Solution to Homework #1

Problem 1

● In order to check the dimensions of the following formula, let us first get the exponents of the fundamental dimensions for the quantities involved in it.

Electrostatic force per unit length of a parallel-plate capacitor = \( f = \frac{\varepsilon_0 wV^2}{2g^2} \)

\( \varepsilon_0 \): Consider the formula for the electrostatic force between two charges.
\[
F = [\text{MLT}^{-2}] = \frac{q_1q_2}{4\pi\varepsilon_0d^2} = \left[ \frac{C}{L^2} \right] \Rightarrow [\text{M}^{-1}\text{L}^{-3}\text{T}^{-2}\text{C}^2] \text{ are the dimensions for } \varepsilon_0.
\]

\( V \): Consider the electric power formula involving voltage and current.
\[
\text{Power} = [\text{ML}^2\text{T}^{-3}] = VI = [\text{T}^{-1}\text{C}] \Rightarrow [\text{ML}^2\text{T}^{-2}\text{C}^{-1}] \text{ are the dimensions for } V.
\]

Now, for the force per unit length formula, we can write the dimensions as follows.
\[
f = \frac{\varepsilon_0 wV^2}{2g^2} = \left[ \frac{\text{M}^{-1}\text{L}^{-3}\text{T}^{-2}\text{C}^2}{\text{L}^2} \right] \left[ \text{M}^2\text{L}^4\text{T}^{-4}\text{C}^{-2} \right] = \left[ \text{MT}^{-2} \right] \Rightarrow \left[ \frac{\text{MT}^{-2}}{\text{L}} \right] \text{ are the dimensions for force per unit length.}
\]

● For a fixed-fixed beam with uniform distributed force of \( q \) per unit length, the maximum deflection occurs at the center of the beam. It is given by
\[
\delta = \frac{fL^4}{384EI}
\]

For a rectangular cross-section beam, \( I = \frac{wt^3}{12} \).

Taking the force from the given electrostatic force formula above, we get
\[
\delta = \frac{12L^3}{384Ewt^3} \cdot \frac{\varepsilon_0 wV^2}{2g^2} = \frac{\varepsilon_0 V^2 L^4}{64Et^3 g^2}
\]

With given proportions, \( t = g = \frac{L}{50} \), we get \( \delta = \frac{50^5 \varepsilon_0 V^2}{64EL} \).

By substituting
\( \varepsilon_0 = 8.8542E-12 \text{ m}^{-3} \text{ kg}^{-1} \text{ s}^4 \text{ A}^2 \)
\( E = 150E9 \text{ Pa} \)
we get,
\[
\delta = \frac{50^5 \varepsilon_0 V^2}{64EL} = \frac{2.8822E - 16V^2}{L}.
\]

For the first plot, we have \( \frac{\delta}{L} = \frac{7.2056E - 15}{L^2} \).

For the second plot, we have
\[ V = \sqrt{\frac{64EL}{50^3 \varepsilon_0}} = \sqrt{\frac{64EL^2}{50^3 \varepsilon_0 L}} = 8.3301E6L \]

The plots and the Matlab script used for it are given below.

```matlab
% Matlab script for MEAM 550, Spring 2004 // GKA
L = 100E-9:0.01:10;
deltabyL = 7.2056E-15./L.^2;
V = 8.3301E6*L;
figure(1)
clf
loglog(L,deltabyL,'-r','LineWidth',2);
grid
hold on
xlabel('Length scale = L (m)');
ylabel('
\delta / L');
figure(2)
clf
loglog(L,V,'-r','LineWidth',2);
xlabel('Length scale = L (m)');
ylabel('Voltage (V)');
grid
```

It is clear from this plot and the equation above that the ratio of the maximum beam deflection to the length of the beam, at a given voltage, decreases rapidly with increasing length scale. Hence, the electrostatic force is favorable at small length scales.
The equation used to plot the above graph makes it clear that the required voltage to get the same fraction of deflection increases linearly with the length scale. So, it is impractical to get substantive deflection with electrostatic force at larger length scales. On the other hand, at the MEMS scale and below, it is quite attractive.

**Problem 2**

The resistive power loss is given by, with dimensions, \( P = I^2 R \) \([ML^2T^{-3}]\) where \( I \) is the current and \( R \) is the resistance. Power loss per unit volume will have dimensions \([ML^{-1}T^{-3}]\). This means that the ratio of power lost due to Joule heating per unit volume increases as the size scale decreases. Thus, from this perspective it is not attractive to miniaturize systems where power is critical. But there may be other reasons to compensate for this excessive power loss in the resistors due to the inevitable Joule heating. Or, this power could be recovered or put to use in some other way.