**ABSTRACT**

Multicarrier communication systems have become ubiquitous, mainly due to the popularity of OFDM in which carriers are separated in frequency by the inverse of the symbol duration. Recently, more spectrally efficient modulations based on non-orthogonal carriers (non-OFDM) have been put forward and shown numerically to have the same performance as OFDM employing up to 40% less bandwidth. This work addresses the issue of analytically deriving the minimum frequency separation which does not affect the minimum distance between multicarrier symbols. In doing so, it shows that the probability of error remains unaffected up to a certain degree of spectral superposition of the carriers, so that the BER of non-OFDM remains the same as that of OFDM. Simulations and comparisons to previous numerical results are used to illustrate this conclusion.

**INTRODUCTION**

OFDM: \( \Delta f T = 1 \)

non-OFDM: \( \Delta f T < 1 \)

**PROBLEM FORMULATION**

Single carrier communication

\[ C = \{ x_m \}, \quad x_m \in C \rightarrow M \text{-symbols constellation} \]

\( s_m(t) = \Re\{x_m g(t)e^{j2\pi f_c t}\} \rightarrow \text{band pass signal of } x_m \)

\( g(t) \rightarrow \text{pulse shape} \)

Multicarrier communication

\( x_n \in \mathbb{C}^N \rightarrow N \times 1 \text{multicarrier symbol} (N \text{ carriers}) \)

\( s(t) = \sum_{n=0}^{N-1} \Re\{x_n(t)\} e^{j2\pi f_c (n-k)} e^{-j2\pi f_c (n-k)t} \rightarrow \text{band pass signal of a sequence } x_n(t) \) of multicarrier symbols

\[ \psi(t) = [ e^{j2\pi f_1 t} \ldots e^{j2\pi f_{N-1} t} ]^T \rightarrow \text{carrier vector} \]

\( \Delta f \rightarrow \text{carriers spectral separation} \)

AWGN channel

\[ r(t) = s(t) + n(t) \]

\( v(t) \rightarrow \text{white, zero mean, Gaussian, PSD } \frac{N_0}{2} \)

MLE:

\[ \min_{x_m} P[r(x_m); x_m] = \int_0^T \left| r(t) - x_m(t) \right|^2 dt \]

**SPECTRAL SEPARATION LOWER BOUND**

**THEOREM 1**

In a multicarrier system composed of \( N = 2 \) carriers spectrally separated by \( \Delta f \) transmitting symbols from a constellation \( C \) using rectangular-shaped pulse of length \( T \),

\[ \min_{x_n} D_{ij}^T (k) = \min_{x_n} d_{ij}^* \text{min}_{x_n} D_{ij}^* (k) \leq \sin(\Delta f T)^2 \]

\[ \min_{x_n} D_{ij}^T (k) \leq t_{ij}^* \sin(\Delta f T)^2 \]

Lemmas:

**LEMMA 1** (Identical symbols case)

\[ D_{ij}^T (k) = \min_{x_n} D_{ij}^* (k) = t_{ij}^* \sin(\Delta f T)^2 \]

**LEMMA 2** (Distinct symbols case)

\[ D_{ij}^T (k) \geq \min_{x_n \neq x_n} D_{ij}^* (k) = t_{ij}^* \sin(\Delta f T)^2 \]

**LEMMA 3** (Sufficiency lemma)

\[ \Delta f T = \frac{\pi}{\sin(\Delta f T)} \]

**PROOF OF THEOREM 1**

Proof of Theorem 1. From Lemma 1, \( \min_{x_n} D_{ij}^T (k) \leq \sin(\Delta f T)^2 \).

For \( \Delta f T > 0 \),

\[ \min_{x_n} D_{ij}^T (k) \geq d_{ij}^* \cos(\Delta f T) \]

and since \( \cos(\Delta f T) < 1 \),

\[ \min_{x_n} D_{ij}^T (k) \leq \sin(\Delta f T)^2 \]

Moreover, Lemma 3 guarantees that close to any \( \Delta f T \) there is a \( \Delta f T \) for which \( \sin(\Delta f T) \leq \frac{1}{2} \).

**SIMULATIONS**

Constellations

\[ \begin{array}{c|c|c|c}
\text{Constellation} & \text{Square (M = 16)} & \text{Hexagonal (M = 16)} & \text{Random (M = 8)} \\
\hline
\end{array} \]

Minimum distance \( \Delta f T \)

\[ \min_{x_n} D_{ij}^T \]

Theorem 1: \( \Delta f T > 0.6933 \)