

Practical limits on the biomagnetic inverse process determined from *in vitro* measurements in spherical conducting volumes

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Abstract. A technique of locating current dipoles in spherical conducting volumes by determining the location of the magnetic field maximum and inverting the magnetic field equations was developed and the expected localisation errors were predicted. AC current dipoles were placed in spheres of uniform conductivity. Each dipole's magnetic field was measured and its location was calculated by determining the angle between the magnetic field null and maximum and using an iterative inverse solution to the magnetic field equations. Absolute agreement between predicted magnetic field strengths and actual magnetic field measurements was within 5%. A study of the effect of signal to noise ratio and number of data points in the analysis indicates that dipole localisation of ~ 1 mm is achievable for a signal to noise ratio greater than 10 decibels ($S/N > 10$ db).

1. Introduction

In the rush to apply biomagnetometer technology to the measurement and localisation of epileptic foci and other phenomena in the brain, the fundamental limits of this measurement technique have generally been neglected. Recently Barth *et al* (1984, 1985), Janday and Swithenby (1987) and others have realised that in order to understand the results of human measurements, the physics of the phenomena and the difficulty of making small signal measurements in magnetically noisy laboratory environments must be understood. The current study systematically determines the limits and errors expected in the localisation of dipoles using superconducting magnetometers. For the results presented here the brain was modelled by a sphere filled with a solution of uniform conductivity and the source was an ideal current dipole. The solution to the inverse problem for a real, multiloop magnetic gradiometer was derived, the expected observation errors were estimated, and a controlled *in vitro* experiment verified the predictions parametrised to signal to noise ratio and the number of measurement points.

2. Analytic discussion

The magnetic field equations were inverted and dipole localisation errors were estimated based on expected measurement errors in gradiometer radial location and in the angle between the magnetic field null and maximum. In this paper, the word 'null' refers to the point on the line between the field maximum and field minimum at which the field is zero. The theory for the magnetic field seen by a point magnetometer outside a conducting sphere was derived by Grynszpan and Geselowitz (1973) and later by

Cuffin and Cohen (1977). The biomagnetometer used in this experiment is a second-order gradiometer with fourteen sensor loops of 1 and 2 cm diameter on a 5 cm baseline. The multiple loops, their size, winding direction and relative locations were taken into account in the magnetic field computations. The magnetic field was assumed to be uniform over the area of each loop. This assumption was confirmed by excellent agreement between the calculated gradient magnetic field and absolute magnetic field measurements.

The biomagnetometer output is defined as the sum of contributions from all of the loops:

$$G = \sum C_L B_L \quad (1)$$

where G is the magnetometer output in T and C_L is the weight for the L th loop based on the loop area and its winding direction. The sum of the weights is equal to zero. B_L is the radial component of the magnetic field in T as seen by the L th loop. The following formula gives the magnetic field at each loop:

$$B_L = 10^{-7} P a r_L^{-3} \gamma^{-1.5} \sin \theta \sin \phi \quad (2)$$

$$\gamma = 1 - (2a r_L^{-1} \cos \theta) + (a r_L^{-1})^2 \quad (3)$$

where P is the current dipole size in Am and a is the dipole location on the Z axis of a spherical coordinate system whose centre is located at the centre of the conducting sphere. Because of symmetry, the dipole can always be forced to lie on the Z axis. The magnetic field is zero when measured on the Z axis. r_L is the radial distance to the L th gradiometer loop, θ is the angle of the sensing loop axis with the Z axis and ϕ is the angle in the XY plane to the projection of the sensing loop.

The approach towards dipole localisation used by most investigators is to assume a perfectly spherical conducting volume, and to minimise a least-squares fit to magnetic field measurements in a multidimensional space. For the cases where the theory accurately describes the system under measurement, this method can give excellent results. However, the results from this method can be quite sensitive to the goodness of fit of the calculated curve to the data. In this experiment another approach was used to determine dipole location, using Newton's method. This approach only requires information from the plane containing the dipole field null and maximum.

It can be shown that θ_{\max} , the angle between a dipole magnetic field zero and maximum, uniquely determines the dipole location (' θ_{\max} technique'). For the analysis presented in this experiment the θ_{\max} location technique was used in one dimension. This method, however, can easily be extended to the two-dimensional surface bounding the head. Equally important, this method requires no normalisation and is computationally fast.

