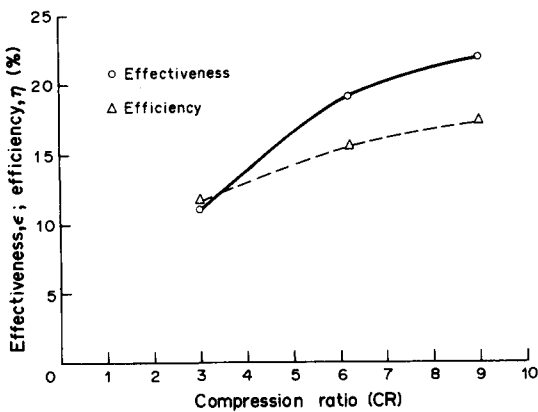Fig. 7. Exergy flow diagram: CR = 6.2, $\Phi = 0.25$.

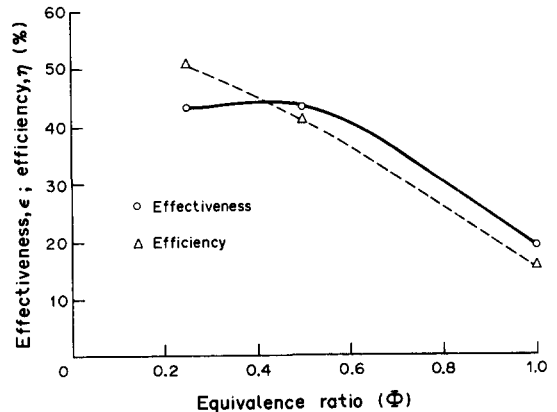Fig. 8. Effectiveness and efficiency as a function of compression ratio.

Fig. 9. Effectiveness and efficiency as a function of equivalence ratio.

a bottoming cycle, and provide some excess air ($\sim \Phi = 0.5$ here).

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