Motion Planning

Mobile Robots and Manipulators
Motion Planner

straight line segments to subgoals

Trajectory Generator

smooth trajectory

Controller

Signals that can be sent to joint controllers/drivers

joint velocities/torques
Motion Planning

Let $A$ be a single rigid object — the *robot* — moving in a Euclidean space $W$, called *workspace*, represented as $\mathbb{R}^N$, with $N = 2$ or $3$.

Let $B_1, ..., B_q$ be fixed rigid objects distributed in $W$. The $B_i$'s are called *obstacles*.

Assume that both the geometry of $A, B_1, ..., B_q$ and the locations of the $B_i$'s in $W$ are accurately known. Assume further that no kinematic constraints limit the motions of $A$ (we say that $A$ is a *free-flying object*).

The problem is: Given an initial position and orientation and a goal position and orientation of $A$ in $W$, generate a *path* $\tau$ specifying a continuous sequence of positions and orientations of $A$ avoiding contact with the $B_i$'s, starting at the initial position and orientation, and terminating at the goal position and orientation. Report failure if no such path exists.
Trajectory generation

Given end points (and possibly intermediate points) and velocities, generate a smooth trajectory between them while avoiding obstacles and obeying constraints.
Mobile Robot Motion Planning

Motion Planning

Known Environments (Model)  Unknown Environments

- Explicit motion plans  - Sensor based motion planning
- Implicit motion plans

Usually combines
Motion Planning
Trajectory Generation
Visibility Graphs
Assumes world model
Explicit

Figure 4. This figure shows the visibility graph in a two-dimensional configuration space with polygonal C-obstacles. The nodes of the graph are the initial and goal configurations and the C-obstacles’ vertices. Two nodes are connected by a link if the straight line segment joining them does not intersect the C-obstacles’ interior. (Hence, the links of the graph also include the edges of the C-obstacles.) The plain links connect C-obstacles’ vertices. They are independent of the initial and goal configurations, and form the roadmap. The dashed links connect the initial and goal configurations to the roadmap. Any path in the visibility graph determines a semi-free path in configuration space. The graph contains the shortest semi-free path (according to the Euclidean metric in $\mathbb{R}^2$). This path is shown in both plain and dashed bold lines.
What if the robot is not a point?
Configuration space

obstacle free

Robot

invalid

Robot

obstacle free

Robot

Obstacle

Robot

Robot
Allowable Robot positions (for $\theta=0$ deg)
Allowable Robot positions (for nonzero $\theta$)
Configuration Space
Finding shortest distance paths in configuration space

- Configuration space is six-dimensional for a single rigid body
- V-graphs in 3 and higher-dimensional space is difficult
- No natural metric in non-Euclidean space
  - 3 and higher-dimensions

- General problem is much harder
  - Robot with $n$ links: $6n$-dimensional space
  - Mobile robot: $3n$-dimensional space
Implicit Method: Potential Field

Basic idea

- Create attractive potential field to pull robot \((R)\) toward a goal

\[
V_{\text{goal}} = k \left[ d(R, \text{goal}) \right]^2
\]

- Create repulsive potential field to repel robot \((R)\) from obstacles

\[
V_{\text{obs}} = \frac{c}{d(R, \text{obs})}
\]

- In two-dimensional space (robot is a point, goal/obstacles are points)

\[
d(R, \text{goal}) = \sqrt{\left( x - x_{\text{goal}} \right)^2 + \left( y - y_{\text{goal}} \right)^2}
\]

\[
d(R, \text{obs}) = \sqrt{\left( x - x_{\text{obs}} \right)^2 + \left( y - y_{\text{obs}} \right)^2}
\]

- Remember: Force on a particle is given by \(f = -\text{grad} \ (V)\)
Basic idea
Potential Field: Goal \((x=0.5, y=0.5)\)

Contour plot of \(V_{goal}\)

\[
V_{goal} = \left[ (x - x_{goal})^2 + (y - y_{goal})^2 \right]^{\frac{1}{2}}
\]
Potential Field: Obstacle \((x=0.0, y=0.0)\)

\[
V_{obs} = \frac{1}{\left[(x - x_{goal})^2 + (y - y_{goal})^2\right]^{3/2}}
\]

Contour plot of \(V_{obs}\)
\[ V_{\text{obs}} + 100 \ V_{\text{goal}} \]
More Complicated Example
Trajectory for $V_{obs} + 500 V_{goal}$
Application to Articulated Arms

Eight-Jointed Planar Arm
Sensor Based Motion Planning

Unknown environments

Specific to sensor

- Tactile sensor
- Range sensor
Sensor Based Motion Planning

Exploratory strategy to build map

Map built from a range sensor
Sensor Based Motion Planning

1. The robot follows a straight line segment to the goal.
2. When it hits an obstacle (at the hitting point), it follows its boundary while keeping track of the straight line segment.
3. When it returns to the hitting point, it follows the boundary to the point on the boundary that is on the line segment and closest to the goal.
4. It then resumes the straight line segment path to the goal.

Always finds a path (if it exists)
Homework Problem

- Robot should reach goal
- Robot should not penetrate obstacle
- Robot path should be smooth

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