θ_{\max} is found by taking the derivative of the gradient field (equations (1)-(3)) with respect to θ and setting the result to zero.

$$0 = dG/d\theta = \sum C_L dB_L/d\theta \quad (4)$$

or

$$0 = \sum (C_L r_L^{-4} \gamma^{-2.5}) (\gamma r_L \cos \theta - 3a \sin^2 \theta). \quad (5)$$

In these equations there is no longer a ϕ dependence. The dipole location is calculated using Newton's method or a similar technique.

The measurement errors at θ_{\max} were predicted by restating the implicit definition of the dipole location (equation (5)) as follows:

$$F(a, r_L, \theta_{\max}) = \sum (C_L r_L^{-4} \gamma^{2.5}) (\gamma r \cos \theta_{\max} - 3a \sin^2 \theta_{\max}). \quad (6)$$

The RMS error in the determination of the dipole location is given by

$$da = \left(\frac{(\partial F / \partial r_L)^2 (dr_L)^2 + (\partial F / \partial \theta_{\max})^2 (d\theta_{\max})^2}{(\partial F / \partial a)^2} \right)^{0.5}. \quad (7)$$

Figure 1 shows the transfer function between θ_{\max} and the radial dipole location (full curve). For this plot the bottom of the biomagnetometer is 10 cm from the centre of the sphere. The broken curve is the estimated error in dipole location for an error in the radial magnetometer location of 2 mm and an error in θ_{\max} of 2° . This figure suggests the possibility of measuring the dipole location to within a few mm.

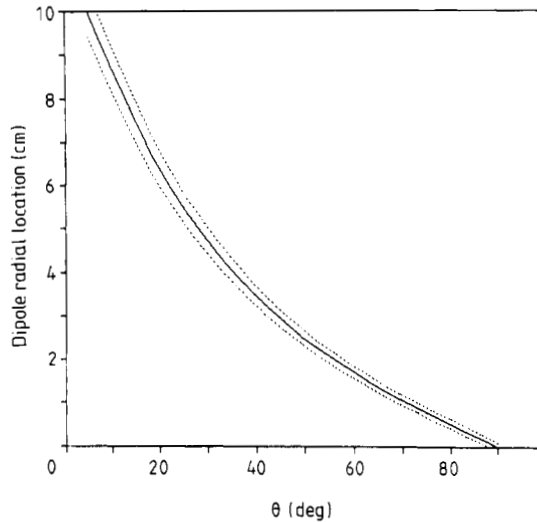


Figure 1. The full curve is the transfer function between θ_{\max} and the dipole radial location for a biomagnetometer placed 10 cm from the centre of the conducting sphere. The broken curves are the predicted RMS location errors assuming the accuracy in the magnetometer radial location is 2 mm and the accuracy in determining θ_{\max} is $\pm 2^\circ$.

3. Experiment

The objectives of this experiment were to determine the accuracy in localisation achievable under ideal measurement conditions and to quantify the error in localisation as a function of signal to noise ratio (S/N) and the number of data points used by the localisation algorithm.

The brain models used for this experiment were spherical flasks of variable radius filled with 24 ppT NaCl solution. In these spheres a current dipole was suspended at known locations with respect to the centre of the sphere. Flasks with radii of 3.9 cm (250 ml), 4.9 cm (500 ml) and 6.2 cm (1000 ml) were used. As expected from the theory, measured magnetic fields were independent of sphere radius.

The current dipole was constructed of 30 gauge silver-plated copper wire in a twisted pair (5 turns/cm). This wire was threaded through epoxy tubing and secured in its centre with epoxy cement. If proper care was taken in the twisting of the wire, no stray magnetic fields were detected by the biomagnetometer. The dipole was mounted in a rubber stopper attached to a bakelite rod and rubber seals. No electrolyte solution was allowed in the neck of the flasks. Dipole sizes of 5–10 mm were used.

