AN SVD-BASED MIMO EQUALIZER APPLIED TO THE AURALIZATION OF AIRCRAFT NOISE IN A CABIN SIMULATOR

Luiz CHAMON, Giuliano QUIQUETO, Sylvio BISTAFÁ, Vítor NASCIMENTO

Polytechnic School of the University of São Paulo
AIRCRAFT NOISE

Main noise sources:  
- Propulsion  
- Structure-borne noise  
- Air conditioning, pressurization...

Noise characteristics:  
- high energy in low frequencies;  
- slow decay above 500Hz;  
- narrow peaks around de 100Hz (propulsion);  
- smaller high frequencies peaks.

Noise levels:  
- Between 70 - 80 dBA
CABIN SIMULATOR

- 20 seats (economy class)
- Air conditioning
- Acoustic reproduction
SVD AND THE PSEUDOINVERSE

\[ A = U \Sigma V^* \]

\[ (A \in \mathbb{C}^{M \times N}, U \in \mathbb{C}^{M \times M}, V \in \mathbb{C}^{N \times N}, \Sigma = diag\{\sigma_i(A)\} \in \mathbb{R}^{M \times N}) \]

- The inverse, when it exists, is given by \( A^{-1} = V \Sigma^{-1} U^* \)

- Otherwise, a pseudoinverse can be evaluated using

\[ A^+ = V \Sigma^+ U^* \]

\( (\Sigma^+ = diag\{\sigma_1^{-1}(A), \ldots, \sigma_n^{-1}(A), 0, \ldots, 0\}, \text{so that } \sigma_{n+1}(A) < k < \sigma_n(A) ) \)
\[ M(j\omega) = H(j\omega)\text{diag}\{G(j\omega)\}S(j\omega)\text{col}\{1\} \]

\[ E(j\omega) = M(j\omega) - S(j\omega)T(j\omega) \]

\[ HG = T \iff \begin{cases} |HG| = \text{col}\{1\} \\ \phi(HG) = \text{col}\{-\omega \tau_m\} \end{cases} \]

For \( M < N \), \( G^0 = H^*(HH^*)^{-1}T \)

For \( M = N \), \( G^0 = H^{-1}T \)

For \( M > N \), \( G^0 = (H^*H)^{-1}H^*T \)
REGULARIZATION

The problem: matrix inversion!

Proximity between actuators or sensors, symmetry and reverberation can worsen the conditioning.

A well known solution:  \( G_R = (H^*H + \beta I)^{-1}H^*T \) (regularization)

- Equivalent to summing \( \beta \) to the eigenvalue of \( H^*H \)
  - Simple and practical

- Depends on the evaluation of \( H^*H \)

- \( \beta \) must remain small so as to keep the inversion effective

\[ \kappa_2(H^*H + \beta I) \approx [\kappa_2(H)]^2 - \frac{(\sigma_{max}^2 - \sigma_{min}^2)}{\sigma_{min}^4} \beta \]


**DECOUPLING EQUALIZERS**

\[ G_{SVD} = H^+ T = V \Sigma^+ U^* T \]

\[ \kappa_2(H_k) \leq \kappa_2(H) - \frac{\sigma_{\text{max}} (k - \sigma_{\text{min}})}{\sigma_{\text{min}} k} \]

- Numerically robust solution: there is an implicit algorithm for the SVD
- Provides a (optimal) rank \( r \) approximation of a matrix
- PCA
- The decoupling depends on a coordination of the actuators
- SVD is unique for every matrix
TRANSFER FUNCTION MATRIX

- Single-loop identification
- 30 seconds e-sweeps

![Graph showing magnitude and phase vs. frequency](image)
AVERAGE SPECTRUM FOR DIFFERENT $k$

- Target
- $k = 0.01$
- $k = 0.05$
- $k = 0.1$

Sound Pressure Level (dB SPL)

Frequency band (Hz)
COMPARISON BETWEEN SEATS AND CENTRAL AISLE

- Target
- Microphones average
- Central aisle average

Sound Pressure Level (dB SPL) vs. Frequency band (Hz)
$L_{eq}$ AT EACH SEAT FOR DIFFERENT $k$

- Target
- $k = 0.01$
- $k = 0.05$
- $k = 0.1$

Leq (dB SPL) vs Seat

1 dB
CONCLUSIONS AND FUTURE WORK

Based on the optimal characteristics of the factorization, an SVD-based method was proposed for designing an equalizers bank.

Empirical results proved the solution to be robust in finite precision.

The mismatch increased in low and high frequencies due to the difficulty to reproduce in these ranges.

Future work includes relative thresholds (fixed condition number), saturation and adaptive singular values.
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CONTACT

Luiz CHAMON
Noise & Vibration Group – University of São Paulo
Mechanical Engineering Department
Av. Prof. Mello Moraes, 2231, sala TS08
05508-900 - São Paulo/SP - Brazil
Tel: +55 11 3091-9670

email: chamon@usp.br