The flask was held in a fixed position relative to the biomagnetometer by a non-magnetic plexiglass fixture rigidly mounted to the magnetometer holder. Figure 2 shows a diagram of the fixture. The distance from the sphere's centre to the magnetometer and the angular position of the dipole relative to the magnetometer were selectable. Before each measurement the magnetometer and the fixture were levelled to $\pm 0.5^\circ$. A laser was used to locate the centre of the sphere and to measure the dipole and biomagnetometer radial locations to an accuracy of ± 1.0 mm.

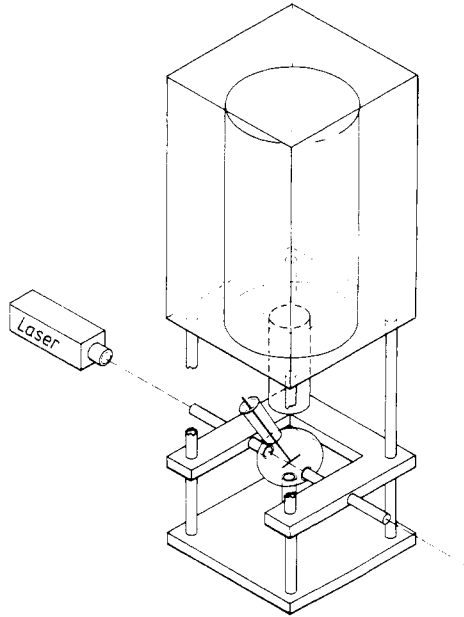


Figure 2. The *in vitro* fixture used to hold the conducting sphere rigid relative to the biomagnetometer and to allow laser alignment.

The angular calibration was done by rotating the flask and dipole until the null in the magnetic field was found (i.e. measured field equal to zero). The null point was defined to be $\theta = 0^\circ$. This zero point could be determined to an accuracy of $\pm 0.5^\circ$.

A frequency synthesiser was used to energise the current dipole and provide references to the two phase-lock amplifiers. Dipole current was measured across a monitoring resistor by one phase-lock amplifier. The output of the biomagnetometer was monitored by the second phase-lock amplifier. A PDP 11/34 computer was used to digitise, average, display and record the resultant output of the narrowband processing.

Using a spectrum analyser, the signal to noise ratio was determined as the ratio of the power in the measured signal to the power of the background measurement noise in a 1 Hz bandwidth.

The magnetic field amplitude data were taken as a function of the angle, θ , from the magnetic field null. A cubic polynomial was fitted to the region of the field maximum. The angle, θ_{\max} , that corresponded to the polynomial maximum was entered as a parameter in the iterative method used to find the dipole location. Using the calculated dipole location, it was possible to predict the absolute magnitude of the magnetic field that should be measured by the biomagnetometer. The measured magnetic fields were compared with the predicted magnetic field, and the calculated dipole locations were compared with the measured dipole locations. Figures 3 and 4 present results of this analysis for typical dipoles. The data in figure 3 have a S/N of approximately 50 dB, and the data in figure 4 have a S/N of approximately 25 dB. In both figures the data have been normalised to a 1 Am source size. The increased scatter of the data in figure 4 is due to the smaller signal-to-noise ratio. The arrow on the X axis points to the maximum determined by the fitting technique. For both of these data sets the error in location is less than 1.0 mm. The analytically predicted errors were 1.3 mm and 0.9 mm, respectively. The curves shown connecting the data points in these figures are plots of predicted absolute magnetic field. The predicted gradient fields agree with the experimentally measured fields to about 1% for these dipoles, and to within 5% for all dipoles used in the experiment. While the absolute magnetic field measurement is not required to do the localisation, it is a validation of the assumptions made, dipole construction and the measurement methods used.

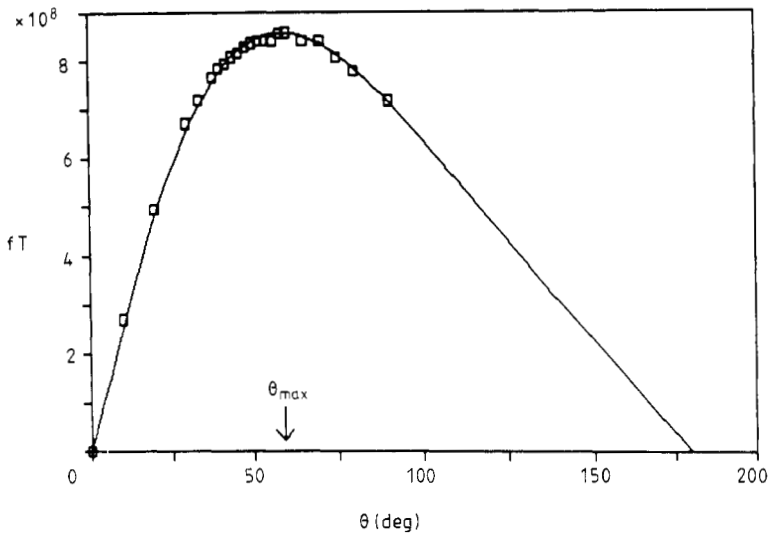


Figure 3. This graph presents the data points for a representative dipole (S/N ~ 50 db) plotted with the predicted magnetic gradient fields. The arrow shown on the X axis points to the calculated θ_{\max} . All of the data are normalised to a dipole source size of 1 Am. (Experiment parameters: magnetometer radial location: 9.05 cm; dipole radial location: 1.7 cm; predicted RMS errors: ± 1.3 mm; calculated θ_{\max} : 58.7°; calculated error: 0.3 mm; average agreement with theory: 0.99.)

A study was done determining the effects on dipole localisation of varying the S/N from 10 to 55 db and of varying the number of data points used by the algorithm from 4 to 10. 4, 6, 8 and 10 data points were randomly selected from the data sets and processed by the localisation algorithm. One hundred trials of each were done for the signal to noise ratios of 10, 15, 25 and 55 db. Table 1 presents the mean localisation

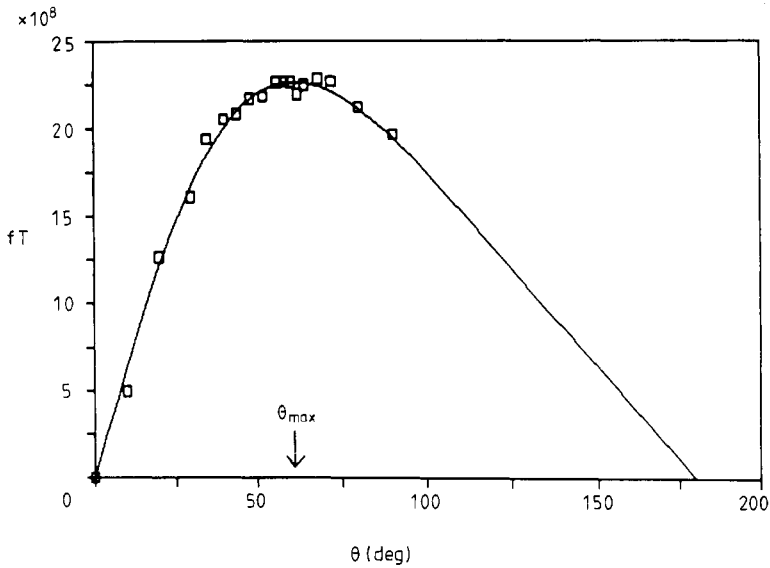


Figure 4. This graph presents the data points for another representative dipole ($S/N \sim 25$ db) plotted with the predicted magnetic gradient fields. The arrow shown on the X axis points to the calculated θ_{max} . All of the data are normalised to a dipole source size of 1 Am. (Experiment parameters: magnetometer radial location: 5.75 cm; dipole radial location: 1.0 cm; predicted RMS errors: ± 0.9 mm; calculated θ_{max} : 61.1° ; calculated error: 0.9 mm; average agreement with theory: 1.00.)

Table 1. Mean dipole location error and the RMS deviation for 100 trials.

S/N	Four points (mm)	Six points (mm)	Eight points (mm)	Ten points (mm)
10 db				
Mean	3.98	2.11	1.15	0.68
Standard deviation	2.61	1.71	1.03	0.62
15 db				
Mean	2.67	1.59	1.36	0.94
Standard deviation	2.43	1.27	1.03	0.63
25 db				
Mean	2.64	1.49	1.06	0.74
Standard deviation	2.55	1.119	0.85	0.62
55 db				
Mean	0.97	0.91	0.73	0.66
Standard deviation	0.76	0.49	0.35	0.32

error and its standard deviation as a function of signal to noise ratio and the number of data points used in the analysis. In the experiment generating this data the dipole was located at a distance of 1.0 cm from the centre of the sphere. The bottom of the biomagnetometer probe was located 5.75 cm from the centre of the sphere.

4. Conclusions

Using a localisation technique based on the angle between null and maximum of its magnetic field, some limits on current dipole localisation have been determined. As

predicted by the theory, sphere size has no effect on the measured magnetic field. Agreement between the measured magnetic fields and the analytically predicted fields was excellent. Current dipoles were located to an accuracy of about 1.0 mm when at least eight data points were used with a $S/N > 10$ db. Measurement for $S/N < 10$ db gave poor results. If the $S/N > 40$ db, four points gave a localisation accuracy to better than 1.0 mm.

The next step in the *in vitro* protocol is to perform similar measurements with non-spherical flasks. This will provide information about the validity of using a spherical model for the non-spherical head and will also provide a test of the potential applicability of the localisation technique described here to clinical measurements using the spherical model.

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Résumé

Limites pratiques du processus inverse biomagnétique, déterminées à partir de mesures *in vitro* dans des volumes conducteurs sphériques.

Les auteurs ont développé une technique de localisation des dipôles de courant dans des volumes conducteurs sphériques en déterminant la localisation du maximum du champ magnétique et en inversant les équations du champ magnétique. Ils ont prévu les erreurs de localisation attendues. Les dipôles de courant alternatif étaient placés dans des sphères de conductibilité uniforme. Le champ magnétique de chaque dipôle a été mesuré et sa position a été calculée en déterminant l'angle entre le champ magnétique nul et maximum et en utilisant une solution inverse itérative pour les équations du champ magnétique. L'accord absolu entre les intensités prévues et réelles du champ magnétique s'est révélé à mieux que 5% près. Une étude de l'effet sur l'analyse du rapport signal sur bruit et du nombre de données, montre que la localisation du dipôle à 1 mm près est possible pour un rapport signal sur bruit supérieur à 10 décibels.

Zusammenfassung

Praktische Beschränkungen auf den biomagnetischen inversen Prozess bestimmt durch *in vitro*-Messungen in leitenden kugelförmigen Volumina.

Entwickelt wurde ein Verfahren zur Lokalisierung von Dipolen in leitenden kugelförmigen Volumina durch Bestimmung des Ortes, an dem das Magnetfeld maximal ist. Die erwarteten Lokalisationsfehler werden vorausgesagt. AC-Dipole wurden in Kugeln mit gleichförmiger Leitfähigkeit angeordnet. Das Magnetfeld eines jeden Dipols wurde gemessen und der Ort wurde berechnet durch Bestimmung des Winkels zwischen dem Magnetfeld Null und dem maximalen Feld und durch Anwendung einer iterativen inversen Lösung auf die Magnetfeldgleichungen. Die Übereinstimmung zwischen der vorhergesagten Magnetfeldstärke und den tatsächlichen Messungen des Magnetfeldes lag innerhalb von 5%. Eine Untersuchung des Einflusses des Signal-Rausch-Verhältnisses und der Anzahl der Datenpunkte in der Analyse zeigte, daß eine Dipol-lokalisation von ~1 mm erreichbar ist für ein Signal-Rausch-Verhältnis größer als 10 decibel.

References

- Barth D S, Beatty J and Sutherling W 1984 *Naval Res. Rev.* 20-8
- Barth D S, Sutherling W and Beatty J 1985 *Proc. 5th World Conf. on Biomagnetism* ed H Weinberg, G Stroink and T Katila (New York: Pergamon) pp 237-47
- Cowen R 1985 *Res. Resources Report IX* (4) 1-5
- Cuffin B N and Cohen D 1977 *IEEE Trans. Biomed Eng.* **BME-24** 372-81
- Grynszpan F and Geselowitz D B 1973 *Biophys. J.* **13** 911-25
- Janday B S and Swithenby S 1987 *Phys. Med. Biol.* **32** 105-14